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INDUSTRIAL ELECTRICITY

PART II

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ELECTRICAL ENGINEERING TEXTS

INDUSTRIAL ELECTRICITY

PART II

BY

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PREFACE

This volume is intended as a continuation of "Industrial Electricity," Part I, which is devoted almost entirely to direct-current circuits and direct-current machinery. Since a large proportion of power is now generated as alternating current, and since all communication circuits also utilize alternating currents, the first chapters of the present volume are devoted to the fundamental principles and the simple laws of alternating currents and alternating-current circuits. Only the simplest mathematics is used. Considerable emphasis is placed on the effect of inductance, capacitance, and frequency in the flow of alternating current. A chapter is devoted to the construction and uses of the ordinary alternating-current instruments, particular attention being given to the testing and adjusting of the induction watthour meter. Since, with few exceptions, alternating-current power is generated and distributed through polyphase systems, a chapter is devoted to the study of the relations existing among currents and voltages in such systems.

With the foregoing as a foundation, the construction and the operating characteristics of alternators, polyphase induction motors, single-phase motors, synchronous motors, and converters are discussed and briefly analyzed. The relations of the characteristics of these types of power machinery to their industrial applications are also discussed to a considerable extent.

The last three chapters are devoted to general industrial applications of electricity. In Chap. XII, the more common electrical illuminants, particularly the incandescent lamp, and simple photometric measurements are described. In Chap. XIII an attempt is made to present the underlying principles of electron emission and its applications to the electrical industry. In this chapter the Fleming valve, the kenotron rectifier, the three-electrode vacuum tube, and the applications of the vacuum tube to electrical communication, particularly radio telephony, are discussed in a simple but reasonably comprehensive manner.

Also the principles underlying broadcasting and a brief description of the wiring diagrams of typical receiving sets are included. In the last chapter another important industrial application of electricity is given; that is, the methods and general rules which should be followed in the installation of electric wires in buildings in order that interruptions of service, fire hazard, and personal injury may be minimized.

In the preparation of this volume, it was found advantageous to utilize certain of the material, more particularly figures, from the author's "A Course in Electrical Engineering, Volume II, Alternating Currents." The scope of this volume, however, is intended to be somewhat broader and much less analytical than that of the author's "Alternating Currents."

The author is indebted to Mr. Robert F. Field of the Cruft Laboratory, and Instructor in Physics and Electric Communication Engineering at Harvard University, who is the author of Chap. XIII, "Electron Tubes;" to Mr. Raymond T. Gibbs, Instructor in Electrical Engineering at the Harvard Engineering School, who is author of Chap. XIV, "Interior Wiring;" and, particularly, to Prof. H. E. Clifford, the Consulting Editor of these Electrical Engineering Texts, for his suggestions in the preparation and arrangement of this volume and for his careful review of the manuscript.

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October, 1925.

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INDUSTRIAL ELECTRICITY

PART II

CHAPTER I

ALTERNATING CURRENT AND VOLTAGE

1. General Field of Use of Alternating Current.—Over 90 per cent. of the electrical energy generated at the present time is generated as alternating current. This is not due primarily to any superiority of alternating over direct current in its applicability to industrial and domestic uses. In fact, there are many instances where direct current is absolutely necessary for industrial purposes, such as for example, charging storage batteries, for electrolytic processes, magnetite lamps, operating street railways and electric locomotives, etc. Where direct-current power is used for these purposes, it is usually generated as alternating current in some large steam or hydro-electric station, is transmitted some distance and converted into direct current.

The reasons for generating nearly all electrical energy as alternating current are as follows:

1. *Alternators have no commutators. Hence, units of large capacity may be operated at high speeds.* This adapts them to the high speeds at which turbines operate most economically. It is possible to operate a 30,000-kw. unit at 1,800 r.p.m., whereas it is difficult to operate a 1,000-kw. direct-current generator at a speed of 1,000 r.p.m because of commutation difficulties. Also, the size of generator for a given capacity becomes less with increase of speed. Figure 1 shows a Westinghouse 70,000-kv-a.,

65,000-kw. installation composed of three units operating in tandem.

The size of the units and the floor space occupied by the installation are both small when the large kilowatt rating is considered.

2. *Because alternators have no commutators, they can generate energy at comparatively high voltages.* For example, in small plants which generate for local use only, the electrical energy may be generated at 2,300 volts, which is a comparatively low voltage. Where the power is to be distributed over a moderate

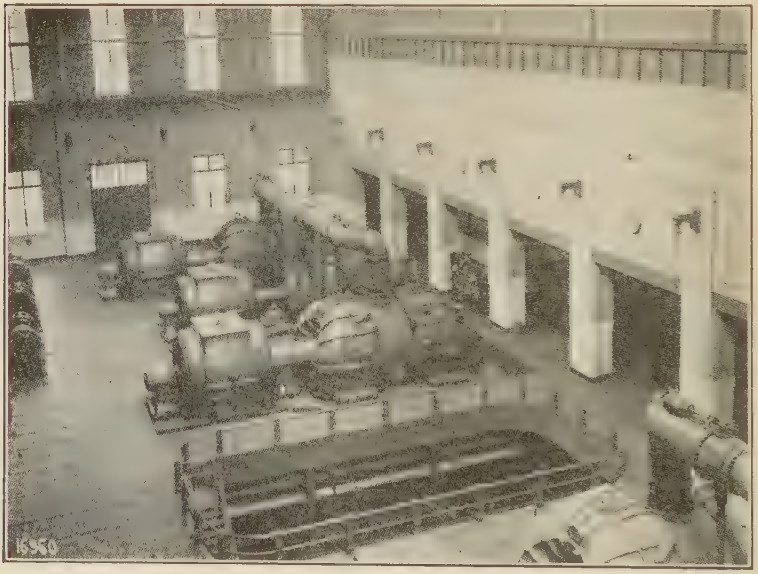


Fig. 1.—70,000-kv-a., 65,000-kw., 3-unit, cross compound, Westinghouse turbo-generator installation. Colfax Plant of Duquesne Light Co., Cheswick, Pa.

area and also where the voltage is to be stepped up for transmission purposes, it is customary to generate at either 6,600, 11,000 or 13,200 volts. Although generators for voltages as high as 20,000 volts are built, their insulation becomes difficult, and it is therefore more economical and safer to generate at the lower voltages. Because of commutators, it is difficult to generate direct-current power at voltages even as high as 1,500 volts per commutator.

3. *With alternating current the voltage may be raised and lowered economically by means of transformers.* This, together with the fact that high voltages may also be generated, permits economical transmission of power over long distances. The weight of copper necessary to transmit a given amount of power a given

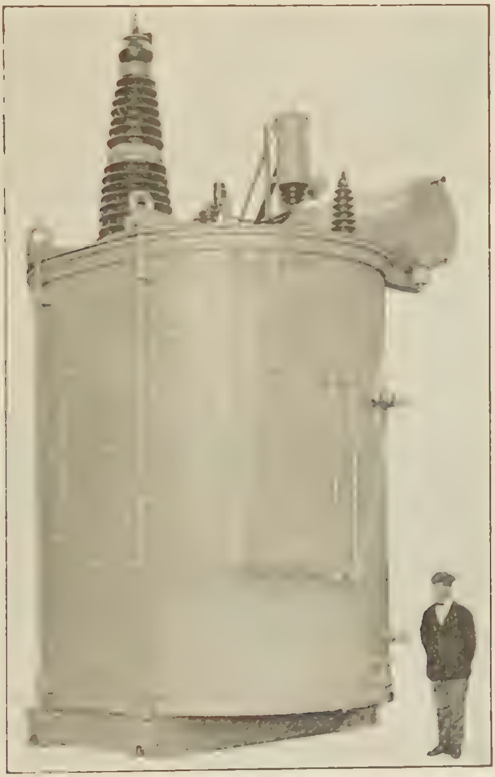


Fig. 2.—20,000 kv-a., General Electric outdoor transformer used to step voltage down from 220,000 volts to 72,000 volts, on Big Creek-Los Angeles line.

distance with a fixed loss varies *inversely as the square of the transmission voltage* (see Chap XI). For example, when the transmission voltage is *doubled*, the weight of copper is *quartered*, other factors being equal. At 100,000 volts, the weight of copper required to transmit a given amount of power a given distance

with a fixed loss is *one one-hundredth* that required if 10,000 volts be used.

There is no efficient method of raising and lowering direct-current voltages. Therefore, direct current is not suited for transmission.

Obviously, transformers are also used to step down the high transmission voltages so that the electrical energy is at the proper voltage for industrial uses.

Figure 2 shows one of the 20,000-kv-a., 220,000-volt transformers used to step down the voltage on the Big Creek-Los Angeles 280-mile transmission line.

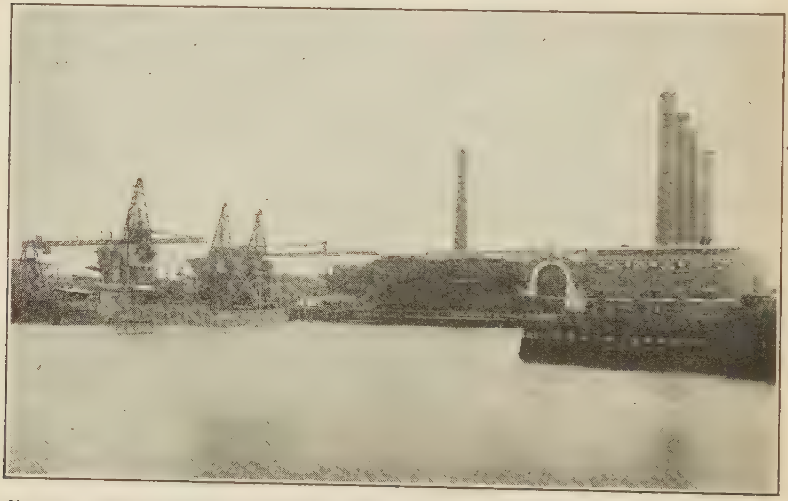


Fig. 3.—L Street power plant of the Edison Electric Illuminating Co. of Boston. Notice wharf, coal-handling machinery, coal pile in background.

4. *Because of the long distances over which alternating-current power may be transmitted economically, it is possible to generate electrical energy in large quantities in a single station and to distribute it over a comparatively large territory.* The large boilers, automatic stokers, superheaters, recording instruments, etc., which are possible in large stations, result in high boiler-room efficiency. Large turbines have an economy which may be three or four times better than that of the steam units in a small plant.

The generator has an efficiency of 95 to 96 per cent. in the larger sizes. Then again, as the boilers and large turbo-units require few attendants per kilowatt, the labor and superintendence charges per kilowatt are small.

For these reasons, it is often more economical to generate power with large units, to transmit it long distances and even to convert it into direct current, than to generate the direct current at the place where it is to be utilized.

It must be remembered, however, that the reduced generating costs may be balanced by the distribution costs resulting from high investment charges in lines, cables, sub-stations, machinery, etc., in addition to the labor and maintenance costs of this distribution system.

Figure 3 shows the L Street station of the Edison Electric Illuminating Company of Boston, which supplies an area of approximately 700 square miles, representing a population of 1,500,000 people, with electrical energy and from which the electrical energy is distributed over a radius of 35 miles.

5. *For constant-speed work, the alternating-current induction motor is cheaper in first cost and maintenance than the direct-current motor.* This is due to the fact that this type of motor has no commutator. On this account, it is frequently more desirable to generate alternating-current power even in an isolated plant if the power is to be utilized largely for operating motors. The alternating current, in such cases, has no advantages over direct current except in the more economical operation of the alternating-current motors.

Alternating current owes its importance to the following: It can be generated economically, at comparatively high voltages, in large-capacity units operating at turbine speeds. By means of transformers the voltage can be raised and lowered efficiently, so that it is possible to transmit large amounts of power for long distances and at voltages best adapted to given operating conditions. Generation of large amounts of energy in large central stations permits high generating economies. For constant-speed work the induction motor is more economical than the direct-current motor.

2. Sine Curve.—Alternating currents and voltages are not steady like direct currents and voltages but vary from instant

to instant. These time variations of current and voltage usually follow sine or cosine laws.

Hence, before studying alternating currents and voltages, an investigation of some of the properties of sine curves should be made.

If the values of the sine of an angle be plotted as ordinates and degrees be plotted as abscissas, a curve similar to that shown in Fig. 4 is obtained. Values of the angle x are plotted along the axis of abscissas and the corresponding values of the sine x are plotted as ordinates. The values of the sines are found on page 410 (also see page 407 for angles greater than 90°). For example, sine 40° is +0.6428. The ordinate at 40° is +0.6428; the ordinate at 250° is −0.9397, since sine 250° = sine (180°−

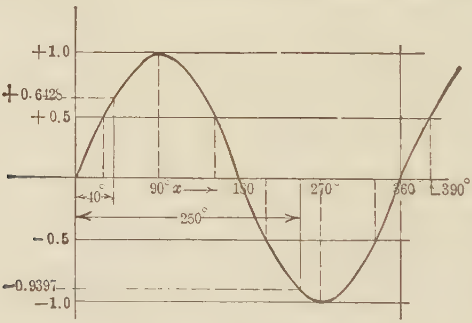


FIG. 4.—Sine curve.

250°) = sine (− 70°) = − 0.9397 (see 29, page 407). A study of Fig. 4 shows that sine 30° = 0.5, sine 150° = 0.5, sine 210° = − 0.5, sine 330° = − 0.5, etc. Obviously, the maximum positive ordinate of the curve is +1.0 (sine 90°) and the maximum negative ordinate is − 1.0 (sine 270°). The curve is symmetrical about its zero axis; when 360° is reached all possible values of the sine have been represented. Beyond 360° the curve merely repeats itself. Thus, sine 390° = sine 30°, etc.

A simple method of constructing a sine curve is shown in Fig. 5. A circle, having for its radius the desired maximum value, A, of the sine curve is drawn. Its circumference, starting at one end of the horizontal diameter 6-0, is divided into any number of equal arcs, 0-1, 1-2, etc. The horizontal line ab is drawn in such a manner that if extended, it would coincide in direction with the

horizontal diameter 6-0. The distance *ab* is made equal to the desired length of base for the completed curve. The line *ab* is divided into the same number of equal segments, 0-1, 1-2, etc., as is the circumference of the circle, and the segments are numbered correspondingly.

The point *O* on the circle is projected horizontally to intersect the ordinate at *O* on line *ab*; point 1 on the circle is correspondingly projected to intersect at 1′ the ordinate at point 1 on line *ab*; etc. The smooth curve drawn through these intersections gives a sine curve. Obviously, both the circumference of the circle and the line *ab* (Fig. 5) are divided into degrees, each arc or segment being equal to 30°.

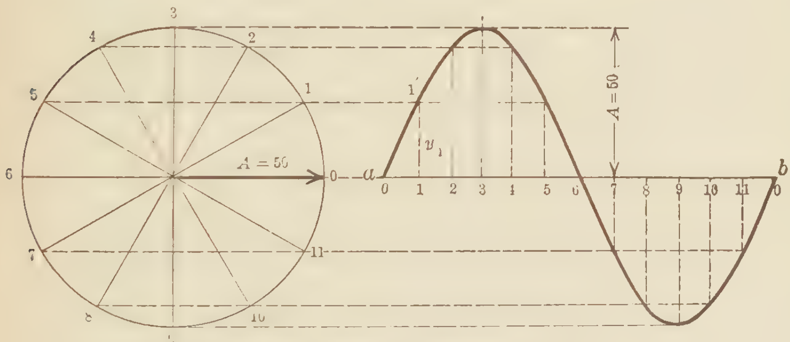


FIG. 5.—Graphical method of constructing a sine curve.

Let A be the maximum value of the sine curve. Then, the ordinate y at any angle x,

$$y = A \sin x. \tag{1}$$

For example, if $A = 50$ (Fig. 5), the ordinate y_1 at 30°, or at (1),

$$y_1 = 50 \sin 30° = 25, \text{ etc.}$$

3. Cosine Curve.—The cosine curve has the same form as the sine curve, but when the angle x is 0, the cosine curve has its maximum positive ordinate rather than its zero ordinate, since cosine 0° is unity. A cosine curve is shown in Fig. 6. This curve may be plotted using the cosine tables (page 410). For example, cosine 40° = 0.7660, cosine 250° = cos (180–250°) =

$-\cos 70° = 0.3420$ (see 30, page 407). These values are shown in Fig. 6.

The cosine curve may be plotted graphically in a manner similar to that used for the sine curve (Fig. 5), except that the numbering on the circle commences at intersection 3, the upper end of the vertical diameter. Thus "3" is changed to "0," "4" to "1," etc., and these points are projected horizontally to intersect the ordinates on line ab having corresponding numbers.

The cosine curve is merely the sine curve moved 90° to the left.

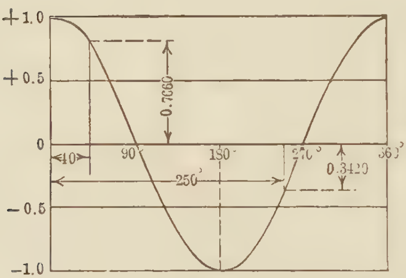


FIG. 6.—Cosine curve.

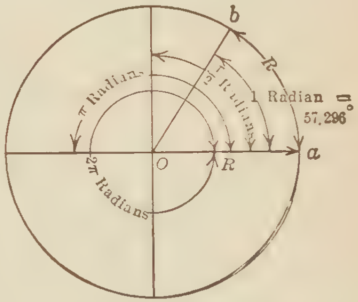


FIG. 7.—Angular measure in radians.

4. Radian.—The most common unit of angular measure is the degree. In analytical work it is often necessary to use another unit, the *radian*, as the unit of angular measure.

A radian is an angle subtended by an arc of a circle whose length is equal to the radius.

For example (in Fig. 7) the angle aob is subtended by the arc ab. The length of the arc ab is equal to the radius R or oa. Hence, angle aob is *one radian.* Obviously, a radian is slightly less than 60°. It is actually equal to $360°/\pi$ or 57.296° (to 5 figures).

Since the circumference of a circle is equal to 2π times the radius, the circumference of a circle subtends 2π radians. Hence, $360° = 2\pi$ radians. It follows that $180° = \pi$ radians, and $90° = \pi/2$ radians. Hence the sine $\pi/2 = 1.0$; cosine $\pi/2 = 0$; cosine $\pi = -1.0$; etc.

Example.—Find the sine of $4\pi/3$ radians.

2π radians equal 360°
4π radians equal 720°
$4\pi/3$ radians equal 240°
$\sin 240° = -\sin 60° = -0.866$ (see Fig. 4). *Ans.*

5. Combining Sine (or Cosine) Curves.—Sine (or cosine) curves may be combined by adding or subtracting their respective ordinates at each point along the axis of abscissas. If the curves have the same scale of abscissas, the curve resulting from thus

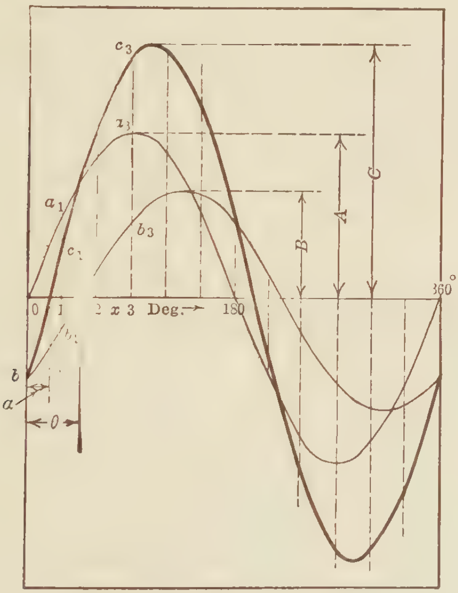


FIG. 8.—Addition of two sine curves.

adding or subtracting is also a sine (or cosine) curve, having the same scale of abscissas as the component curves.

For example, in Fig. 8, a sine curve of maximum value A crosses the zero axis in a positive direction at zero degrees. A second sine curve of maximum value B crosses the zero axis in a positive direction at θ degrees ($\theta = 45°$ in Fig. 8) to the right of 0. These curves are said to differ in phase by θ degrees. Let it be required to determine the curve resulting from the addition

of curves A and B. Ordinates 1, 2, 3, etc., are erected alcng the zero or x axis, sufficiently close together to give enough points for a smooth resultant curve. At ordinate 0, the ordinate of the A curve is zero, and the ordinate of the B curve is $-0b$. Hence, the algebraic sum of the two curves at this point is $-0b$, and b is a point on the resultant curve. At 1 the A curve has a positive value of a_1 and the B curve a negative value $-b_1$. The resultant ordinate c_1 is found by subtracting b_1 from a_1. At 3 both curves have positive values and the resultant ordinate c_3 is found by adding b_3 to a_3. Thus the resultant curve of maximum value C is found by adding algebraically the ordinates of the two curves A and B for each point on the axis of abscissas.

The resultant curve is a *sine* curve, having the same scale of abscissas as its two component curves A and B. Its maximum value C is less than the arithmetical sum of A and B. It crosses the zero axis in a positive direction α degrees to the right of the A curve and $\theta - \alpha$ degrees to the left of the B curve.

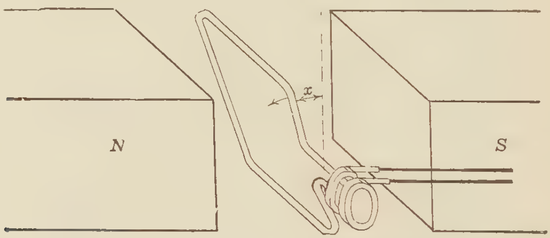


Fig. 9.—Generation of an alternating e.m.f.

6. Simple Alternating Electromotive Force.—It was shown in Part I, Chap. X, that when a single coil rotates at constant speed in a uniform field, as in Fig. 9, an alternating e.m.f. is generated. This e.m.f. is zero when the plane of the coil is perpendicular to the field, and reaches its maximum value when the plane of the coil is parallel to the field (see Part I, page 209, Fig. 185). The value of the e.m.f. at any instant varies with the value of the angle x (Fig. 9), which the plane of the coil makes with the geometrical neutral of the machine at that instant. Figure 10 shows in some detail the generation of an alternating e.m.f. In (a) the coil ab is shown rotating in a counter-clockwise direction and at the instant shown it makes

an angle x with the neutral plane oo. By Fleming's right-hand rule (see Part I, page 208), the direction of induced e.m.f. in a is outwards when it is in this position. The arrow v attached to a is proportional to and gives the instantaneous direction of the linear velocity of the conductor, which is constant.

In order that a conductor cutting magnetic flux may generate an e.m.f., there must be a component of its velocity *perpendicular* to the magnetic field. That is, the flux, the conductor, and the velocity must be mutually perpendicular (see Part I, page 207).

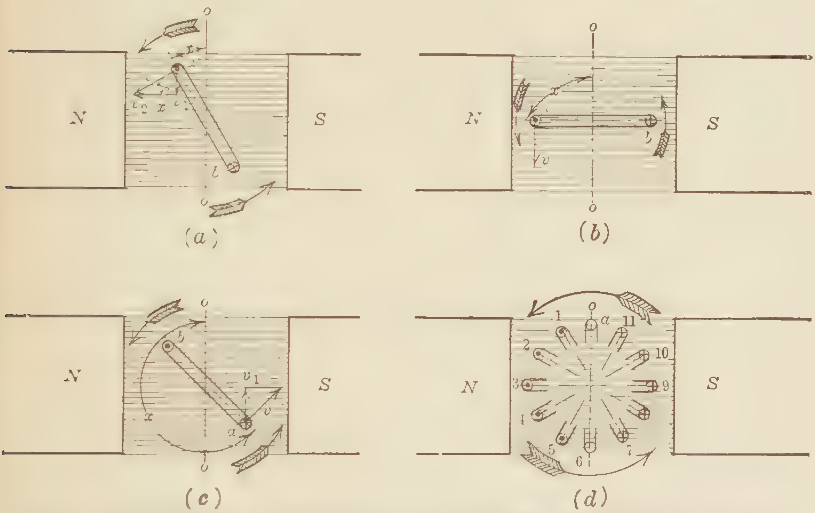


FIG. 10.—Generation of alternating e.m.f. by a simple coil rotating in a uniform magnetic field.

For example, in Fig. 10, when the plane of the coil ab is perpendicular to the magnetic field, conductor a is moving parallel to the magnetic lines. Hence, at this instant, there is no component of velocity perpendicular to the field. Therefore, as is well known, there is no e.m.f. generated at this instant.

In (a) (Fig. 10) the plane of the coil ab makes an angle x with the neutral plane oo. The velocity v is oblique to the direction of the magnetic field. This velocity v has a component v_1 perpendicular to the magnetic field. The induced e.m.f. is obviously proportional to this perpendicular component v_1. It is readily shown that v_1 is proportional to sin x. Hence, the

e.m.f. induced in coil *ab* is proportional to the sine of the angle x, and therefore, varies sinusoidally with time (Fig. 11).

In Fig. 10 (*b*), the angle x is 90°, and $\sin x = 1.0$, its maximum positive value. Therefore, at 90° (Fig. 11) the e.m.f. has its maximum value. In Fig. 10 (*c*), the angle x is 225°, its sine is negative, and the value of the e.m.f. is negative, as shown in Fig. 11. Figure 10 (*d*) shows the direction of the induced e.m.f. in either conductor *a* or *b* at successive instants during the rotation. It is clear that an alternating e.m.f. is induced in coil *ab*.

Figure 11 shows graphically the value of the induced e.m.f. in conductor *a*, it being assumed that the maximum value (occurring at (*b*) in Fig. 10) is 10 volts and that the e.m.f. is positive when it is acting outwards. The values of angle x and the numbers

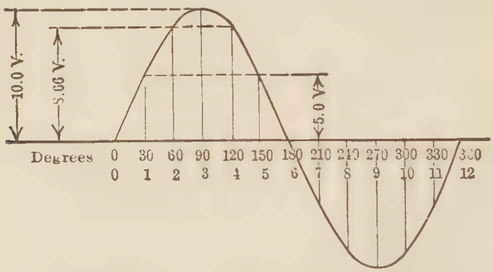


Fig. 11.—Sinusoidal variation of an alternating e.m.f.

corresponding to the positions of *a*, shown in Fig. 10 (*d*), are given along the scale of abscissas. When angle x is 30° (1 in (*d*)), the e.m.f. is 10 sin 30°, or 5.0 volts, as shown in Fig. 11, etc.

Thus, when a single coil rotates at uniform velocity in a uniform field, it generates an e.m.f. which varies as the sine of the angle which the plane of the coil makes with a plane perpendicular to the direction of the magnetic field.

7. Time Variation of Alternating Electromotive Force.—Figure 12 shows the e.m.f. curve of Fig. 11. In Fig. 12, however, it is assumed that the coil (Figs. 9 and 10) rotates at 60 r.p.s. It therefore completes one revolution in $\frac{1}{60}$ sec.

Also, in $\frac{1}{60}$ sec. it rotates through 2π radians or 360 space-degrees. Hence, two scales of abscissas may be used for the e.m.f. curve, one scale being *space angles* of the coil, expressed in

either degrees or radians, and the other scale being *time*. The scale of abscissas (Fig. 12) is given in both degrees and radians as well as in time. That time as well as the space-angles may be used for the scale of abscissas is due to the fact that at constant speed the space-angle through which the coil rotates is proportional to the time.

For convenience the time axis is often given in *time-degrees* rather than in actual time, such as seconds. For example, instead of stating $\frac{1}{240}$ sec. (Fig. 12), 90 electrical time-degrees might be used.

Hence, it may be said that alternating currents and e.m.fs. vary sinusoidally with *time*.

8. Cycle; Alternation; Frequency.—After the coil (Fig. 9) has rotated through 360 space-degrees, the e.m.f. obviously

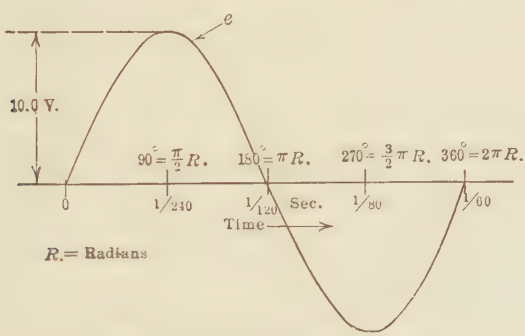


FIG. 12.—Space-degrees and time.

begins to repeat itself. For example when the coil has rotated through 390 space-degrees, its position is the same as its position when the angle x was equal to 30 space-degrees. Hence, the value and direction of the e.m.f. are the same for 390° and 30°. Consequently, when the coil has rotated through 360 space-degrees, the e.m.f. has passed through all values of magnitude, direction, and rate of change possible under the given conditions. At subsequent positions of the coil, these values merely repeat themselves at intervals of 360 space-degrees. Hence, the e.m.f. (or current) undergoes a complete *cycle* of values for every 360 space-degrees through which the coil rotates. Therefore,

one cycle corresponds to 360 electrical time-degrees as shown in Fig. 13. One alternation corresponds to 180 electrical time-degrees. The number of *cycles per second* is the *frequency.*

In a two-pole generator, the e.m.f. goes through one cycle for every revolution of the coil. Hence, the cycles per second or the frequency, is equal to the revolutions per second of the coil.

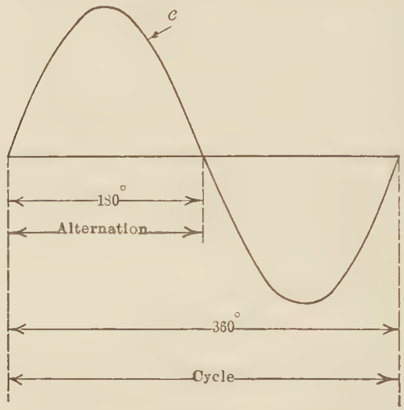


FIG. 13.—Alternation and cycle.

Example.—The armature of a two-pole alternator rotates at 1,500 r.p.m. What is the frequency?

The revolutions per second,

$$s = \frac{1{,}500}{60} = 25.$$

The frequency,

$$f = 25 \text{ cycles per second.} \quad \textit{Ans.}$$

It is obvious that with a two-pole generator one space-degree for the coil corresponds to one time-degree for the e.m.f.

9. Frequency, Poles and Speed.—Figure 14 (*a*) shows a four-pole machine. For simplicity a single conductor *a* of a coil is shown rotating, rather than the complete coil. As soon as this conductor has passed a north and a south pole, that is, after it has passed from 1 to 5, it has completed one electrical cycle or 360 electrical time-degrees, as is shown in Fig. 14 (*b*). Mechanically, it has completed one-half a revolution, or 180 space-degrees, so that in one revolution, or 360 space-degrees, the e.m.f.

in the conductor will have completed two cycles, and will have gone through 720 electrical time-degrees. Therefore, in this case *one space-degree* corresponds to *two electrical time-degrees.* That is, for every space-degree that the coil passes through, the voltage wave completes two electrical time-degrees. This conductor needs to make only 30 r.p.s. or 1,800 r.p.m. in order to generate a 60-cycle e.m.f. Likewise, for a 25-cycle e.m.f., this

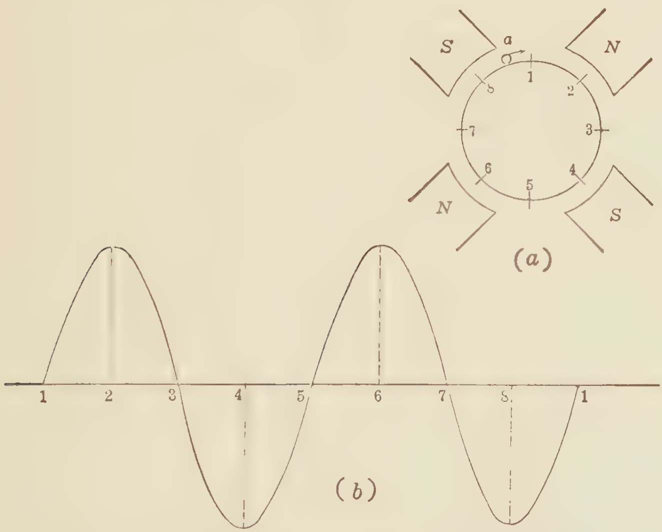


FIG. 14.—Two cycles per revolution in 4-pole alternator.

conductor needs to revolve at only 12.5 r.p.s. or 750 r.p.m. For a given frequency, as the number of poles increases, the mechanical speed decreases proportionately. The relation between speed, poles and frequency may be written in the form of an equation:

$$f = \frac{P \times S}{2 \times 60} = \frac{P \times S}{120}, \tag{2}$$

where f is the frequency in cycles per second, P is the number of poles, and S is the speed in revolutions per minute.

The table shows the relation of speed, frequency and number of poles for a few typical cases.

Poles	Speed, r.p.m.	
	60 cycles	25 cycles
2	3,600	1,500
4	1,800	750
6	1,200	500
8	900	375
40	180	75

Example.—A 60-cycle, engine-driven alternator has a speed of 120 r.p.m. How many poles has it?

Using equation (2) and solving for P,

$$P = \frac{120f}{S} = \frac{120 \times 60}{120} = 60 \text{ poles.} \quad \textit{Ans.}$$

In practice, nearly all alternators have stationary armatures and rotating field structures, and the above equations apply.

10. Commercial Frequencies.—In this country, frequencies are standardized at 60 cycles and at 25 cycles per second, although other frequencies are used. In California and in Mexico, for example, 50 cycles is used on some of the large transmission systems. In the early days of alternating-current development, 133 cycles was common, but few, if any, plants now generate at this frequency. The principal advantage of higher frequencies is that transformers require less iron and copper, and so are lighter and cheaper. The flicker of lamps is not perceptible at 60 cycles, but at 25 cycles it is very evident. On the other hand, the voltage drop in transmission lines and in apparatus varies almost directly as the frequency, so that better voltage regulation throughout the system is obtained with low frequency. Power apparatus, such as induction motors, synchronous converters, alternating-current commutator motors, etc., operates better at low than at high frequencies. With one or two exceptions, however, the operation is satisfactory at 60 cycles per second. A power and lighting company would ordinarily operate at 60 cycles per second, because the flicker of lamps at 25 cycles per second is objectionable and the transformers at this lower fre-

quency are heavier and more costly than they are at the higher frequency. On the other hand, an electric company generating strictly for power purposes would ordinarily use 25 cycles. This frequency is used by the New York, New Haven and Hartford R. R. for its electric locomotives; on the Norfolk and Western Ry. for operating electric locomotives; and by the Boston Elevated Ry. Co. for transmitting high-voltage power to its direct-current sub-stations. In Europe, frequencies as low as 15 and even 12.5 cycles per second are common.

11. Electromotive Force Induced in Conductors of Alternator Armatures.—Thus far, e m.fs. have been considered as being due to the rotation of a single flat coil in a magnetic field of uniform intensity. The e.m.f. is sinusoidal under these conditions because the component of conductor velocity perpendicular to the field varies as the sine of the angle through which the coil rotates (see Fig. 10). Commercial alternators must be quite different structurally from the simple alternators just described, since alternators built with a single coil and having no iron in the armature would have so small a power output as to be impracticable. In most commercial alternators the field structure rotates, rather than the armature. This eliminates the slip-rings for carrying the power current and simplifies insulation (also see page 130). The conductors are embedded in slots on the armature surface and the resulting air-gap is short (see page 149, Fig. 136). Ordinarily, the winding of any one phase is distributed over a belt consisting of several adjacent slots (see page 142, Fig. 138).

To illustrate the manner in which an e.m.f. varying sinusoidally with time may be induced in the conductors of an alternator armature, a single conductor embedded in a slot is shown in Fig. 15. The armature is stationary and the field structure is shown as moving from left to right. For simplicity the armature surface is represented as being flat. The pole-faces are chamfered in such a manner that the flux density along the air-gap is sinusoidal, the shortest air-gap, hence the greatest flux density, occurring at the center of the poles. The effect of the slot is neglected. The value of *flux density* for each point along the air-gap is given by the ordinates of curve B (Fig. 15). For example, if the maximum density, which occurs at the center of

the pole, is 7,000 gausses, the density at point a, 30 space-degrees to the right of the point where the flux density is zero, is 7,000 sin 30° or 3,500 gauses. Obviously, the total flux is given by the product of the average height of curve B and the area of the pole-face. Exact sinusoidal distribution of flux does not ordinarily occur in commercial machines, but it is frequently approximated.

The equation for induced e.m.f. is $Blv\ 10^{-8}$ volts (see page 155, and Part I, page 207). Although the conductor itself (Fig. 15)

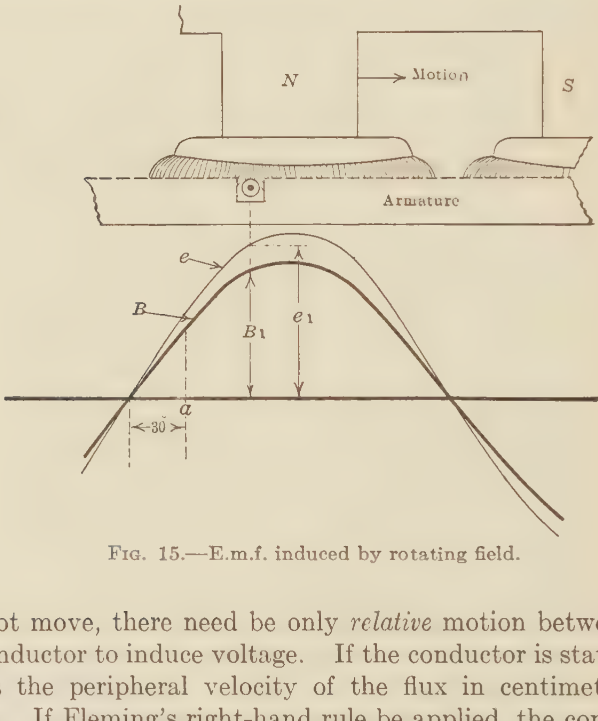


FIG. 15.—E.m.f. induced by rotating field.

does not move, there need be only *relative* motion between flux and conductor to induce voltage. If the conductor is stationary, v gives the peripheral velocity of the flux in centimeters per second. If Fleming's right-hand rule be applied, the conductor, although stationary, must be considered as moving from right to left, since it moves in this direction *relative to the flux*. When under the north pole, the direction of the induced e.m.f. is found to be outwards, as is shown by the dot. Since in the foregoing equation l and v are constant, the induced e.m.f. in each conductor *must be proportional to the flux density B of the field in which the conductor finds itself*. When the conductor is midway

between pole centers, the flux density B is zero and the e.m.f. e is zero; when the pole center is directly over the conductor, the flux density B is a maximum and the e.m.f. e is a maximum. At the position shown in Fig. 15 the conductor is in a field whose density is B_1 gausses and the e.m.f. induced at that instant is e_1 volts.

It follows that if the flux density at each point along the gap varies as the sine of its electrical space-angle, the e.m.f. induced in each armature conductor will vary sinusoidally with time. In practice, the flux often departs considerably from sinusoidal distribution, but other factors tend to make the e.m.f. wave sinusoidal, when an entire phase is considered (see page 143).

12. Equations of Alternating Voltages and Currents.—It was shown in Par. 6 that the variation of an alternating e.m.f. with time could be expressed as a sine function of either a space-angle or of time. Ordinarily, the voltage is expressed as a function of time and for mathematical reasons the time-angle is expressed primarily in radians rather than in degrees.

For example, let it be required to express a 60-cycle alternating e.m.f. as a sine function of time, the maximum value being 150 volts.

$$\begin{aligned} e &= E_m \sin \omega t \qquad (3) \\ &= 150 \sin \omega t, \end{aligned}$$

where e is the value of the e.m.f. at any time t, E_m is the maximum value of the e.m.f., and with a given frequency ω is a constant. The time t is given in seconds.

The value of the constant ω must be determined.

If the frequency is 60 cycles per second, the e.m.f. completes one cycle, or 2π radians, in $1/60$ sec. Therefore, at the time $t = 1/60$ sec. the angle ωt equals 2π radians.

That is

$$\omega\,(1/60) = 2\pi$$
$$\text{or } \omega = 2\pi 60 \text{ or } 377.$$

Therefore, the complete equation of the e.m.f. is,

$$e = 150 \sin 2\pi 60t = 150 \sin 377t. \qquad (\mathrm{I})$$

The symbol ω is equal to $2\pi f$ where f is the frequency. ω is also the number of radians through which an e.m.f. or current passes in one second. For example, the foregoing e.m.f. com-

pletes 2π radians every cycle. There are 60 cycles per second. Hence, the radians per second are $60 \times 2\pi$ or 377. ω is often called angular velocity (in radians per second) (see page 406).

Example.—Find the value of the foregoing e.m.f. wave when the time is equal to 0.003 sec.

Substituting 0.003 in I,

$$e = 150 \sin (377 \times 0.003) = 150 \sin 1.131 \text{ (radians).}$$

1.13 is obviously the time-angle in radians.

Since 2π or 6.283 radians equal 360°, the angle in degrees,

$$\omega t = \frac{1.13}{6.283} 360 = 64.7°.$$

$$\sin 64.7° = 0.904 \text{ (page 411).}$$

Hence, $e = 150 \times 0.904 = 135.6$ volts. *Ans.*

Example.—A 25-cycle alternating current, having a maximum instantaneous value of 75 amp., varies sinusoidally with time. The value of time may be taken as zero when the current is zero and increasing in a positive

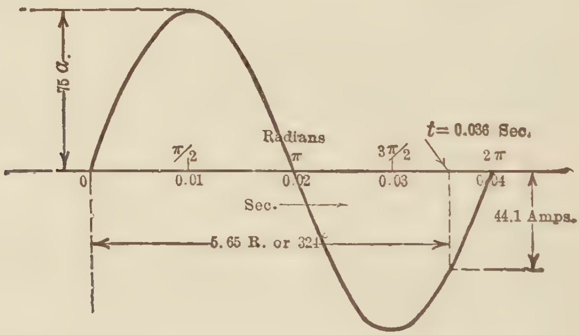


Fig. 16.—Instantaneous value of a 25-cycle current.

direction. Find (*a*) the equation of the current; (*b*) the time when it first reaches its positive maximum value, (*c*) the value of current when the time is 0.036 sec.

(*a*) $\omega = 2\pi 25 = 157$.

Hence the equation of the current,

$$i = 75 \sin 2\pi 25 = 75 \sin 157t. \quad \textit{Ans.}$$

(*b*) This current is plotted in Fig. 16. Obviously, it first reaches its positive maximum value when the time-angle is 90° or $\pi/2$ radians. Hence,

$$2\pi 25t = \frac{\pi}{2}$$

$$t = \left(\frac{\pi}{2}\right)\frac{1}{2\pi 25} = \frac{1}{100} \text{ or } 0.01 \text{ sec.} \quad \textit{Ans.}$$

This result is obviously correct since $\frac{1}{25}$ sec. is required to complete one cycle. Therefore, $\frac{1}{4} \times \frac{1}{25}$ or 0.01 sec. is required to complete one-fourth of a cycle.

(c) $i = 75 \sin (157 \times 0.036) = 75 \sin 5.65$ (radians)

$$\frac{5.65}{6.283} 360° = 324°$$

From (27), page 407,

$$-\sin 324° = \cos (90° + 324°) = \cos 414°$$
$$= \cos (414° - 360°) = \cos 54°$$
$$= 0\ 5878$$
$$75 \times (-0.5878) = -44.1 \text{ amp} \quad \textit{Ans.}$$

Figure 16 shows this value of current.

The angle could have been found by proportion. It requires 0.04 sec. for the current to complete one cycle or 360°. In 0.036 sec. the angle is

$$\frac{0.036}{0.04} 360° = \frac{9}{10} 360° = 324°$$

13. The Alternating-current Ampere.—Figure 17 (*a*) shows an alternating-current sine wave, having a maximum value of 1.414 amp. At first thought it might seem that the value in amperes of such a wave should be based on the *average* value. If the wave over one complete cycle is considered, the average value is zero, as there is just as much negative as positive current. A direct-current ammeter, if connected to measure this current, would indicate zero, as such an instrument reads *average* values.

The value of an alternating current is not based on its average value, but on its *heating* effect, and may be defined as follows:

An alternating-current ampere is that current which, flowing through a given ohmic resistance, will produce heat at the same rate as a direct-current ampere.

Assume that a resistance unit is immersed in a calorimeter and that when a direct-current ampere is sent through this resistance the temperature of the water is raised 20° in 10 min. An alternating-current ampere, if sent through this same resistance unit, will raise the temperature of the water by the same amount in the same time, other conditions such as radiation, etc., being the same. That is, both currents produce heat at the same rate.

The heating effect varies as the *square* of the current (i^2R). Therefore, the value in amperes of the wave of current in Fig. 17

(*a*) must be based upon its *squared* values. Figure 17 (*b*) shows the current wave of Fig. 17 (*a*) plotted, together with its squared values. That is, each ordinate of the i-wave is squared and these values plotted to give the i^2-wave shown. The maximum value of this new wave will be 2.0 $(= \overline{1.414})^2$ since the maximum value of the original current wave is 1.414 or $\sqrt{2}$. The squared wave also lies entirely above the zero axis, because the square of a negative value is positive.

This squared wave has a frequency twice that of the original wave and has its horizontal axis of symmetry at a distance of 10.

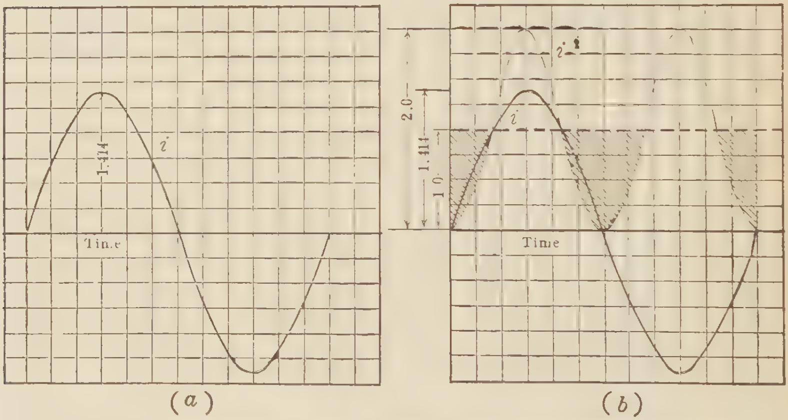


FIG. 17.—Maximum and effective values of sine-wave alternating current.

unit above the zero axis, as shown in Fig. 17 (*b*). The average value of this squared wave is 1.0 amp., as shown by the dotted line, because the areas above the dotted line will just fit into the shaded valleys below the dotted line. Therefore, if an equivalent rectangle were made from this wave, its height would be 1.0 unit. This value, 1.0, is the *average of the squares* of the current wave. Average heating varies as the average of the square of the current so that the heating effect of this current over one cycle is proportional to the area of this rectangle. A direct-current ampere flowing through the same resistance would produce the same heating as this current does, since its squared value is one ampere.

The current in Fig. 17 (*a*), which has a maximum value of 1.414 amp., therefore, has a heating value of 1.0 *amp.* This value of the current is called the *root-mean-square* (r.m.s.) or *effective* value of the current. The ordinary alternating-current ammeter indicates this root-mean-square or effective value of current. Therefore, an alternating-current ampere, sine wave, which produces heat at the same rate as a direct-current ampere, has a *maximum* value of 1.414 ($= \sqrt{2}$) amp. In fact, for any sine-wave current, the ratio of the *maximum* to the *effective* value is equal to the $\sqrt{2}$ or 1.414. The ratio of effective to maximum value is $1/1.414 = 0.707$.

Example.—An alternating current has a maximum instantaneous value of 57 amp. What will an alternating-current ammeter read when connected in the circuit?

$$\frac{57}{1.414} = 40.3 \text{ amp.} \quad \textit{Ans.}$$

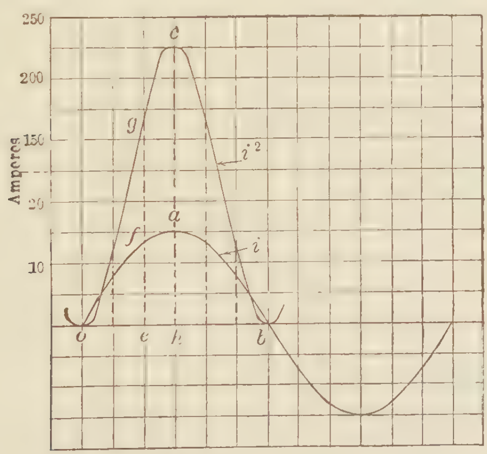


FIG. 18.—Determination of effective current.

14. Graphical Determination of Effective Current.—The method of obtaining the effective value of any current, whether or not it varies sinusoidally with time, is shown in Fig. 18. The curve *oab* is a sine curve of current having a maximum value of 15 amp. Ordinates such as *efg*, *hac*, etc., are erected at regular intervals over one-half a cycle. The value of each ordinate, in amperes, of the curve *oab* is squared. For example, the ordinate

ef to scale is 13 amp., hence the ordinate *eg* is 169 amp. The current squared curve *ogcb* is plotted as shown. Its maximum ordinate obviously must be $\overline{15}^2$ or 225 amp. The average value of this curve is found. There are two simple methods of doing this. The values of several equally spaced ordinates are added and the sum is divided by the number of ordinates. This gives the average value of the squared curve if a sufficient number of ordinates be used.

The area of the curve *ogcbo* in square inches may be found with a planimeter. This area divided by the base *ob* in inches gives the average height of the curve in *inches*. To find the average value in amperes multiply this average height in inches by the scale of ordinates in amperes per inch. For example, assume that curve *ogcbo* has an area of 1.69 sq. in. and the base is 1.5 in. The scale of ordinates for the curve is 100 amp. to the inch. Its average value then becomes

$$\text{Av. } i^2 = \frac{1.69}{1.5}100 = 112.7 \text{ amp.}$$

Therefore, this average squared current over a half-cycle will produce the same heating in a given resistance as a direct-current whose squared value is 112.7 amp. Hence, from definition, the effective alternating current is $\sqrt{112.7} = 10.61$ amp.

The ratio of maximum to effective current is $15/10.61 = 1.414$. For simplicity, the foregoing method was given for a sine curve of current, but the method is applicable if the current varies other than sinusoidally with time. Ordinarily, with non-sinusoidal currents, the ratio of the maximum to the effective value is other than $\sqrt{2}$.

15. Average Current or Voltage.—Although the average current or voltage over one cycle is zero, the average for one alternation is obviously not zero. To find the average current (Fig. 18), it is only necessary to find the average value of the curve *oab* in amperes. This may be accomplished either by averaging ordinates or finding the area, dividing by the base and multiplying by the ordinate scale. For example, the area bounded by the line *oabo* (Fig. 18) is found to be 0.717 sq. in , the base *ob* is 1.5 in., and the scale is 20 amp. to the inch. Hence, the average current is $\frac{0.717}{1.5}20 = 9.56$ amp.

With a sine wave the average value is $2/\pi$ times the maximum value. Hence, the ratio of the effective value to the average value is $\frac{1/\sqrt{2}}{2/\pi} = 1.11$. The ratio of effective to average value is called the *form factor* of the curve.

The ratio of average to effective value is 1/1.11 or 0.9.

For example, with the foregoing current, whose maximum value is 15 amp., the average value for a half-cycle is $\frac{2}{\pi}$ 15 or 9.56 amp. The ratio of effective to average current is 10.61/9.56 = 1.11.

16. Ohm; Volt.—If a resistance of one ohm, as measured with direct-current, has no inductance and is so designed that alternating current in flowing through it does not produce any secondary effects, such as eddy-currents, or skin effect, it offers a resistance of one *ohm* to alternating current.

When an alternating-current ampere flows through such a resistance, the drop across its terminals is equal to one alternating-current *volt*. Hence, the relation between *maximum* and *effective* volts is the same as the relation between *maximum* and *effective* amperes. For a sine wave, the maximum voltage is $\sqrt{2}$, or 1.414, times the effective voltage.

17. Phase Relations.—The current and voltage in the ordinary alternating-current system have the same fundamental frequency under normal operating conditions, although they do not necessarily pass through their corresponding zero values at the same instant. Figure 19 (*a*) shows two sine-wave currents, one having an effective value of 8 and the other of 12 amp. Their maximum values are accordingly $8\sqrt{2}$ or 11.3 amp. and $12\sqrt{2}$ or 17.0 amp., respectively. Both currents pass through zero, increasing positively, at the same instant, and are therefore said to be *in phase* with each other.

Figure 19 (*b*) shows two sine-wave currents of 8 and 12 amp. but not passing through zero at the same instant. The 8-amp. current passes through zero, increasing positively, later than does the 12-amp. current. It must be remembered that time is increasing from left to right. If the 12-amp. current is passing through its zero value at two o'clock, the 8-amp. current is passing through its corresponding zero value some time later,

for any value of time to the right of 2.00 is later than two o'clock. Therefore, the 8-amp. current *lags* the 12-amp. current.

The time of lag shown in Fig. 19 (*b*) corresponds to 60° and is represented by the angle θ. Therefore, the 8-amp. current lags the 12-amp. current by an angle θ or by 60°. Or the 12-amp. current may be said to *lead* the 8-amp. current by an angle θ or by 60°.

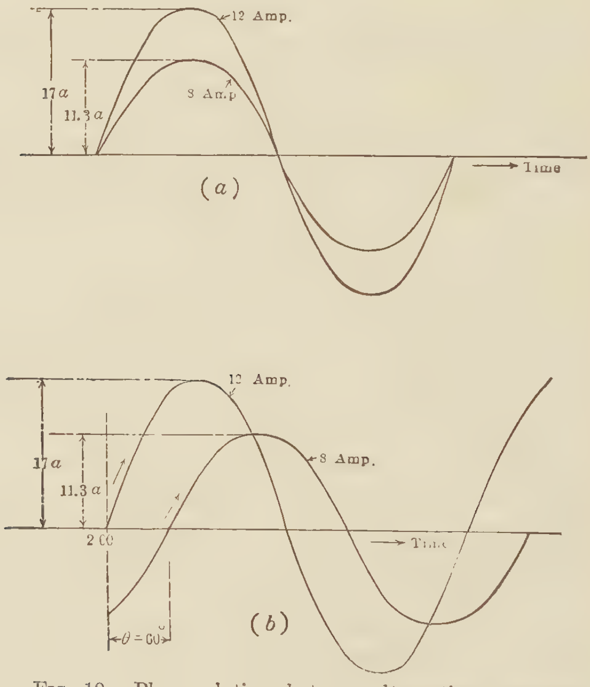


FIG. 19.—Phase relations between alternating currents.

In Fig. 19 (*a*) the two currents are *in phase* with each other. In Fig. 19 (*b*) the two currents have a *phase difference* of 60°. These phase differences may exist between currents and voltages, between two or more voltages, or between two or more currents.

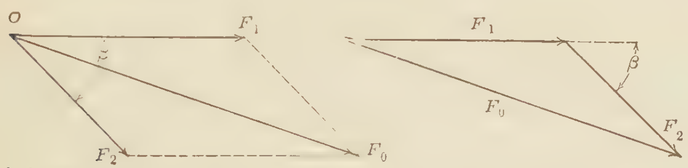
18. Scalars and Vectors.—Quantities in general are divided into two classes, scalars and vectors.

A scalar is a quantity which is completely determined by its magnitude alone. Examples of scalar quantities are dollars,

energy, gallons, mass, temperature, etc. Such quantities are added algebraically. For example, two dollars plus five dollars equals seven dollars.

A vector has direction as well as magnitude. A common example of a vector is force. When a force is under consideration, not only its magnitude but its direction as well must be considered. When two or more forces are added, they are not necessarily added algebraically, but must be combined in such a way as to take into consideration their directions as well as their magnitudes.

To find the relations existing among currents and voltages in direct-current circuits, a knowledge of scalar quantities only is required. To find the relations existing among currents and voltages in alternating-current circuits a knowledge of vector quantities is required.



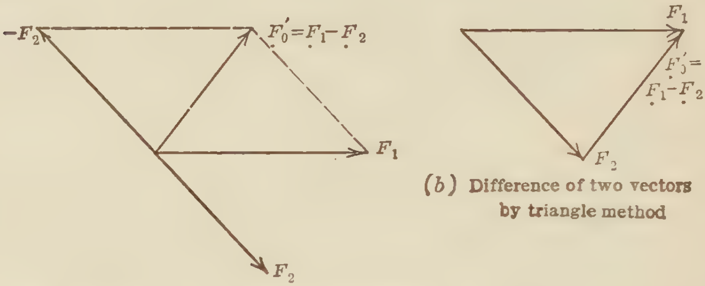
(*a*) Sum of two vectors by parallelogram method (*b*) Sum of two vectors by triangle method

Fig. 20.—Addition of vector quantities.

Figure 20 (*a*) shows two forces acting at the point O and represented by the vectors F_1 and F_2. The length of each of these vectors, to scale, is equal to the *magnitude* of the force which it represents. The direction of each of these vectors shows the *direction* in which the force acts. β is the angle between F_1 and F_2. Their sum, F_0, or the single force which would have the same effect on their point of application, O, as F_1 and F_2 acting in conjunction, is called their *resultant*. F_o is one diagonal of the parallelogram having F_1 and F_2 as adjacent sides.

Figure 20 (*b*) shows a triangle having F_1 and F_2 as two of its sides, F_1 and F_2 being parallel to, and acting in the same directions as, F_1 and F_2 of Fig. 20 (*a*). The exterior angle between F_1 and F_2 is therefore equal to β. The third side of the triangle F_0 is equal in magnitude and direction to F_0 of Fig. 20 (*a*). Therefore, the resultant of two vectors may be found by

means of a triangle properly constructed, of which two sides are the two component vectors and the third side is their sum. Such a triangle is called a *triangle of forces.* It is usually simpler to use the triangle of forces than to use the parallelogram of forces.



(*b*) Difference of two vectors by triangle method

(*a*) Difference of two vectors by parallelogram method

FIG. 21.—Subtraction of vector quantities.

To subtract one vector from another, reverse the first vector and add it vectorially to the second vector. For example, in Fig. 21 (*a*) it is desired to subtract F_2 from F_1. F_2 is reversed giving $-F_2$. F_0', the vector sum of F_1 and $-F_2$, found by completing the parallelogram, is equal to $F_1 - F_2$. Vectors may be subtracted by the triangle method as shown in Fig. 21 (*b*). The vector F_0', connecting the ends of the two vectors F_1 and F_2 whose difference is desired, is their vector *difference.*

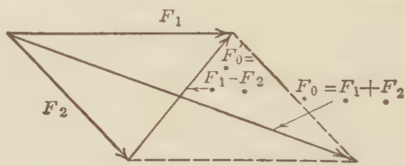


FIG. 22.—Sum and difference of two vectors.

If a parallelogram (Fig. 22), having vectors F_1 and F_2 as adjacent sides, be completed, one diagonal F_0 of the parallelogram is the vector *sum* of F_1 and F_2. The other diagonal F_0', of the parallelogram, is the vector *difference* of F_1 and F_2.

A vector is often indicated by placing a dot under its symbol. For example, in Fig. 22

$$\underset{\cdot}{F}_0 = \underset{\cdot}{F}_1 + \underset{\cdot}{F}_2$$

shows that F_0 is the *vector* sum of F_1 and F_2 and not their algebraic sum.

When more than two vectors are added, the resultant of two is first found and this resultant is combined with a third vector, etc. This is illustrated in Fig. 23, in which three vectors F_1, F_2, and F_3 are added.

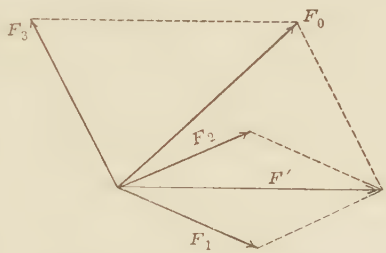


FIG. 23.—Sum of three vectors.

F_1 and F_2 are first combined and the resultant F' is found. F' is then combined with F_3, giving F_o as the sum of all three vectors, F_1, F_2, and F_3. That is,

$$\dot{F}_0 = \dot{F}_1 + \dot{F}_2 + \dot{F}_3.$$

F' is an intermediate vector and therefore does not appear in the ultimate result.

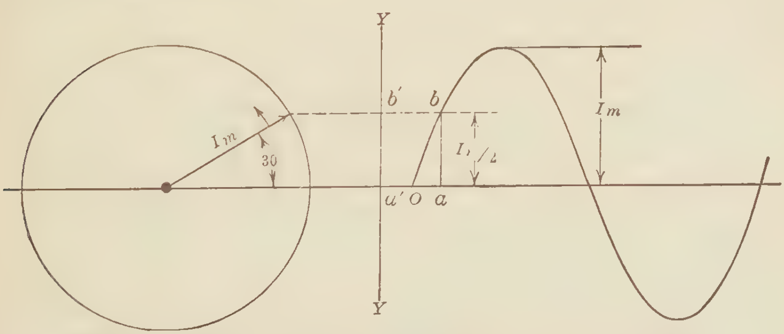


FIG. 24.—Instantaneous values of current from a rotating vector.

19. Vector Representation of Alternating Quantities.—It was shown in Fig. 5 that a sine wave could be drawn by projecting from the ends of successive radii to meet corresponding equally spaced ordinates. Thus, the end of radius 1 was projected to

meet ordinate $11'$ at $1'$ to give the ordinate of the sine curve at the angle 01.

It follows that if the values of current and voltage are represented by sine curves, the value of the current or voltage may be found at any instant by projecting a radius upon a vertical line.

This is illustrated in Fig. 24. A current has a maximum value I_m. This value I_m is laid off as a radius. As I_m rotates in a counter-clockwise direction, values of the sine curve are found by projecting I_m in its successive positions corresponding to 1, 2, 3, etc. (Fig. 5). As one cycle is completed for every revolution of the radius I_m, it follows that the radius I_m rotates at a speed in revolutions per second equal to the frequency of the current in cycles per second. For example, if the current I_m has a frequency of 60 cycles, the radius I_m must make 60 complete revolutions per second, and in a counter-clockwise direction. Counter-clockwise rotation has been adopted internationally as the positive direction of rotation.

When the radius I_m is at the right-hand horizontal position, the value of the current is zero. When I_m has advanced 30°, the point b on the current-wave has been reached. The value of the current at this instant is ab, or what is the same thing, the current value is given by the distance $a'b'$, the projection of I_m upon the *vertical* axis. At this particular instant, the distance $ab = a'b' = I_m/2$, since $\sin 30° = 0.5$.

The value at any instant of a sine wave of alternating current or voltage is, therefore, given by the projection on a vertical line of a vector equal in length to the maximum value of the current or voltage and rotating at a speed in revolutions per second equal to the frequency of the current or voltage. The angular velocity of the vector in radians per second is equal to 2π times the frequency.

20. Vector Representation with Phase Difference.—Let it be required to represent two currents I_1 and I_2, having maximum values of I_1' and I_2' and differing in phase by 60°, I_2 lagging I_1 by 60° as shown by the sine curves (Fig. 25). Assume that I_1 has an *effective* value of 12.0 amp. and I_2 an *effective* value of 8.0 amp. The maximum value of I_1 will be $12.0\sqrt{2} = 17.0$ amp. and the maximum value of I_2 will be $8.0\sqrt{2} = 11.3$ amp.

Since I_1 is at its zero value and increasing in a positive direction when the time is equal to zero, it may be considered as being generated by the radius vector I_1' rotating in a counter-clockwise direction and making an angle of zero degrees with the horizontal axis when the time is zero.

Since I_2 lags I_1 by 60°, I_2 will not be going through its zero value until the radius vector I_1' has advanced 60° in a counter-clockwise direction. Therefore, I_2 will be generated by the radius vector I_2', making an angle of 60° with I_1' in a clockwise direction.

At the instant when the radius I_1' is in the horizontal position, the value of I_1 is zero. At this same instant, the radius I_2' will

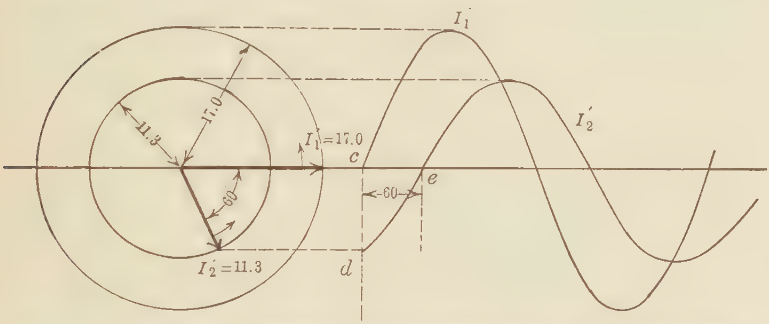


Fig. 25.—Current waves produced by two current vectors differing in phase by 60°.

not have reached its horizontal position, the value of the current being represented by *cd* (Fig. 25). In fact, the radius I_2' does not reach its horizontal or zero position until I_1' has advanced 60° beyond the horizontal. Further, the horizontal distance *ce* is 60°, the same as the phase angle between the two rotating vectors.

Therefore, these two current-curves can be constructed in their proper phase relation by means of two rotating vectors, having lengths of 17.0 and 11.3 amp., having equal angular velocities, and differing in phase by 60° (Fig. 25).

21. Addition of In-phase Currents.—Figure 26 shows two currents, having *effective* values of 8 and 12 amp., uniting to flow in a common wire. If these two currents were direct currents,

then by Kirchhoff's first law (see Part I, page 78), the current I_3 could have only two possible numerical values, $12 + 8 = 20$ amp. if the two currents flow in the same direction, and 12 minus $8 = 4$ amp. if they flow in opposite directions.

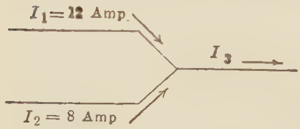


FIG. 26.—Alternating currents meeting at a junction.

If the two currents (Fig. 26) are alternating, their sum I_3 may be equal numerically to *any* value *from* 20 *amp.* to 4 *amp.*, depending on the phase relation existing between I_1 and I_2.

Figure 27 shows the instantaneous values of these two currents plotted over one cycle with the currents in phase. The 8-amp. current has a maximum value of $8\sqrt{2}$ or 11.3 amp. and the 12-

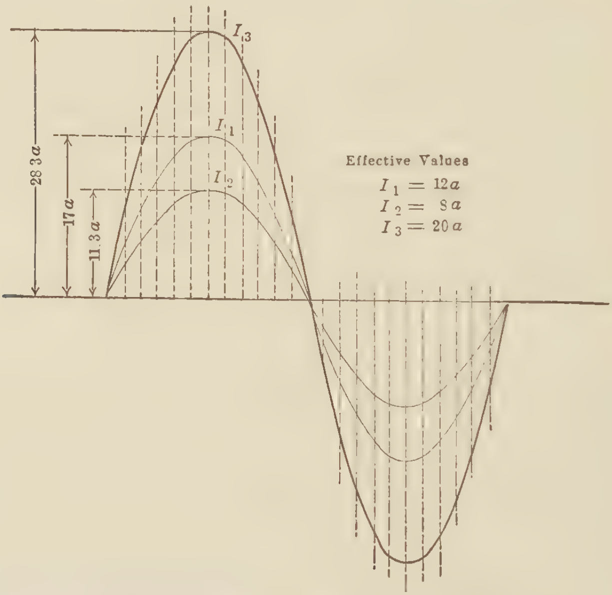


FIG. 27.—Addition of two currents in phase.

amp. current has a maximum value of $12\sqrt{2}$ or 17.0 amp. These values are shown in Fig. 27. Since both these currents have the same frequency, hence the same scale of abscissas, the curve found by adding their ordinates at each point will be a sine curve

(see Par. 5). The resulting current curve I_3 will have a maximum ordinate of $17.0 + 11.3$ or 28.3 amp., and its zero points will coincide with those of the two component curves. The effective value of the resultant current I_3 is $28.3/\sqrt{2}$ or 20.0 amp.

Hence, if two alternating currents of the same frequency are in phase their sum is merely the algebraic sum of the component currents. This is also true of voltage.

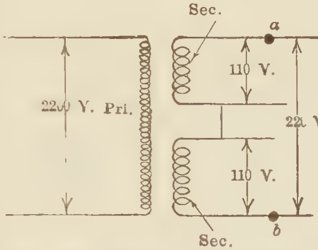


FIG. 28.—5-kv-a., 2,200/220-volt transformer windings.

Example.—In Fig. 28 is shown a 5-kv-a. (see page 214), 2,200/220-volt, 60-cycle transformer. There are two secondary coils, the voltage across each of which is 110 volts. As connected in Fig. 28, these two voltages are in phase with each other. What is the voltage across their open ends *ab*?

$$E_{ab} = 110 + 110 = 220 \text{ volts.} \quad \textit{Ans.}$$

22. Addition of Currents Differing in Phase.—Let it be required to add the two currents having *effective* values of I_1

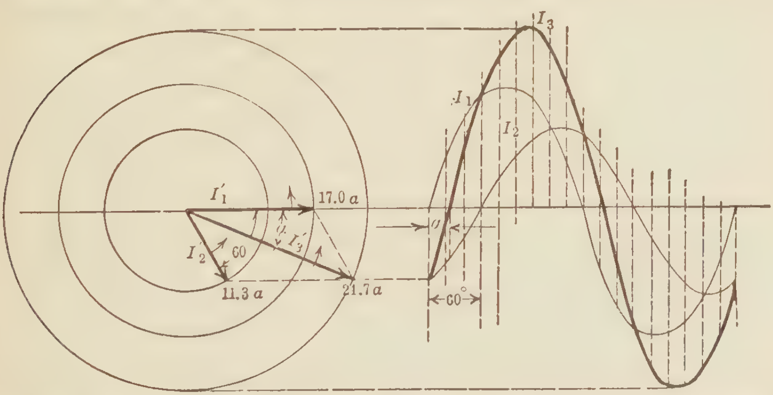


FIG. 29.—Relation of vector addition of vectors to scalar addition of ordinates.

and I_2 (Fig. 27) when they differ in phase by 60°, with I_2 lagging (also see Fig. 19). Let the current I_1 be determined graphically by the rotating vector I_1' (Fig. 29), whose maximum value is $12.0\sqrt{2}$ or 17.0 amp. When time is zero, the current I_1 is assumed to have its zero value and to be increasing positively.

Hence, at this instant vector I_1' is in the horizontal position, as shown. Since I_2 lags I_1 by 60°, it is determined graphically by the rotating vector I_2', whose maximum value is $8.0\sqrt{2}$ or 11.3 amp., and which lags I_1' by 60°.

The sum of the two currents in Fig. 29 is found in the same manner as was done in Fig. 27. Ordinates are erected and points on the resultant curve are found by adding algebraically at each ordinate the values of the two currents. This same procedure was also followed in Fig. 8, page 9.

As a result of this addition a third curve I_3 is found. I_3 lags I_1 by an angle α and leads I_2 by an angle $60°\text{-}\alpha$. If the maximum value of I_3 be determined by measurement, its value will be found to be 24.7 amp.

This corresponds to an effective value of $24.7/\sqrt{2}$ or 17.45 amp. Therefore, the sum of two sine-wave alternating currents, having effective values of 12 and 8 amp. and differing in phase by 60°, is 17.45 amp.

If the rotating vectors, I_1' and I_2' (Fig. 29) be added vectorially by completing the parallelogram, a third vector I_3' results. This vector I_3' has a length corresponding to 24.7 amp., the exact value of the maximum of the resultant current as found by the addition of ordinates. If a curve be plotted using I_3' as the rotating vector, projecting as before, it will coincide with I_3 obtained by the addition of ordinates for the 12- and 8-amp. waves. The angle α by which the radius vector I_1 leads I_3 equals the angle α by which the current curve I_1 leads the current curve I_3.

Hence, the problem can be solved without going through the somewhat lengthy process of plotting the curves and adding their ordinates. It is merely necessary to lay off the maximum values of the currents 60° apart and add them vectorially, just as forces are combined. The resulting vector will be the maximum value of the curve obtained by adding the curves I_1 and I_2.

In practice, one generally has to do with effective values such as are read on ammeters rather than with maximum values. If the effective values of the currents be added in this same manner, their vector sum is the sum of the two alternating currents in effective amperes, since adding effective values merely means adding each of the vectors of the parallelogram (Fig. 29), reduced to $1/\sqrt{2}$ times the values given.

This is illustrated in Fig. 30, where the 12- and 8-amp. vectors (r.m.s. values) are laid off 60° apart, the 12-amp. vector leading. The scale (Fig. 30) is considerably larger than that used in Fig. 29. By completing the parallelogram, the resultant cur-

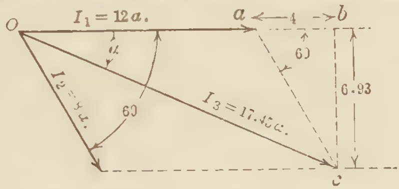


FIG. 30.—Vector addition of currents, using effective values.

rent Oc is obtained. This has a value of 17.45 amp. Its value is readily found as follows:

Project ac upon Oc, where $ac = 8$

$$ab = ac \cos 60° = 4.00$$
$$bc = ac \sin 60° = 6.93$$
$$Oc = \sqrt{(12 + 4.00)^2 + (6.93)^2} = 17.45 \text{ amp.} \quad \textit{Ans.}$$

The angle α can be readily determined.

$$\tan \alpha = \frac{6.93}{12 + 4} = 0.433$$
$$\alpha = 23.4°$$

The significance of combining the two currents I_1 and I_2 into I_3 (Figs. 26, 29, and 30) is illustrated in Fig. 31. Two alter-

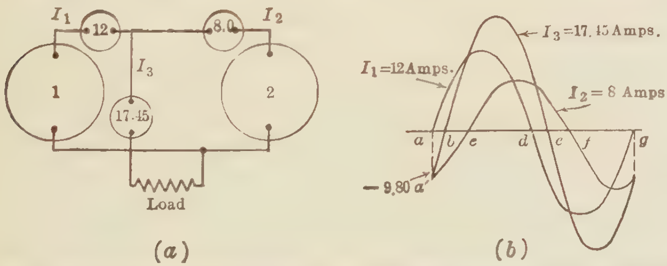


FIG. 31.—Alternators with currents out of phase supplying a common load.

nators 1 and 2 are connected in parallel to supply current to a common load. Machine 1 supplies an effective current of 12 amp. and machine 2 supplies an effective current of 8 amp. That is, an ammeter connected in the armature lead from 1 reads

12.0 amp. and an ammeter connected in the armature lead of *2* reads 8.0 amp. The machines are so adjusted that the 12-amp. current leads the 8-amp. current by 60°.

Let it be required to find the effective current I_3 taken by the load and the value of the current taken by the load at various instants.

The effective current I_3 taken by the load is obviously 17.45 amp., lagging I_1 by 23.4°, as was just determined.

In Fig. 31 (*a*) the current from either alternator is positive when it flows from the upper terminal. The conditions existing when the time is zero are shown in Fig. 31 (*b*). The value of I_1 is zero at this instant and the value of I_2 is aa' or -9.80 amp. (11.3×-0.866). Since 1 delivers no current at this instant, 2 must be supplying all the load current of 9.80 amp. At this instant the load current is negative. That is, current is returning to the positive terminal of 2. During the time represented by the distance *ab* (Fig. 31 (*b*)), generator 2 is supplying negative current to the load and generator 1 is supplying positive current. That is, the two machines are in opposition during this period. More negative current is supplied by 2 than positive current by 1, hence the load current in this period is negative. At the instant *b*, the positive current supplied by 1, is just equal to the negative current supplied by 2. Hence, the load current is zero at this instant. Actually, current equal to the value of the ordinate at *b* is leaving the upper terminal of 1 and is entering the upper terminal of 2, but not flowing to the load. In the period *bc* the positive current delivered by 1 exceeds the negative current delivered by 2 and positive current flows to the load. In the period represented by *cd*, both machines act in conjunction to deliver positive current to the load. In the period *de*, 2 delivers positive current and 1 delivers negative current, but the positive current exceeds the negative current and the net load current I_3 is positive. At the instant *e* the positive current delivered by 2 is just equal to the negative current delivered by 1, so that the net load current is zero. That is, at the instant *e* current is merely leaving the upper or positive terminal of 2 and entering the upper terminal of 1 but not flowing to the load. In the period *ef* the negative current delivered by 1 exceeds the positive current delivered by 2, and the load current I_3 becomes negative. In the period *fg*

both machines act in conjunction to deliver negative current to the load.

From the foregoing, it is obvious that over the cycle the current at some instants is merely flowing between machines (at b and e); at other instants either one machine or the other is supplying the entire load (at a, c, d, f); during some periods the two machines are acting in conjunction to supply the load current (cd, fg); at other periods they are acting in opposition to supply the load current, the net load current being the difference of the machine currents (ac, df). The net effect over the cycle is,

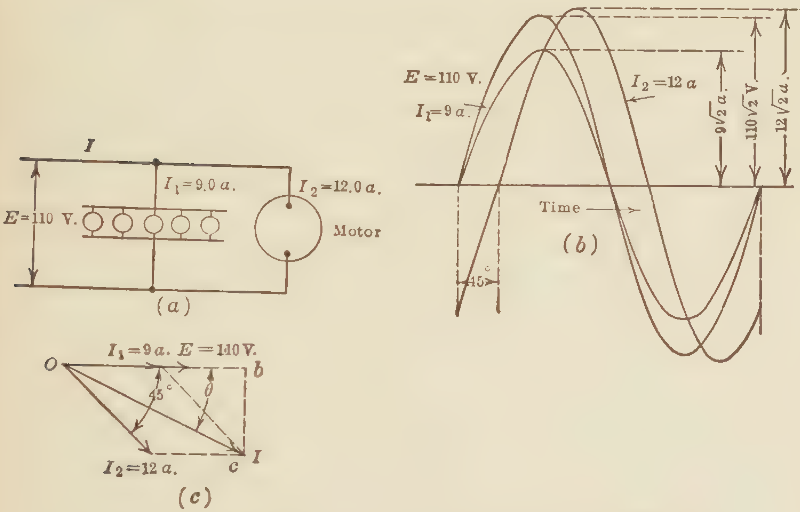


FIG. 32.—Vector addition of currents having 45° phase difference.

however, that an ammeter connected in machine 1 reads 12.0 amp.; an ammeter connected in machine 2 reads 8.0 amp., and an ammeter connected in the load reads 17.45 amp.

It is obvious that the sum of any number of alternating currents depends upon their phase relations as well as upon their magnitudes.

The vector addition of currents is further illustrated by the following example:

Example.—A lamp load (Fig. 32 (a)) takes 9.0 amp. (r.m.s.) from 110-volt (r.m.s.) mains and a single-phase motor takes 12.0 amp. (r.m.s.). The current taken by the lamps is in phase with the line voltage and the current

taken by the motor lags the line voltage by 45°. Find the resultant current I and the angle θ by which it lags the line voltage.

The instantaneous values of current and voltage are plotted in Fig. 32 (b), the 9.0-amp. current being in phase with the voltage and the 12.0-amp. current lagging the voltage by 45°. The currents and voltage are shown vectorially in (c), a different scale from that in (b) being used. The resultant current I is the diagonal of the parallelogram having I_1 and I_2 as two adjacent sides. The value of I is determined as follows: I is projected on I_1 extended.

$$ab = ac \cos 45° = 12.0 \times 0.707 = 8.48$$
$$bc = ac \sin 45° = 12.0 \times 0.707 = 8.48$$
$$I = \sqrt{(Oa + ab)^2 + (bc)^2}$$
$$= \sqrt{(9.0 + 8.48)^2 + (8.48)^2} = 19.43 \text{ amp.} \quad \textit{Ans.}$$
$$\tan \theta = \frac{bc}{Ob} = \frac{8.48}{17.48} = 0.485$$
$$\theta = 25.9°. \quad \textit{Ans.}$$

23. Addition of Voltages.—Since alternating voltages vary sinusoidally with time in precisely the same manner as for currents, they are added in exactly the same manner as currents. This is illustrated by the following example:

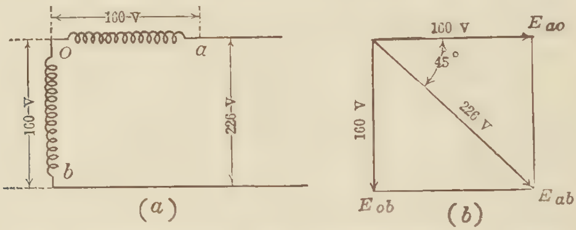


FIG. 33.—Vector addition of two equal voltages having 90° phase difference.

Example.—Each of two alternator coils Oa and Ob (Fig. 33 (a)) is generating an e.m.f. of 160 volts. These voltages differ in phase by 90°. Determine the voltage across their open ends if they are connected together at O as shown.

Let E_{ao} and E_{ob} (Fig. 33) (b)) represent the respective voltages across coils aO and Ob. Combining these two vectorially, the voltage E_{ab} is obtained. As E_{ao} and E_{ob} are at right angles, their resultant is readily found.

$$E_{ab} = \sqrt{E_{ao}^2 + E_{ob}^2} = \sqrt{160^2 + 160^2} = 226 \text{ volts.} \quad \textit{Ans.}$$

It must be kept constantly in mind that alternating voltages and currents must be combined vectorially.

The only occasion when arithmetical addition is permissible is when the voltages or the currents are in phase.

CHAPTER II

SINGLE-PHASE ALTERNATING-CURRENT CIRCUITS

24. Circuit Containing Resistance Only.—The simplest alternating-current circuit is one which contains resistance only. In such a circuit the only voltages involved are the impressed e.m.f. and the ohmic drop.

Figure 34 shows a resistance R connected across the terminals of an alternator. An alternating current i, varying sinusoidally with time, flows through this resistance. Let the maximum

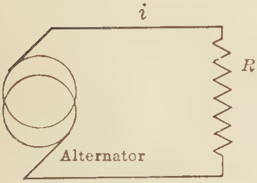


FIG. 34.—Resistance load connected to alternator.

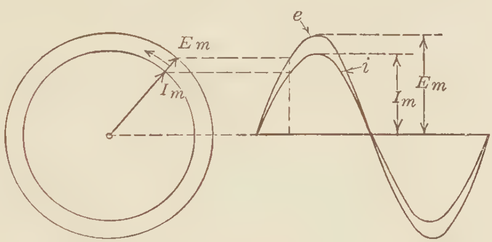


FIG. 35.—Instantaneous and vector values of current and voltage *in phase.*

instantaneous value of the current be I_m amperes. The equation of the current is:

$$i = I_m \sin \omega t = I_m \sin 2\pi ft, \tag{I}$$

where f is the frequency in cycles per second.

The current, having a maximum value I_m, is shown plotted for one cycle (Fig. 35). From the definition of alternating-current voltage (see page 25), the impressed voltage must be utilized wholly in sending the current through the resistance R. Therefore, the impressed voltage must be equal to the resistance-drop. The circuit voltage e at *any* instant is equal therefore to the iR drop.

Thus:

$$\begin{aligned} e &= iR \\ &= I_m R \sin \omega t \text{ (from (I))} \end{aligned} \tag{4}$$

It is obvious that the voltage e is *in phase* with the current i, for when $t = 0$, $\sin \omega t = 0$ and both the current and the voltage are simultaneously going through their zero values and increasing in magnitude. When $\omega t = \pi/2$ or $90°$, $\sin \omega t = 1$, and both the current and the voltage are simultaneously at their maximum values. The voltage curve e, in phase with the current curve i, is also shown in Fig. 35.

Since both current and voltage go through their zero, maximum, and other corresponding values together, the radius vectors which determine their curves must be in phase with each other. These radius vectors are equal to the maximum values of the current and voltage (I_m and E_m) as shown in Fig. 35. (Also see Fig. 25, page 31.) If maximum values are considered, the vector diagram of the circuit is given by the two rotating vectors in Fig. 35. That is, the current and voltage are *in phase* with each other.

It is obvious that if each of these vectors is reduced in the ratio of 1 to $\sqrt{2}$, it will represent *effective* rather than maximum value. The vector diagram of the circuit when effective values are considered will then be given. This vector diagram is shown in Fig. 36 (*a*). The current and voltage vectors are arbitrarily shown as horizontal. They may be shown in any position, as in (*b*), provided they are in phase with each other.

FIG. 36.—Vector diagram with current and voltage in phase.

From equation (4) the maximum value of the current and hence of the voltage occurs when $\omega t = 90°$, and $\sin \omega t = 1$. At this instant

$$E_m = I_m R.$$

Dividing both E_m and I_m by $\sqrt{2}$, gives their *effective* values. Thus,

$$\frac{E_m}{\sqrt{2}} = \frac{I_m R}{\sqrt{2}} \quad \text{or} \quad E = IR, \tag{5}$$

where E and I are *effective* values of voltage and current. It also follows from equation (5) that

$$I = \frac{E}{R}, \tag{6}$$

which is obviously Ohm's law.

With resistance only in the circuit, the current equals the impressed voltage divided by the resistance. That is, the current obeys Ohm's law just as for a direct-current circuit.

Example.—A 12-ohm, non-inductive resistance is connected across a 110-volt (r.m.s.), 60-cycle circuit (Fig. 37). (*a*) What current flows in the resistance? (*b*) What is the maximum instantaneous value of both voltage and current? (*c*) What is the equation of the impressed voltage? (*d*) What is the equation of the current? From (6).

(*a*) $I = \frac{110}{12} = 9.17$ amp. *Ans.*

(*b*) $E_m = 110\sqrt{2} = 155.5$ volts. *Ans.*
$I_m = 9.17\sqrt{2} = 12.97$ amp. *Ans.*

(*c*) $e = 110\sqrt{2}\sin 2\pi 60t = 155.5 \sin 377t$. *Ans.*

(*d*) $i = 9.17\sqrt{2}\sin 2\pi 60t = 12.97 \sin 377t$. *Ans.*

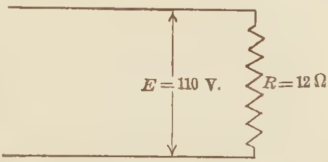


FIG. 37.—Resistance across 110 volts, alternating.

25. Effect of Inductance on Alternating-current Flow.—It was shown in Part I (page 168) that inductance has no effect on the flow of direct current, *so long as the current is steady.* If, however, the current changes, inductance does have an effect. If the current increases, inductance opposes this increase. If the current decreases, inductance opposes this decrease.

Except at the instants when it is at either its maximum positive or maximum negative value, an alternating current is varying continuously. Hence, except at the instants when the current has its maximum positive and negative values, an e.m.f. of self-induction exists in the alternating-current circuit. This e.m.f., as well as the impressed voltage, has its effect on the flow of current, and must be taken into consideration. Hence, self-inductance has an effect on the flow of alternating current, even when an ammeter in the circuit shows that the current has a constant r.m.s. value.

This fact may be illustrated by a simple experiment. Connect a double-pole, double-throw (D.-P., D.-T.) switch (Fig. 38), so that one set of clips is connected to a 110-volt, direct-current supply and the other set to a 110-volt (r.m.s.), 60-cycle, alternating-current supply. Across the blades of the switch connect a lamp bank in series with an iron-cored inductance coil. This inductance coil may be made by winding 100 to 150 turns of No. 10 or 12 magnet wire over cardboard or fiber tubing, about 2.5

in. (6.35 cm.) diameter. The iron core may be made by taping together either iron wire or sheet-steel strips. The core should be of such a size that it almost fills the tubing, but its diameter is not so large that it cannot slide readily in and out of the tube.

When the switch is thrown to the direct-current side, the lamps will come to full brilliancy almost immediately. To be sure a small time lag occurs, due to the momentary opposition of the e.m.f. of self-induction to the increase of current (see Fig. 39 (*a*)). When the direct current reaches its steady value, aside from the small resistance-drop in the winding of the inductance coil, the lamps will burn as brilliantly as if they were connected directly across the 110-volt mains.

A slight flicker may be produced in the lamps, however, by withdrawing suddenly the iron core, either completely or part

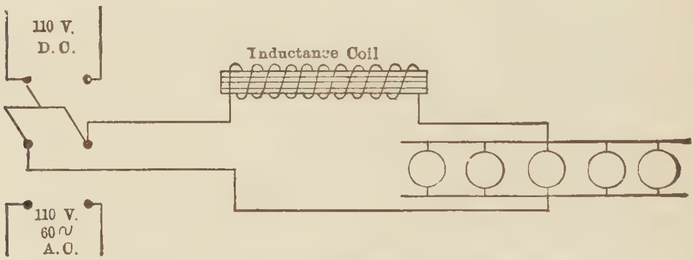


Fig. 38.—Effect of inductance on current flow.

way out of the coil. The flux linking the turns of the coil is thus suddenly reduced and an e.m.f. is generated in the winding because of the change of flux. The direction of this e.m.f. is such as *to oppose* the core being withdrawn, hence it is in conjunction with the line current which also opposes the core being withdrawn. Therefore, the lamps brighten during the short time that the core is being withdrawn (see Part I, page 166).

If the core is suddenly thrust into the coil, the lamps are dimmed momentarily. The direction of the induced e.m.f. now *opposes* the core entering the coil, hence *opposes* the line current which tends to draw the core within the coil. Hence, the lamps are dimmed for the instant.

Aside from these transient effects, however, the lamps burn equally brilliant for all positions of the core, and they are practi-

cally as bright as if they were connected directly across the blades of the switch.

If the core be well within the coil and the switch be thrown to the alternating-current side, the lamps will barely glow or may be entirely extinguished. The same effective voltage is impressed across the circuit as with the direct current and the circuit has practically the same *resistance* as before. Therefore, the large reduction in the current must be due to the choking effect of the inductance.

The fact that inductance alone is responsible for the dimming of the lamps may be demonstrated by slowly withdrawing the core from the coil. The lamps increase in brightness the further the core is withdrawn. This is due to the decrease in the inductance caused by the reduction in the flux per ampere linking the turns of the coil. This change in brightness is not a momentary effect as with direct current.

If the frequency be lowered, to 25 cycles for example, with a given position of the plunger or core the lamps burn more brilliantly, due obviously to the lesser effect of the inductance because of the decreased rate of change of current.

It will be observed from these experiments that inductance chokes or opposes the flow of alternating current. It therefore has an important effect on the value of the alternating current flowing in a circuit and must be taken into consideration, as well as the resistance.

26. Circuit Containing Inductance Only.—From the preceding paragraph it is clear that inductance must oppose the flow of the alternating current. The variation of the alternating current with time causes an e.m.f. to be induced. This induced e.m.f. *opposes* the change of current. No such e.m.f. exists when the circuit contains resistance only. Therefore, in determining the current, this e.m.f. of self-induction must be taken into consideration.

Figure 39 (a) shows the rise of current in a direct-current circuit containing resistance and inductance, when a steady voltage is impressed (see Part I, page 169, Fig. 152). The current rises slowly to its ultimate value. On the other hand, when the current attempts to decrease in the circuit, the inductance tends to prevent this decrease (Fig. 39 (b)) (see Part I, page 171, Fig.

153). In other words, if inductance is present in a circuit, it always *opposes* any change in the current. With a steady direct current, however, the inductance has no effect. If in Fig. 39 (*a*) the voltage across the inductance be decreased to a very small value or even made zero when the current reaches the value a, the current will no longer continue to increase, but rather will begin to decrease. Under these conditions there is not sufficient time for the current to approach its Ohm's law value, and the maximum value which it does reach is much less than the value which would be given by Ohm's law. If the voltage across a circuit increases and decreases with time and the voltage begins to decrease before the current has had time to reach its Ohm's law value, the current is no longer given by the impressed voltage divided by the resistance, as with a steady direct current. This

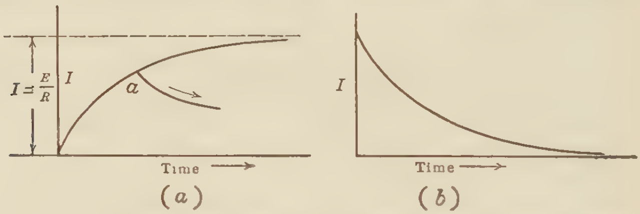


FIG. 39.—Increase and decrease of current in an inductive circuit.

effect occurs in alternating-current circuits. The impressed voltage is varying sinusoidally with time. With inductance in the circuit, the current does not have time to reach its Ohm's law value either positively or negatively before the voltage reverses its direction. The current change is opposed by the e.m.f. of self-induction, which at any instant is equal in volts to $-Li/t$, where L is the inductance in henrys and i/t is the rate in amperes per second at which the current is changing at that instant. The minus sign signifies that this voltage is opposing the change in the current (see Part I, page 167).

Figure 40 shows a current curve i. Starting at a the current is *changing* at its maximum rate in a positive direction. Therefore, at this instant the e.m.f. of self-induction must be at its negative maximum value as is shown by the ordinate of the curve $-e$. At point b, the current is at its maximum positive value. A tangent drawn to the curve at this instant is horizontal and,

therefore, at this instant the current is not changing at all. Hence, the e.m.f. of self-induction is zero as is shown by the curve $-e$ crossing the zero axis. At c the current is changing at its maximum rate negatively and the e.m.f. self-induction must be maximum positive. Continuing in this manner the voltage curve $a'b'c'$ is obtained. It will be observed that this is a sine curve and is *lagging* the current by 90°.

This is the only voltage in the circuit which *opposes* the change of current. It corresponds to the back e.m.f. of a motor in that the line must supply a voltage opposite and equal to the back e.m.f. before any current can flow into the armature. This

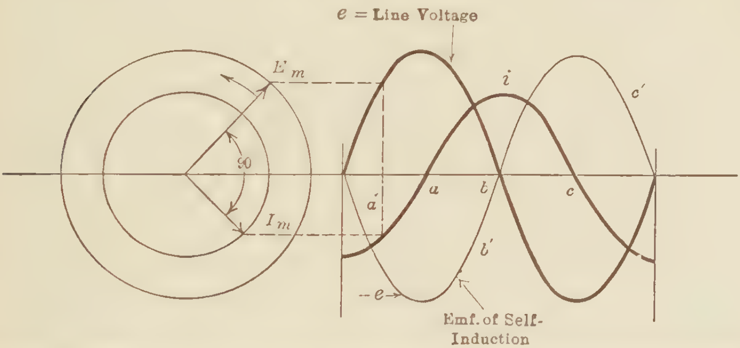


FIG. 40.—Current and voltages existing in an alternating-current circuit containing inductance only.

same condition exists in the alternating-current circuit. Before any current can flow in a circuit containing inductance, but no resistance, a voltage opposite and equal to the e.m.f. of self-induction must be supplied by the line.

Therefore, in Fig. 40 the voltage e, which is the line voltage, is opposite and equal to the e.m.f. of self-induction.

It will be noted that the *impressed* voltage *leads* the current by 90°, or the current *lags* this voltage by 90°. With inductance only in the circuit, the current *lags* the impressed voltage by 90°. (In practice it is impossible to obtain a pure inductance, as inductance must necessarily be accompanied by a certain amount of resistance.)

In Fig. 40 are shown the radius vectors E_m and I_m which determine the sine curves e and i. Since the impressed voltage

e leads the current i by 90°, the radius vector E_m must lead the radius vector I_m by 90° as shown (see Fig. 25, page 31). Therefore, the radius vectors E_m and I_m together give the vector diagram of this circuit if *maximum* values are under consideration. If a vector diagram involving *effective* values be desired, it is only necessary to reduce E_m and I_m in the ratio of $1:\sqrt{2}$. Such a diagram to a larger scale is shown in Fig. 41. The voltage vector E has arbitrarily been taken along the horizontal axis. The current vector I lags E by 90°.

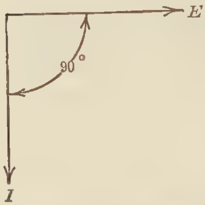


FIG. 41.—Vector diagram with inductance only in circuit.

It is possible to determine the choking effect of inductance quantitatively. The e.m.f. of self-induction is obviously proportional to the *inductance* and to the *rate of change of current*. The rate of change of current is proportional to the *current* and to the *frequency or to* ω which is 2π times the frequency. It may be shown that the e.m.f. of self-induction $-E$ in an alternating-current circuit is given by

$$(-E) = -L(2\pi f)I = -L\omega I \text{ volts.} \tag{7}$$

The line voltage E which is opposite and equal to this e.m.f. of self-induction is given by

$$E = 2\pi fLI = L\omega I = IX_L, \tag{8}$$
$$\text{where } X_L = 2\pi fL.$$

Solving for the current

$$I = \frac{E}{2\pi fL}, \tag{9}$$

Since $2\pi f = \omega$,

$$I = \frac{E}{L\omega} = \frac{E}{X_L} \tag{10}$$

$2\pi fL = L\omega = X_L$ is the *inductive reactance* of the circuit. Since the current is equal to the voltage E divided by the inductive reactance X_L, the inductive reactance must be expressed in *ohms*.

Example.—Figure 42 shows a pure inductance of 0.2 henry connected across 110-volt (r.m.s.), 60-cycle mains. What current flows?

$$X_L = 2\pi 60 \times 0.2 = 75.4 \text{ ohms.}$$

Using Eq. (10), $I = \dfrac{110}{75.4} = 1.46$ amp. (r.m.s.) *Ans.*

Figure 43 shows the vector diagram of this circuit. In this diagram the current vector I is arbitrarily shown along the horizontal axis. The line voltage $E = IX_L = 110$ volts *leads* the current by 90°. Compare this diagram with Fig. 41. Both diagrams are correct since each shows the voltage leading the current by 90°.

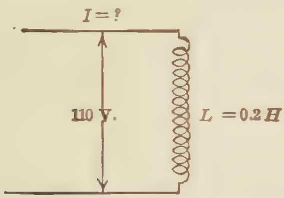


FIG. 42.—Circuit containing inductance only.

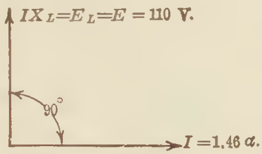


FIG. 43.—Vector diagram for circuit containing inductance only.

Example.—A 25-cycle current of 4.0 amp. flows through an inductance of 0.05 henry. What is the voltage across the inductance and what is its phase relation to the current?

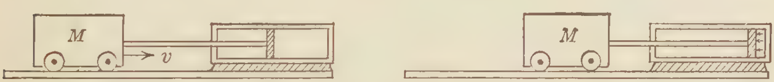
$$X_L = 2\pi\ 25 \times 0.05 = 7.85 \text{ ohms.}$$

Using Eq. (8),

$$E = 4.0 \times 7.85 = 31.4 \text{ volts.} \quad \textit{Ans.}$$

E leads I by 90°. *Ans.*

27. Mechanical Analogue of Inductance.—It is clear that inductance always opposes *change* of current. It opposes increase in current. If a current is already established it opposes any decrease in current. In Part I (see page 171) it was stated that



(a) Pressure Zero; Velocity Maximum (b) Pressure Maximum; Velocity Zero

FIG. 44.—Mechanical analogue of inductance.

inductance in the electrical circuit corresponds to inertia in mechanics. The mechanical analogue of inductance may be illustrated by a system consisting of a mass M (Fig. 44) on frictionless wheels and rails working in conjunction with a piston and frictionless cylinder. The mass is connected rigidly to the piston rod. The cylinder is filled with a compressible fluid such as a gas and is closed at both ends. When the piston is at the center of the cylinder, the gas pressures on both sides of the piston

are made equal. The entire system is frictionless so that no energy is dissipated. (With pure inductance no energy is dissipated, as will be shown later.)

Electromotive force in the electric circuit corresponds to pressure in mechanics and hydraulics (see Part I, page 53). Current (rate of flow of quantity) in the electric circuit corresponds to velocity in mechanics. A study of Fig. 44 shows that when the system is set in oscillation, the mass, because of its momentum, will compress the gas in one end of the cylinder and the gas in the other end will be rarefied. When the kinetic energy of the mass is all utilized in so changing the gas pressure, its velocity becomes zero. The compressed gas which is then at its maximum pressure expands and so accelerates the mass in the reverse direction. The velocity of the mass M reaches a maximum at the center of the stroke and then begins to decrease owing to the compression of the gas in the other end of the cylinder. The maximum velocity v of the mass occurs at the instant when the piston is at the center of the cylinder as in (a). At this instant the resultant pressure acting on the piston is zero, since the pressures on both sides are the same. Also, at this instant the kinetic energy of the mass is a maximum and equal to $W = \frac{1}{2} Mv^2$ where M is the mass and v its velocity.

At the instant when the moving system reaches the right-hand extremity of its movement (Fig. 44 (b)) its velocity is zero. The maximum pressure during this half of the stroke occurs at this instant, since the gas between the piston and the right-hand cylinder head is compressed into its minimum volume. The gas on the opposite side of the piston is at its maximum volume and lowest pressure. At this instant the kinetic energy of the mass is zero, since its velocity is zero.

A study of Fig. 40 shows that when the current (electric velocity) is a maximum, the e.m.f. (electric pressure) is zero. At this instant the energy stored in the magnetic field is a maximum. ($W = \frac{1}{2}Li^2$. See Part I, page 173.) When the current (electric velocity) is zero (Fig. 40) the e.m.f. (electric pressure) is a maximum.

The inertia of the mass also opposes motion. When the gas, in expanding, attempts to impart velocity to the mass, inertia *opposes* the increase in velocity. When the gas, by compressing,

attempts to decrease the velocity of the mass, the inertia of the mass opposes this decrease. In the electric circuit, inductance likewise oppposes both increase and decrease of current.

Therefore, an oscillating mechanical system having inertia, and an alternating electric system having inductance are similar as regards the relations among velocity and current, pressure and voltage, and energy.

28. Circuit Containing Capacitance Only.[1] –In Fig. 45 is shown a cylindrical tank T_1, of considerable volume, which may be supplied with water through a large pipe P_1 connected into its bottom. A smaller cylindrical tank T_2, whose bottom is at

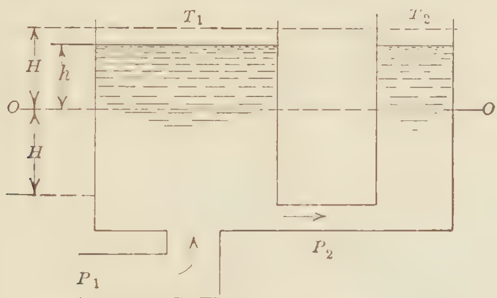


FIG. 45.—Flow of water between tanks.

the same level as that of T_1, is connected to T_1 by a frictionless pipe P_2.

If water is supplied to T_1 by the pipe P_1, the water level in T_2 will always be the same as the water level in T_1 if the inertia of the water be neglected. That is, the hydrostatic pressure in the two tanks is always the same. If a condenser is connected across electric mains, it develops a counter e.m.f. q/C, where q is its charge in coulombs and C is its capacitance in farads (see Part I, page 195, equation 73). At every instant this counter e.m.f. of the condenser must be equal to the line voltage, provided the connecting leads have negligible resistance.

Let the horizontal plane oo be the datum plane, that is, the plane from which the water levels in the two tanks are determined. Assume that by controlling the rate of flow of water

[1] See Part I, Chap. IX, for a discussion of capacitance. Also see Part I, p. 196, for a description of condensers.

through P_1, the water level h in T_1 referred to the datum plane oo is made to vary sinusoidally with time. The *rate of increase* of water level h will be a maximum when h is zero. (Examine any sine curve.) Therefore, the rate *of flow* of water through the pipe P_2 is a maximum at this instant. That is, the hydraulic current is a maximum when the pressure referred to the datum plane is zero. The direction of flow is also positive in that the water flows from tank T_1 to tank T_2.

As the height h in T_1 increases, its *rate of increase* diminishes and the rate of flow of water in pipe P_2 diminishes. The rate of flow of water in pipe P_2 becomes *zero* when the height h of water in tanks T_1 and T_2 reaches its *maximum* value H. At this instant the pressure in tank T_2 is a maximum. Thus it is seen that the current flow in P_2 reaches its maximum value a quarter cycle before the pressure in tank T_2 reaches its maximum value. That is, the current or flow of water may be said to *lead* the pressure by 90°.

When the water level in T_1 reaches its maximum value, the flow of water in P_2 is zero for an instant. When the water level in tank T_2 begins to diminish, the current in P_2 flows from tank T_2 to T_1. That is, its flow is reversed in sign and becomes negative. This negative direction of flow continues, not merely until the water level h is zero, but until h is at its maximum negative value $(-H)$. The flow then becomes zero for an instant, after which it again begins to flow in the positive direction.

Electricity is stored in a condenser in much the same manner that water is stored in the tank T_2. For example, when a direct-current voltage is impressed across the plates of a condenser (Part I, page 193), there is an initial rush of current which charges the condenser to line potential. After this there is no further flow of current if the line voltage remains constant, since the condenser develops a counter e.m.f. q/c opposite and equal to the line voltage. If the condenser plates are now short-circuited, making the voltage across the plates zero, current flows out of the condenser.

Figure 46 (a) shows an alternating voltage e impressed across the plates of a condenser C. When the voltage starts from its zero value at a (Fig. 46 (b)), and increases positively, current

flows *into* the condenser. Therefore, this current is positive, just as the flow of water was positive (Fig. 45) when the pressure in tank T_1 was increasing. As long as the voltage across the condenser plates continues to *increase*, current must flow *into* the condenser from the positive wire and this current will be positive in sign. When point b is reached, the increase of voltage ceases and the current becomes zero, just as the flow of water ceases (Fig. 45) at the instant of maximum pressure H.

Between b and c the voltage is decreasing so that current is flowing *out of* the condenser into the positive line, and as the current flow has reversed, the sign of the current is now negative. This corresponds to the reversal of water flow in P_2 (Fig. 45),

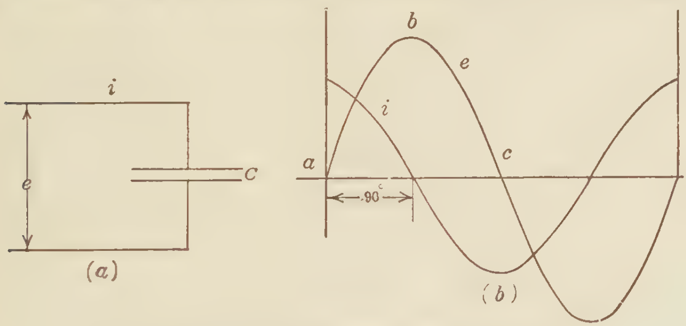


FIG. 46.—Circuit containing capacitance only.

when the pressure in tank T_1 was decreasing. After e passes through zero at c, the e.m.f. is negative and charges the condenser in the opposite direction, so the current still remains negative. This continues until the voltage reaches its negative maximum. At this point the current reverses and again becomes positive.

An examination of Fig. 46 shows that when an alternating voltage is impressed on a condenser, the current *into* the condenser *leads* the voltage by 90°. It may appear remarkable that current can lead its voltage in this manner, but in Fig. 45 the current flow through pipe P_2 reached its maximum value before the pressure in tanks T_1 and T_2 reached their maximum values.

It will be seen from the foregoing that alternating current does not actually flow conductively through the insulation of the condenser. A perfect condenser offers an infinite resistance to alternating, as well as to direct current. However, with alternat-

ing current the condenser is alternately charged and discharged, so that a quantity of electricity flows into the positive plate, and then out again, etc. It is this quantity of electricity which flows to charge and to discharge the condenser which constitutes the alternating current. An ammeter placed in the line to such a condenser indicates a current. This is analogous to water flowing in the pipe P_2 (Fig. 45), when the water level in tank T_1 is alternately raised and lowered, even if tank T_2 is not leaky. A suit-

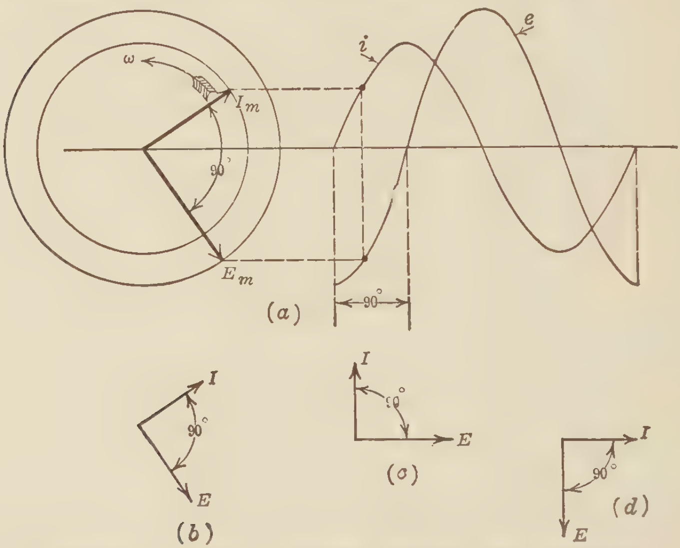


Fig. 47.—Current and voltage and vector diagrams with capacitance only in circuit.

able indicating instrument, if such could be designed, would assume a steady deflection with the alternating water flow in the pipe P_2.

Figure 47 (a) shows the vectors E_m and I_m rotating at angular velocity ω, from which the sine curves of voltage and current are derived for the circuit containing capacitance only. Since the current leads the voltage by 90°, the vector I_m leads E_m by 90°. If a vector diagram involving maximum values of voltage and current is desired, it is given by these two rotating vectors E_m and I_m, the current leading the voltage by 90°. A vector diagram involving *effective* values of voltage and current is

ordinarily more useful. To obtain this diagram it is merely necessary to divide E_m and I_m (Fig. 47 (a)) by $\sqrt{2}$, and the vector diagram given in Fig. 47 (b), involving E and I in effective values, is obtained. Merely for convenience, either the current or the voltage vector may be shown horizontal. In Fig. 47 (c) the voltage E is shown horizontal and the current vector leads it by 90°. It would have been equally correct to have shown the current vector horizontal and the voltage vector lagging it by 90° (Fig. 47 (d)).

29. Quantitative Values of Current in Circuit Containing Capacitance Only.—With inductance only in circuit, the current can be determined quantitatively, provided the voltage, the inductance, and the frequency be known. In a similar manner, the current in a circuit containing capacitance only can be determined, provided the voltage, the capacitance, and the frequency be known.

Obviously, the current is proportional to the voltage, since the quantity which charges the condenser each half-cycle is proportional to the voltage ($q = Ce$). Also, the current must be proportional to the capacitance, since the quantity which charges the condenser each half-cycle is also proportional to the capacitance ($q = Ce$). The current must also be proportional to the frequency, since both the positive and the negative quantity per second is proportional to the number of times that the condenser is charged per second. Actually, the current in r.m.s. amperes is equal to the product of the voltage, capacitance, and angular velocity ω. Thus

$$I = 2\pi fCE = EC\omega \tag{11}$$

where E is in volts (r.m.s.), C is in *farads* (not microfarads), and ω is $2\pi f$ where f is the frequency in cycles per second.

Example.—How many amperes will a 5.0 microfarad (μf.) condenser take when connected across 100-volt, 60-cycle mains? Draw a vector diagram for this circuit.

$C = 0.000005$ farad
$\omega = 2\pi 60 = 377$
$I = 100 \times 0.000005 \times 377 =$ 0.1 885 amp. *Ans.*

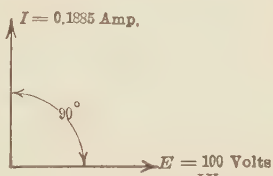


Fig. 48.—Vector diagram of a circuit containing a $5\mu f$ condenser.

The vector diagram is shown in Fig. 48, where the 0.1885-amp. vector is shown leading the 100-volt vector by 90°.

In eq. (11), $C\omega$ is in the nature of conductance or mhos, since it is *multiplied* by the voltage to obtain the current. In order that it may be combined with reactance or ohms its reciprocal must be used. That is, if $C\omega$ gives mhos, $1/C\omega$ gives ohms.

The current then becomes,

$$I = \frac{E}{\dfrac{1}{C\omega}} \tag{12}$$

$1/C\omega$ is called the *capacitive* or *condensive reactance* of the circuit, just as with inductance the reactance is called the inductive reactance. Condensive reactance is expressed by the symbol X_C.

Thus,

$$X_c = \frac{1}{C\omega} \text{ (ohms).} \tag{13}$$

Example.—In the foregoing problem find the condensive reactance of the circuit and also the current.

$$X_C = \frac{1}{0.000005 \times 377} = \frac{1}{0.001885} = 530 \text{ ohms.} \quad \textit{Ans.}$$

$$I = \frac{100}{530} = 0.1885 \text{ amp.} \quad \textit{Ans.}$$

Since most commercial condensers are rated in microfarads, rather than in farads, it simplifies computations to use the following equation:

$$X_c = \frac{1{,}000{,}000}{C'\omega} \tag{14}$$

where C' is in microfarads.

Example.—Find the condensive reactance of a 40μf. condenser when the frequency is 50 cycles per second.

Using eq. (14)

$$X_C = \frac{1{,}000{,}000}{40 \times 2\pi 50} = 79.6 \text{ ohms.} \quad \textit{Ans.}$$

It follows from (12) and (13) that

$$I = \frac{E}{X_C} \tag{15}$$

$$E = IX_C \tag{16}$$

$$X_C = \frac{E}{I} \tag{17}$$

Example.—In the foregoing problem, what 50-cycle voltage across the 40μf. condenser will maintain a current of 2.0 amp.?

Since X_C = 79.6 ohms, from (16),

$$E = 2.0 \times 79.6 = 159.2 \text{ volts.} \quad \textit{Ans.}$$

This voltage *lags* the current by 90°.

POWER IN ALTERNATING-CURRENT CIRCUITS

30. Alternating-current Power.—So far, the current and voltage relations have been considered in the three fundamental alternating-current circuits consisting of resistance alone, inductance alone, and capacitance alone. Before combining such circuits by connecting them in series and in parallel, the power relations in each individual circuit will be considered. Under steady conditions the power in a direct-current circuit is always given by the product of the volts across the circuit and the current in amperes flowing in the circuit (see Part I, page 61). This same rule applies to alternating-current circuits, provided the *instantaneous* values of amperes and volts are considered. The *average* power, however, is not necessarily the product of the effective volts and effective amperes, the values which are ordinarily measured with instruments. For example, a voltmeter across a circuit reads 120 volts and an ammeter in the circuit reads 4.0 amp. It is only under special conditions that the power is equal to 120 $\times$ 4.0 or 480 watts.

31. Power with Voltage and Current in Phase.—Figure 49 shows a voltage wave e and a current wave i in phase with each other, such as occurs when a circuit contains resistance only. To obtain the power *at any instant*, the amperes and volts at that instant are multiplied together and a new curve p may be plotted, the ordinates being the instantaneous products of e and i.

Assume that at the instant shown at a (Fig. 49) the current represented by the ordinate ab is 12 amp.; the voltage represented by the ordinate ac is 80 volts. The power at that instant is represented by their product, or 960 watts, and is given by the ordinate ad. Point d is one point on the power curve. Other points are similarly found, and the power curve p is plotted. The curve p gives the power in the circuit at any instant. The ordinates of this power curve are positive during the first half-cycle, since both voltage and current are positive throughout this

period. During the second half-cycle both voltage and current are negative. The power must still be positive, however, since the voltage and current are still in conjunction. If both voltage and current reverse simultaneously, the power must still be positive. In other words, during both the first and the second half-cycle the voltage and the current act in conjunction and the power is positive. The ordinates of the power curve are positive, therefore, throughout the cycle.

It will be noted that this power curve is a sine wave having double the frequency of the voltage or current. For every

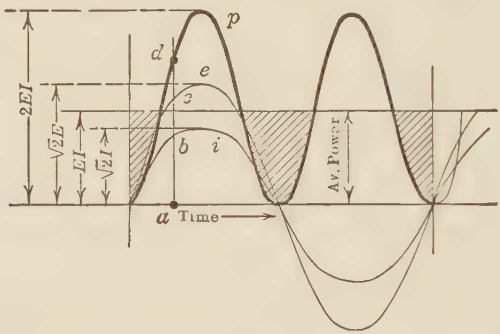


FIG. 49.—Power curve, current and voltage in phase.

cycle of either voltage or current, the power curve touches the zero axis twice, so that in such a circuit the power is zero twice during each cycle. Since the voltage and current waves are in phase, their peaks occur at the same instant, and the corresponding peak of the power curve is, therefore,

$$(\sqrt{2}E)\ (\sqrt{2}I) = 2EI,$$

where E and I are the *effective* values of voltage and current.

This power curve has its horizontal axis of symmetry at a distance EI above the zero axis. Consequently, EI must be the *average* value of the power during each cycle, since the upper half-waves will just fill the shaded valleys below the axis of symmetry of the power curve. When the current and the voltage are *in phase*, the average power is given by their product, as with direct currents.

Example.—An incandescent-lamp load takes 30 amp. from 115-volt, 60-cycle mains. (In this type of load the current and voltage are substantially in phase.) How much power do the lamps consume?

$$P = EI = 115 \times 30 = 3{,}450 \text{ watts.} \quad \textit{Ans.}$$

32. Positive and Negative Power.—In Fig. 50 is shown a 3-cell storage battery whose terminal voltage is 6 volts, as is indicated by the voltmeter connected across its terminals. This battery delivers current to a resistance R connected across its terminals. The e.m.f. of the battery obviously acts in such a direction as to send current *out* of its positive terminal. This is shown by the zero-center ammeter, connected in the line, the deflection being to the right. The current through the resistance

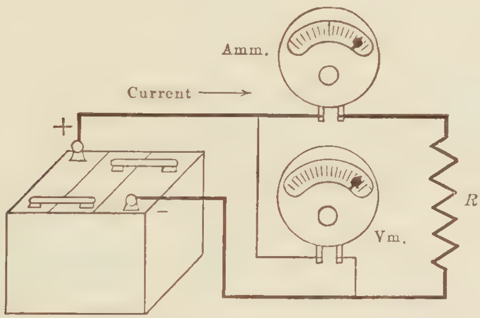


FIG. 50.—Battery delivering energy—its power is positive.

flows from the battery positive terminal to the resistance R connected across the battery terminals. That is, so far as the external circuit is concerned, both the voltage and current are acting in conjunction. If the battery is considered a source of energy, as it is under these conditions, the current is *positive* and the power delivered to the resistance is *positive.*

In Fig. 51, the 6-volt storage battery is now connected across the terminals of a generator, the positive terminal of the generator being connected to the positive terminal of the battery and the negative terminal of the generator being connected to the negative terminal of the battery. (Compare this figure with Fig. 73, Part I, page 74.) If the e.m.f. of the generator is just equal to the e.m.f. of the battery, no current flows and the battery is merely "floating." If the e.m.f. of the generator be lowered,

current flows from the positive terminal of the battery to the positive terminal of the generator, and the ammeter will deflect to the right as before. The current flow is therefore in conjunction with the battery e.m.f. and the power is positive. The battery is delivering energy to the generator and tending to drive it as a motor. That is, so far as the relations of voltage, current, and power are concerned, the conditions are identical with those given in Fig. 50, with resistance only across the battery terminals.

If now, the generator e.m.f. be raised so that it exceeds the e.m.f. of the battery, current will flow from the generator positive terminal *into* the battery positive terminal. This will be indi-

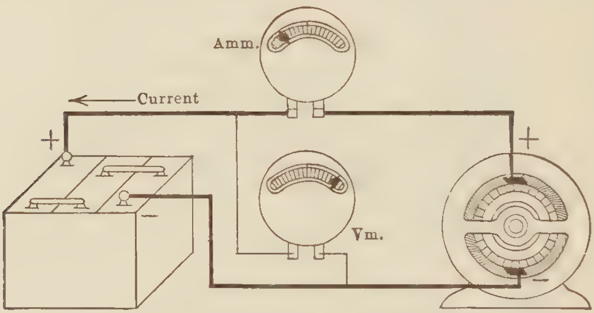


FIG. 51.—Battery receiving energy—its power is negative.

cated by the zero-center ammeter now deflecting to the left, as shown in Fig. 51. The battery e.m.f. has *not* reversed in sign as is shown by the voltmeter connected across the battery terminals still reading up scale. The direction of flow of the *current*, however, has reversed, and is now negative. The battery is now being charged and is, therefore, receiving energy. However, the battery is still considered a source of energy, therefore, it must be *delivering negative* power. The current and voltage are now in opposition, that is, they are of opposite sign, and the power is negative.

(If the battery is considered as a translating device or a device for *receiving* energy, the power is positive when the current flows *in* at its positive terminal, and negative when it flows *out* of its positive terminal.)

33. Power with Inductance Only.—With inductance only in circuit, the impressed voltage leads the current by 90° (Fig. 40). Figure 52 shows curves of voltage and current with inductance only in circuit, the voltage e leading the current i by 90°. To determine the variation of power with time, the current and voltage at each instant are multiplied together, as was done in Fig. 49.

At points a, b, c, d and e (Fig. 52), either current or voltage is zero, and the power must be zero at each of these instants. Between a and b the voltage is positive and the current is negative, and they are therefore acting in *opposition*. That is, the current is flowing against the impressed voltage in a manner similar to that shown in Fig. 51. Also, the product of a positive and a negative quantity is negative. Hence, the power between points a and b must be *negative*. This means that the inductance is delivering energy to the source of supply, just as the generator (Fig. 51), delivered energy to the battery which was considered a source of energy. Between points b and c both the current and the voltage are *positive* and are therefore in conjunction. Consequently, the power between these two points must be *positive*. That is, the source is supplying energy to the inductance. Between c and d the current is positive, but the voltage is now negative. That is, the current and voltage are again in opposition. Therefore, the power is again negative between these two points. Between d and e both the current and the voltage are negative and are therefore in conjunction, so that the power is now positive. The power curve p, obtained by multiplying the voltage and current at each point, is a sine curve having double the frequency of current or voltage. Its axis of symmetry coincides with the axis of current and voltage, therefore, there must be as much of the power curve above the zero axis as there is below this axis, or the positive area above the axis must be equal to the negative area below the axis for each cycle. That is, all

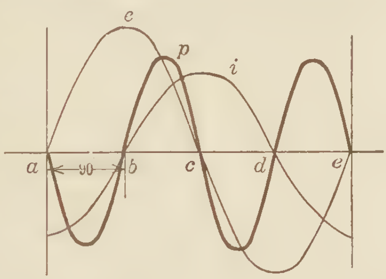


FIG. 52.—Power curve; current and voltage in quadrature, current lagging.

the energy received by the inductance from the source of power is returned by the inductance to the source of power. The average power over each cycle, therefore, is zero. Between points *b* and *c* and *d* and *e* when the power is positive, the source of power is storing energy in the inductance ($\frac{1}{2}Li^2$). Between points *a* and *b* and *c* and *d* when the power is negative, the inductance is returning this energy to the source of power.

The power relations with inductance in the circuit are not unlike a single-cylinder, compressed-air engine having no losses. During part of the cycle, the compressed air entering the cylinder gives energy to the shaft and the flywheel. Subsequently, the flywheel and shaft, due to their inertia, return this energy to the cylinder by compressing the air in the cylinder. It has already been stated that inductance in the electric circuit corresponds to inertia in mechanics.

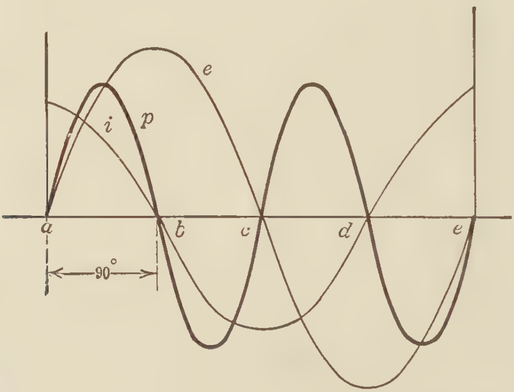


FIG. 53.—Voltage, current, and power curves; circuit containing capacitance only.

34. Power with Capacitance Only.—With capacitance only in circuit, the impressed voltage lags the current by 90° as was shown in Fig. 47. Figure 53 shows curves of voltage and current with capacitance only in circuit, the voltage *e* lagging the current *i* by 90°. The power curve is determined by multiplying at each instant the voltage ordinate by the current ordinate, at that instant.

As in Fig. 52, either voltage or current is zero at the instants a, b, c, d, and e. Hence, at each of these instants the power is zero. Between points a and b, both the voltage and current are positive, hence are acting in conjunction, so that the power is positive. Between points b and c the voltage is positive but the current is negative. That is, voltage and current are in opposition, so that between these points the power is negative. Between points c and d, the voltage and current are both negative, hence are acting in conjunction, and between these points the power is positive. Between points d and e the voltage is negative, the current is positive, and the power is negative.

The power curve p is a sine curve having a frequency double that of voltage or current. Its axis of symmetry coincides with the axis of voltage and current. Hence, the positive area of the power curve must be equal to the negative area, so that over a complete cycle the *net or average power is zero.* Between a and b and c and d when the power is positive, the source of power is storing energy in the condenser ($\frac{1}{2}Ce^2$). Between points b and c and d and e when the power is negative, the condenser is returning energy to the source of supply. With a perfect condenser, such as is assumed, the condenser returns to the source of power during each cycle all the energy which it receives. Therefore, the average power over a complete cycle must be zero.

These power relations in the electric circuit are not unlike the action of a compressed-air cylinder operating on a spring. During part of the cycle, the expansion of the air in the cylinder compresses the spring. If there are no losses, the spring returns all this energy to the cylinder by re-compressing the air during the other part of the cycle. Capacitance in the electric circuit corresponds to elasticity in mechanics.

A comparison of Fig. 53 with Fig. 52 shows that when the power is positive with capacitance, it is negative with inductance, as between points a and b, c and d, etc. That is, during those times when energy is being stored in the condenser, the inductance is delivering energy to the source, etc. By proper adjustments, the power taken by the inductance may be made equal to the power taken by the capacitance at every instant. Since the power taken by the inductance and capacitance is always of opposite sign, the power supplied by the source at every instant

must be zero. As it is impossible to have a perfect inductance or a perfect capacitance, this condition of zero power can only approximately be realized.

35. Power with Phase Difference Other Than 90°.—It has been shown that the power is the product of the effective voltage and current when voltage and current are in phase, and that the power is zero when voltage and current are in quadrature. When voltage and current are neither in phase nor in quadrature, but differ in phase by an angle θ, which is greater than zero and less than 90°, the average power is less than the product of the volts and amperes, but is greater than zero. This is shown in Fig. 54 where the current lags the voltage by an angle θ. (Such a condition occurs when resistance and inductance are connected either

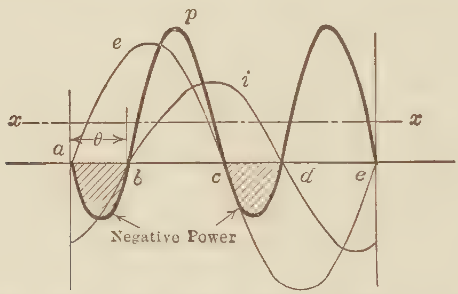


Fig. 54.—Power curve; current lagging voltage by angle θ.

in series or in parallel (see Pars. 36 and 44).) The power curve p is the product of the volts and amperes at each instant. As in Figs. 52 and 53, at a, b, c, d and e, either the voltage or the current is zero and hence the power is zero at each of these instants. Between a and b, and between c and d, voltage and current are in opposition, and the power is negative. Between b and c, and between d and e, the voltage and current are in conjunction, and the power is positive. During the intervals of time bc and de, the source of power delivers energy to the circuit, this energy being represented by the areas under the positive loops of the power curve. During the intervals of time ab and cd, the circuit is delivering energy to the source of power, this energy being represented by the shaded areas. The power relations in such a circuit are not unlike the action of a single-cylinder, internal-

combustion engine. During that part of the cycle when the engine is firing, the cylinder gives energy to the shaft and flywheel. During the compression part of the cycle, the flywheel returns *some* of this energy to the cylinder. Over the complete cycle, however, the cylinder delivers more energy to the shaft than is returned by the flywheel.

The power curve p is a double-frequency curve, as before, but its axis of symmetry, xx, is above the zero axis ae. A study of Fig. 54 shows that the unshaded areas under the power curve and above the zero axis are greater than the shaded areas below the zero axis. Hence, over a complete cycle there is more positive energy than negative energy. That is, the average power is not zero, but is positive, and is less than the product of E and I. It will be shown later that this power

$$P = EI \cos \theta \tag{18}$$

where θ is the phase angle between voltage and current. Cos θ is called the *power-factor* of the circuit. P is the true watts and EI the apparent watts, or volt-amperes.

The power-factor

$$\text{P.F.} = \cos \theta = \frac{\text{true watts}}{\text{apparent watts}} = \frac{P}{EI} \tag{19}$$

The power-factor can never be greater than unity.

Example.—A single-phase motor takes 4.5 amp. at 220 volts, the current lagging. The wattmeter indicates 720 watts. (*a*) What is the power-factor of the circuit? (*b*) By what angle does the current lag the voltage?

(*a*) From equation (19)

$$\text{P.F.} = \frac{720}{220 \times 4.5} = \frac{720}{990} = 0.728. \quad \textit{Ans.}$$

(*b*) Cos θ = 0.728. From Appendix D,

$$\cos 43.3' = 0.7278$$
$$\theta = 43.3°. \quad \textit{Ans.}$$

Alternating-current Circuits with Resistance, Inductance, and Capacitance in Series.

36. Resistance and Inductance in Series.—Thus far only special cases of alternating-current circuits have been considered, such as circuits containing resistance only, inductance only, and capacitance only. It is obvious that these three quantities may be connected in various series and parallel combinations. First consider a resistance R and an inductance L in series (Fig. 55).

A voltage E is impressed across the circuit and as a result a current I flows. Let it be required to determine the relations existing among the current, voltage, power, resistance and inductance. Figure 56 (a) shows a vector diagram for this circuit. As this is a series circuit the current I is the same in both X_L and R at every instant. Hence one current vector suffices for the circuit. This current vector is laid off horizontally to scale. The position of the current vector I is arbitrary. (It is given the position shown merely for convenience.) The voltage across the resistance E_R must be equal to IR as has already been shown (see Par. 24 and Fig. 36, page 40). This voltage $E_R = IR$ is

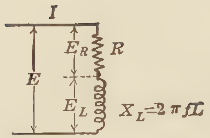


Fig. 55.—Circuit containing resistance and inductance in series.

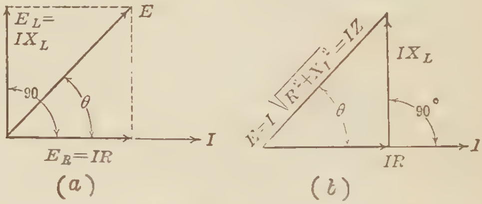


Fig. 56.—Vector diagram for a series circuit containing resistance and inductance.

laid off along the current vector I (Fig. 56 (a)). The voltage across the inductance E_L must be equal to IX_L ($= 2\pi fLI$) and it must also *lead* the current by 90°. This was shown in Par. 26 and in Fig. 43. Therefore, the voltage across the inductance, $E_L = IX_L$, is laid off at right angles to the current and leading it by 90° (Fig. 56 (a)).

The line voltage E must be equal to the sum of its component parts, E_R and E_L. Since E_R and E_L are not in phase, but are in quadrature, the line voltage E is the *vector* sum of E_R and E_L. The vector addition is shown in Fig. 56 (a). It is obvious that the voltage E is the hypotenuse of a right triangle, of which $IR(= E_R)$ and $IX_L(= E_L)$ are sides.

Therefore,

$$E = \sqrt{(IR)^2 + (IX_L)^2}$$
$$= \sqrt{I^2(R^2 + X_L^2)} = I\sqrt{R^2 + X_L^2}. \qquad \text{(I)}$$

Solving equation (I) for the current,

$$I = \frac{E}{\sqrt{R^2 + X_L^2}} = \frac{E}{\sqrt{R^2 + (2\pi fL)^2}} = \frac{E}{Z}. \qquad (20)$$

$Z = \sqrt{R^2 + X_L^2}$ is the *impedance* of the circuit and is expressed in ohms. It is ordinarily denoted by Z. Equation (20) corresponds to Ohm's law for the direct-current circuit. The current in an alternating-current circuit is directly proportional to the voltage across the circuit and inversely proportional to the impedance of the circuit. That is, if the voltage in volts be divided by the impedance in ohms, the value of the current is obtained in amperes.

Also the voltage

$$E = IZ \qquad (21)$$

and the impedance

$$Z = \frac{E}{I}. \qquad (22)$$

An inspection of Fig. 56 shows that the angle θ by which the current lags the voltage may also be determined. Since the tangent of an angle is the ratio of the opposite to the adjacent side (see page 406).

$$\tan\theta = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{2\pi fL}{R}. \qquad (23)$$

Since the cosine of an angle is the ratio of the adjacent side to the hypotenuse, in Fig. 56,

$$\cos\theta = \frac{IR}{\sqrt{(IR)^2 + (IX_L)^2}} = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{Z} \qquad (24)$$

Figure 56 (b) shows the vector addition performed by the triangle method, rather than by the parallelogram method (see page 27). That is, IX_L is added to the end of IR and at right angles to IR, leading. It is obvious that both methods give the same result.

To illustrate the method of computing the current in a circuit having resistance and inductance in series the following example is given.

Example.—A circuit containing 0.1 henry inductance and 20 ohms resistance in series is connected across 100-volt, 25-cycle mains. (a) What is the impedance of the circuit? (b) What current flows? (c) What is the

voltage across the resistance? (*d*) What is the voltage across the inductance? (*e*) Determine the angle by which the voltage leads the current.

$X_L = 2\pi 25 \times 0.1 = 157 \times 0.1 = 15.7$ ohms (from equation (8), page 46).

(*a*) $Z = \sqrt{(20)^2 + (15.7)^2} = \sqrt{646} = 25.4$ ohms. *Ans.*

(*b*) $I = \dfrac{E}{Z} = \dfrac{100}{25.4} = 3.94$ amp. *Ans.*

(*c*) $E_R = IR = 3.94 \times 20 = 78.8$ volts. *Ans.*

(*d*) $E_L = IX_L = 3.94 \times 15.7 = 61.8$ volts. *Ans.*

As a check $\sqrt{(78.8)^2 + (61.8)^2} = 100$ volts.

(*e*) $\tan \theta = \dfrac{X_L}{R} = \dfrac{15.7}{20} = 0.785.$

From page 412, $\theta = 38.1°$. *Ans.*

37. Power with Resistance and Inductance in Series.—In Fig. 55, the power dissipated in the resistance, is I^2R. It has already been shown that the average power taken by a pure inductance over a complete cycle is zero. Hence, the inductance (Fig. 55) consumes no power. All the power taken by the circuit must therefore be accounted for in the resistance. The total power

$$P = I^2R = I(IR). \tag{I}$$

In the vector diagram (Fig. 56)

$$\cos \theta = \frac{IR}{E}$$

$$IR = E \cos \theta$$

Substituting for IR in (I),

$$P = I(E \cos \theta) = EI \cos \theta \tag{25}$$

This equation is the same as equation (18) on page 63. Cos θ is the *power-factor* of the circuit. The cosine of an angle, and hence the power-factor, can never exceed unity. When θ is greater than zero, the cosine is less than unity (see curve, page 8). Therefore, the power-factor is unity when the current and voltage are in phase and is less than unity when they differ in phase.

The power-factor is obviously the true power divided by the volt-amperes (EI), or the apparent power (see equation (19), page 63).

Example.—Determine the power-factor in the foregoing example. The power,

$$P = I^2R = (3.94)^2\,20 = 310 \text{ watts}$$

The volt-amperes,

$$EI = 100 \times 3.94 = 394 \text{ watts}$$

The power-factor,

$$\text{P.F.} = {}^{310}\!/\!_{394} = 0.787. \quad \textit{Ans.}$$

Also,

$$\text{Cos } \theta = \text{Cos } 38.1° = 0.787 \text{ (see page 411).} \quad \text{Check.}$$

38. Resistance and Capacitance in Series.—In Fig. 57 is shown a resistance R connected in series with a condenser C across a voltage E. Let it be required to determine the relations existing among the current, voltage, power, resistance and capacitance in this circuit. Since this circuit is a series circuit, the current is the same throughout the circuit at every instant. Hence one current vector suffices for the circuit. For convenience only, this current vector is laid off horizontally (Fig. 58 (*a*)).

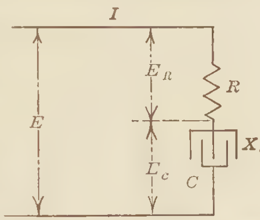


FIG. 57.—Circuit containing resistance and capacitance in series.

The voltage across the resistance must be equal to IR, and it must also be in phase with the current I. Therefore, this voltage is laid off along the current vector as shown in the figure. The voltage across the capacitance is equal to $IX_c(= I/2\pi fC)$ and it *lags* the current by 90° (see Fig. 47, page 52). The line voltage E must be equal to the sum of its component parts, E_R and E_C.

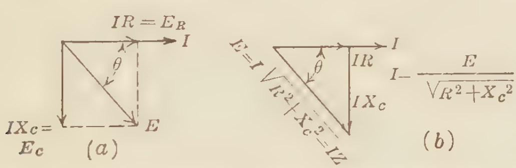


FIG. 58.—Vector diagrams for circuit containing resistance and capacitance in series.

Since E_R and E_C are not in phase, the line voltage E is the *vector* sum of E_R and E_C. The parallelogram method of vector addition is shown in Fig. 58 (*a*) and the triangle method in Fig. 58 (*b*). Since E_R and E_C are in quadrature, the line voltage E is the hypotenuse of a right triangle of which $IR(= E_R)$ and $IX_C(= E_C)$ are the two sides.

Therefore,

$$E = \sqrt{(IR)^2 + (IX_C)^2} = \sqrt{I^2(R^2 + X_C^2)} = I\sqrt{R^2 + X_C^2}.$$

Solving for the current I,

$$I = \frac{E}{\sqrt{R^2 + X_C^2}} = \frac{E}{\sqrt{R^2 + \left(\frac{I}{2\pi fC}\right)^2}} = \frac{E}{Z}. \tag{26}$$

$Z = \sqrt{R^2 + X_C^2}$ is the *impedance* of the circuit when resistance and capacitance are connected in series. It is denoted by Z, and is expressed in ohms (see page 65). It also follows that the voltage

$$E = IZ \text{ and } Z = \frac{E}{I} \tag{27}$$

Since the tangent of an angle is the ratio of the opposite to the adjacent side, an inspection of Fig. 58 (*b*) shows that,

$$\tan\theta = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{2\pi fCR} \tag{28}$$

Since the cosine of an angle is the ratio of the adjacent side to the hypotenuse,

$$\cos\theta = \frac{IR}{\sqrt{(IR)^2 + (IX_C^2)}} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{Z}. \tag{29}$$

In all the foregoing formulas, C must be expressed in *farads*.

The following example is given as illustrative of the method of computing the quantities in a circuit having resistance and capacitance in series.

Example.—A capacitance of 20μf. and a resistance of 100 ohms are connected in series across 120-volt, 60-cycle mains. Determine: (*a*) the impedance of the circuit; (*b*) the current flowing in the circuit; (*c*) the voltage across the resistance; (*d*) the voltage across the capacitance; (*e*) the angle between the voltage and the current.

20 μf. = 0.000020 farad.

$$X_C = \frac{1}{2\pi 60 \times 0.000020} = 133 \text{ ohms (equation (13), page 54).}$$

(*a*) $Z = \sqrt{(100)^2 + (133)^2} = \sqrt{27{,}700} = 166$ ohms. *Ans.*

(*b*) $I = \frac{120}{166} = 0.723$ amp. (equation (26)).

(*c*) $E_R = IR = 0.723 \times 100 = 72.3$ volts. *Ans.*

(*d*) $E_C = IX_C = 0.723 \times 133 = 96.2$ volts. *Ans.*

$E = \sqrt{(72.3)^2 + (96.2)^2} = 120$ volts (check).

(*e*) $\tan\theta = \frac{X_C}{R} = \frac{133}{100} = 1.33$ (equation (28)).

$\theta = 53.1°$. *Ans.*

39. Power with Resistance and Capacitance in Series.—In Fig. 57, the power dissipated in the resistance is I^2R. It has already been shown that the average power taken by a pure capacitance over a complete cycle is zero. Hence the capacitance (Fig. 57) consumes no power. All the power taken by the circuit must therefore be accounted for in the resistance. The total power

$$P = I^2R = I(IR). \tag{I}$$

In the vector diagram (Fig. 58 (*b*)),

$$\cos\theta = \frac{IR}{E}$$

$$IR = E\cos\theta$$

Substituting for IR in (I)

$P = I(E\cos\theta) = EI\cos\theta$ (see equation (25), page 66).

As with inductance and resistance in series, $\cos\theta$ is the power-factor of the circuit.

Example.—Determine the power and the power-factor in the foregoing example.

$$P = I^2R = (0.723)^2 \times 100 = 52.2 \text{ watts. } \quad \textit{Ans.}$$

$$\text{P.F.} = \cos\theta = \frac{R}{Z} = \frac{100}{166} = 0.602. \quad \textit{Ans.}$$

$$\text{Also P.F.} = \frac{P}{EI} = \frac{52.2}{120 \times 0.723} = 0.602 \text{ (check).}$$

40. Circuit Containing Resistance, Inductance, and Capacitance in Series.—Figure 59 shows a resistance R, an inductive reactance X_L, and a condensive reactance X_C, all connected in series. The voltage across the circuit is E volts, the frequency is f cycles per second and the current is I amp. Since this is a series circuit, the current I is the same throughout the circuit. In the vector diagram, the current vector I, for convenience, is laid off horizontal (Fig. 60 (*a*)). The voltage across the resistance $IR(= E_R)$ is laid off in phase with the current since the

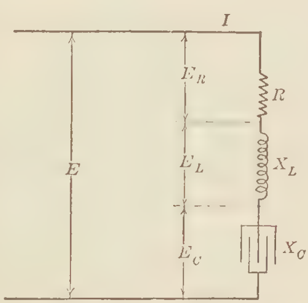


Fig. 59.—Circuit containing resistance, inductance and capacitance in series.

voltage across a resistance must be in phase with the current. The voltage across the inductance $IX_L(= E_L)$ leads the current by 90° as shown by the vector oc. The voltage across the capacitance ($IX_C = E_C$) lags the current by 90° as shown by the vector od. In this particular example the voltage across the inductance IX_L is shown greater than the voltage across the capacitance IX_C.

The line voltage E must be equal to the vector sum of its component voltages. That is, vectorially

$$\dot{E} = \dot{E}_R + \dot{E}_L + \dot{E}_C.$$

This vector addition is most readily accomplished by first combining IX_L and IX_C (see page 29). Since IX_C is less than IX_L

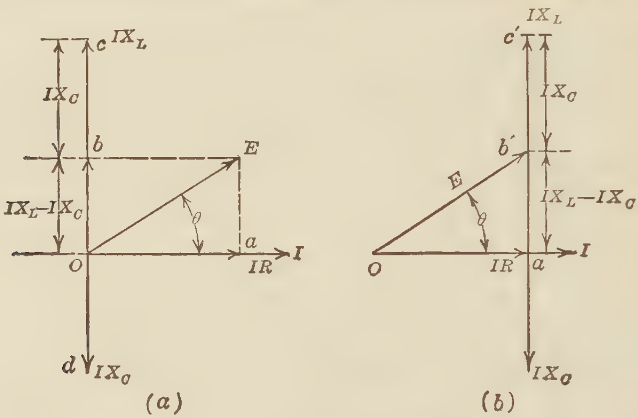


FIG. 60.—Vector diagrams for circuit with resistance, inductance and capacitance in series.

it is subtracted from IX_L. This gives the resultant vector $ob = IX_L - IX_C$ (Fig. 60 (a)). The vector ob is then combined with IR to give the line voltage E. Since E is the diagonal of a rectangle of which IR and ($IX_L - IX_C$) are the sides,

$$E = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= \sqrt{I^2R^2 + I^2(X_L - X_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}.$$

$$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad (30)$$

$$= \frac{E}{Z} \qquad (31)$$

The expression $\sqrt{R^2 + (X_L - X_C)^2} = Z$ is the *impedance* of a circuit having resistance, inductance, and capacitance in series. If X_L is greater than X_C, the quantity within the parenthesis is positive. If X_C is greater than X_L, the quantity within the parenthesis is negative, but since the square of a negative quantity is positive, $(X_L - X_C)^2$ is always positive.

Since $X_L = 2\pi fL$ and $X_C = 1/2\pi fC$, (30) may be written

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}} \tag{32}$$

where f is the frequency in cycles per second.

The phase angle θ is determined most readily by its tangent.

$$\text{Tan } \theta = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}. \tag{33}$$

If X_L is greater than X_C, tan θ is positive and θ is positive (see page 407). The voltage then *leads* the current. If X_L is less than X_C, tan θ is negative and θ is negative. The voltage then *lags* the current.

Also

$$\cos \theta = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{Z}. \tag{34}$$

The following examples illustrate the application of the foregoing relations to a specific problem.

Example.—A series circuit consisting of a resistance of 50 ohms, a capacitance of 25 μf. and an inductance of 0.15 henry is connected across 120-volt, 60-cycle mains.

Find: (*a*) the impedance of the circuit; (*b*) the current in the circuit; (*c*) the voltage across the resistance; (*d*) the voltage across the inductance; (*e*) the voltage across the capacitance; (*f*) the phase angle of the circuit.

$$X_L = 2\pi 60 \times 0.15 = 377 \times 0.15 = 56.6 \text{ ohms.}$$

$$X_C = \frac{1}{2\pi 60 \times 0.000025} = 106 \text{ ohms.}$$

(*a*) $Z = \sqrt{(50)^2 + (56.6 - 106)^2} = \sqrt{(50)^2 + (-49.4)^2} =$
$\sqrt{2{,}500 + 2{,}440} = 70.2$ ohms. *Ans.*

(*b*) $I = \dfrac{120}{70.2} = 1.71$ amp. (equation (31)). *Ans.*

(*c*) $E_R = IR = 1.71 \times 50 = 85.5$ volts. *Ans.*
(*d*) $E_L = IX_L = 1.71 \times 56.6 = 96.8$ volts. *Ans.*

(e) $E_C = IX_C = 1.71 \times 106 = 181.1$ volts. *Ans.*

(f) $\tan\theta = \dfrac{X_L - X_C}{R} = \dfrac{56.6 - 106}{50} = \dfrac{-49.4}{50} = -0.988$ (equation (33)).

$\theta = -44.6°$. Therefore, the current leads. *Ans.*

As a check, the vector sum of the three component voltages may be obtained (Fig. 61).

$E_L - E_C = 96.8 - 181.1 = -84.3$ volts.

$$E = \sqrt{(85.5)^2 + (-84.3)^2} = \sqrt{7{,}310 + 7{,}110} = 120 \text{ volts.}$$

(check.)

The complete vector diagram for this circuit is given in Fig. 61.

Example.—A current of 2.0 amp. flows in a circuit consisting of a resistance of 40 ohms, an inductive reactance of 48 ohms and a condensive reactance of 20 ohms, all connected in series. The frequency is 25 cycles per second.

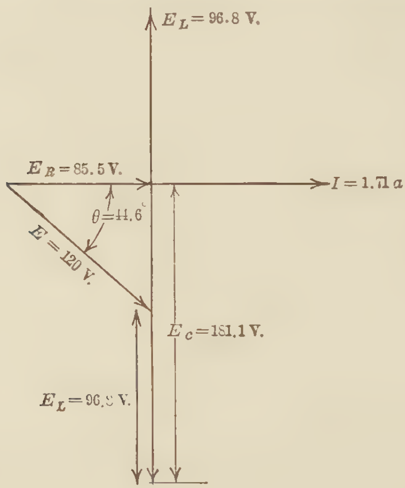


Fig. 61.—Vector diagram for series circuit, giving numerical values.

Find: (a) The impedance of the circuit; (b) the circuit voltage; (c) the phase angle of the circuit; (d) the inductance of the circuit; (e) the capacitance of the circuit.

(a) $Z = \sqrt{(40)^2 + (48 - 20)^2} = \sqrt{1{,}600 + 784} = 48.8$ ohms. *Ans.*

(b) $E = IZ = 2 \times 48.8 = 97.6$ volts. *Ans.*

(c) $\text{Tan}\,\theta = \dfrac{48 - 20}{40} = \dfrac{28}{40} = 0.700 \quad \theta = 35.0°$. *Ans.*

(d) $48 = 2\pi 25L = 157L$

$L = \dfrac{48}{157} = 0.306$ henry. *Ans.*

(e) $20 = \frac{1}{2\pi 25C} = \frac{1}{157C}$

$$C = \frac{1}{3{,}140} = 0.000318 \text{ farad or } 318\mu\text{f.} \quad \textit{Ans.}$$

41. Power in Circuit Containing Resistance, Inductance, and Capacitance in Series.—In the series circuit (Fig. 59) the power dissipated in the resistance,

$$P = I^2R. \tag{I}$$

No power is dissipated in either the inductance or the capacitance, as has already been shown. Hence all the power taken by the circuit is accounted for in the resistance. Equation (I) may be written,

$$P = I(IR).$$

From Fig. 60, $IR = E \cos \theta$.

Hence the power,

$$P = EI \cos \theta$$

as with resistance and inductance in series, or with resistance and capacitance in series.

Example.—Find the power and the power-factor in the problem, page 71.

$$P = I^2R = (1.71)^2 \times 50 = 146 \text{ watts.} \quad \textit{Ans.}$$

$$\cos \theta = \frac{R}{Z} = \frac{50}{70.2} = 0.712 \text{ (equation 34).} \quad \textit{Ans.}$$

$$\text{Also, P.F.} = \frac{P}{EI} = \frac{146}{120 \times 1.71} = 0.712. \quad \text{(check.)}$$

42. Resonance in a Series Circuit.—The general equation (32) for the current in a series circuit shows that for fixed values of resistance and impressed voltage the current is a maximum when the denominator is a minimum. The denominator is obviously a minimum when the expression in the parenthesis under the square-root sign is equal to zero.

That is, in equation (32)

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

the current is a maximum when

$$\left(2\pi fL - \frac{1}{2\pi fC}\right) = 0.$$

The current then becomes

$$I = \frac{E}{\sqrt{R^2 + (0)}} = \frac{E}{R}$$

its Ohm's law value.

Under these conditions

$$2\pi fL = \frac{1}{2\pi fC} \tag{35}$$

and

$$2\pi fLI = \frac{I}{2\pi fC}.$$

$2\pi fLI$ is the voltage across the inductance and $\frac{I}{2\pi fC}$ is the voltage across the condenser.

When the current is a maximum under the foregoing conditions *the voltage across the inductance is equal to the voltage across the capacitance.* As these two voltages are in exact opposition, they balance each other, so that the IR drop is equal to the line voltage. This is illustrated in Fig. 62.

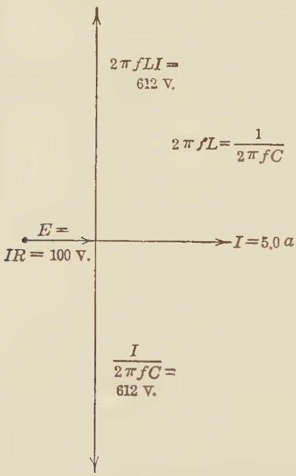


FIG. 62.—Vector diagram for series circuit in resonance.

When these conditions exist, the circuit is said to be in *resonance*. The current is then in phase with the line voltage and the power $P = EI$.

Solving equation (35) for the frequency

$$2\pi fL - \frac{1}{2\pi fC} = 0.$$

$$f = \frac{1}{2\pi\sqrt{LC}}. \tag{36}$$

This is the frequency for which a circuit having fixed values of L and C will be in resonance. It is sometimes called the *natural frequency* of the circuit, because it is the frequency at which the current in the circuit will oscillate, if no external frequency is impressed on the circuit, provided the resistance R is less than $\sqrt{4L/C}$. For example, in a radio telegraph circuit a condenser C, charged to a high voltage, is discharged into an inductance L, of negligible resistance. The frequency of the resulting oscilla-

tions, as determined by the values of L and C, is given in equation (36). Also when a radio receiving set is tuned to the incoming signal, the inductance, and capacitance are so adjusted that the natural or resonant frequency of the set is equal to the frequency of the incoming carrier wave. The voltage across the inductance equals the voltage across the capacitance, when the circuit is in resonance. As the two voltages are in opposition, each may reach a high value, even with moderate line voltage. This is illustrated by the following example.

Example.—A circuit has a resistance of 20 ohms, an inductance of 0.3 henry, and a capacitance of 20 μf.; (*a*) for what value of the frequency will the circuit be in resonance? (*b*) If the current is 5 amp. find the line voltage; (*c*) the voltage across the inductance; (*d*) the voltage across the capacitance; (*e*) the power consumed by the circuit. Draw a vector diagram for the circuit.

(*a*) $f = \dfrac{1}{2\pi\sqrt{0.3 \times 0.000020}} = 65$ cycles. *Ans.*

(*b*) $E = IR = 5 \times 20 = 100$ volts. *Ans.*

(*c*) $E_L = 2\pi fLI = 6.28 \times 65 \times 0.3 \times 5 = 612$ volts. *Ans.*

(*d*) $E_C = \dfrac{I}{(2\pi fC)} = \dfrac{5}{2\pi 65 \times 0.000020} = 612$ volts. *Ans.*

(*e*) $P = EI = 100 \times 5 = 500$ watts. *Ans.*

The vector diagram is shown in Fig. 62.

It will be observed that the voltage across the inductance and that across the capacitance are equal, each being 612 volts, or more than six times the line voltage.

Example.—A radio receiving set has a fixed inductance of 0.08 millihenry. To what value of capacitance should the condenser be adjusted in order that the set may be tuned to an incoming signal whose frequency is 1,000,000 cycles per second. $\left(\text{Wave length, } \lambda = \dfrac{3 \times 10^8}{1{,}000{,}000} = 300 \text{ M.}\right)$

Using equation (35)

$$2\pi fL = \frac{1}{2\pi fC}.$$

$$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 10^{12} 0.00008}$$
$$= 0.000000000317 \text{ farad}$$
$$= 0.000317 \mu\text{f.} \quad \textit{Ans.}$$

It should be noted that the current is a maximum when a series circuit is in resonance.

If in equation (32) (page 71) the resistance is practically zero, and the circuit is in resonance, the current $I = E/0$, or

is infinite. That is, a short-circuit results. Also the voltage across the inductance and that across the capacitance theoretically become infinite. Although this may not harm the inductance, it will be very likely to puncture the condenser. *Therefore, with a power circuit, do not ordinarily connect inductance and capacitance in series unless there be sufficient resistance.*

43. Practical Inductance and Capacitance.—Thus far only pure inductance and pure capacitance have been considered. Obviously it is impossible to obtain either, although pure capacitance may be very closely approximated. The wire with which inductance is wound obviously has resistance. In order to obtain sufficiently high values of inductance, an iron core is usually necessary. Although the core may be laminated, there occur both eddy-current and hysteresis losses in the core. Hence, there is appreciable loss in the circuit and the angle between current and voltage can not be 90° and is usually considerably less than 90°. Angles as large as 86° are obtainable, but ordinarily the angle lies between 80 and 85°.

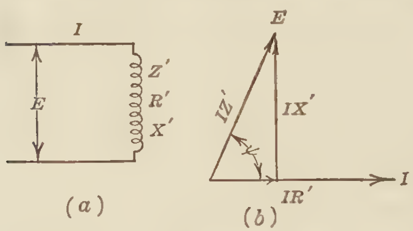


FIG. 63.—Impedance coil.

In considering such a circuit analytically such an impedance may be considered as consisting of a resistance and a pure inductance in series. For example, in Fig. 63 (*a*) is shown an impedance coil Z' having an iron core. Owing to the fact that this coil has resistance and iron losses, the angle between the voltage and current is not 90°, but the current lags the voltage by an angle ψ less than 90°, as shown in Fig. 63 (*b*). The impedance-drop IZ' may be resolved into two components at right angles to each other, IR' along the current vector I, and IX' at right angles to I and leading. R' may then be considered the equivalent resistance of the impedance coil and X' its reactance. A comparison of Fig. 63 (*b*) with Fig. 56 (*b*) (page 64), which is the vector diagram for a circuit consisting of resistance and pure inductance in series, shows that they are identical in character.

Example.—An impedance coil with an iron core when connected across 110-volt, 60-cycle mains takes 4.0 amp. and 30 watts. Determine: (*a*) the

power-factor of this impedance coil; (*b*) the phase angle; (*c*) the equivalent resistance; (*d*) the reactance; (*e*) the inductance.

(*a*) $\text{P.F.} = \frac{30}{110 \times 4.0} = \frac{30}{440} = 0.0682.$ *Ans.*

(*b*) $\cos \psi = 0.0682$

$\psi = 86.1°$ (page 410). *Ans.*

(*c*) The power P is obviously equal to I^2R'

Where R' is the equivalent resistance.

$$30 = (4.0)^2R' = 16R'$$

$$R' = \frac{30}{16} = 1.875 \text{ ohms.} \quad \textit{Ans.}$$

(*d*) The impedance,

$$Z' = \frac{E}{I} = \frac{110}{4.0} = 27.5 \text{ ohms.}$$

The reactance

$$X' = \sqrt{Z'^2 - R'^2} = \sqrt{(27.5)^2 - (1.875)^2}$$
$$= 27.4 \text{ ohms.} \quad \textit{Ans.}$$

(*e*) $X' = 2\pi fL$

$27.4 = 2\pi 60L = 377L$

$L = \frac{27.4}{377} = 0.0728$ henry. *Ans.*

Figure 64 shows an impedance coil Z' connected in series with a resistance r. Let it be required to determine the vector diagram

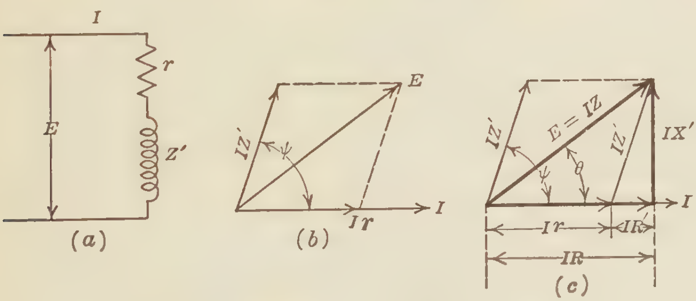


FIG. 64.—Resistance and impedance in series.

for this circuit. Since this is a series circuit the current is the same throughout the circuit. The current vector I is laid off horizontally, as shown in (*b*). The Ir drop is laid off in phase with the current. The voltage across the impedance coil IZ' leads the current by an angle ψ which is less than 90°. This is also shown in the diagram. The line voltage E is obviously the vector sum of Ir and IZ' as shown. The voltage IZ' may be

resolved into two components, IR' along the current vector and IX' leading the current by 90° as shown in (c), where R' is the equivalent resistance of the impedance coil and X' is its reactance.

A vector diagram may be drawn for the circuit in which the total resistance drop is IR where $R = r + R'$, and the reactance drop is IX'. In other words, this circuit may be considered as having a total resistance R and a pure inductive reactance X' in series. Figure 64 (c) should be compared with Fig. 56 (b) (page 64).

In practice, condensers give very nearly pure capacitance. Mica condensers often have phase angles which depart only 2 or 3 min. from 90° and it is only possible to measure these angles by the most refined methods. Condensers made with paraffined paper have phase angles which depart from 90° ordinarily by less than one degree, although in extreme cases the angle may be as high as 2°. If the phase angle of a condenser departs too much from 90°, the power consumed by the condenser raises its temperature and the condenser soon burns out.

PARALLEL CIRCUITS

44. Resistance and Inductance in Parallel.—In practice, parallel circuits are more common than series circuits because of the almost universal use of the parallel or multiple system of transmission and distribution. The determination of the total current when a number of circuits are connected in parallel is ordinarily not difficult.

In the series circuit the same current flows through each member of the circuit. In the parallel circuit the voltage is the same across each member of the circuit. The method of finding the total current with resistance and inductance in parallel is illustrated by the following example:

Example.—Determine: (a) the total current I in the circuit shown in Fig. 65, consisting of a 10-ohm resistance and an inductance of 0.03 henry connected in parallel across 100-volt, 50-cycle mains; (b) the power taken by the circuit; (c) the power-factor of the circuit; (d) the power-factor angle of the circuit.

(a) The current taken by the resistance

$$I_R = \frac{100}{10} = 10 \text{ amp.}$$

The reactance of the inductive branch

$$L = 2\pi 50 \times 0.03 = 314 \times 0.03 = 9.42 \text{ ohms.}$$

The current in this branch

$$I_L = \frac{100}{9.42} = 10.6 \text{ amp.}$$

The vector diagram is shown in Fig. 65 (*b*). The voltage $E = 100\ V.$ is common to both branches of the circuit, and is laid off horizontally for convenience. The current I_R in the resistance (10 amp.) is in phase with the voltage. The current I_L in the inductance (10.6 amp.) *lags* the voltage by 90°. The total current I is their vector sum. Since I_R and I_L differ in phase by 90°, the resultant current I is the square root of the sum of their squares.

$$I = \sqrt{(10)^2 + (10.6)^2} = \sqrt{212.4} = 14.57 \text{ amp.} \quad \textit{Ans.}$$

(*b*) The average power taken by the inductance is zero. All the power taken by the circuit must be accounted for in the resistance. Hence,

$$P = EI_R = 100 \times 10 = 1{,}000 \text{ watts.} \quad \textit{Ans.}$$

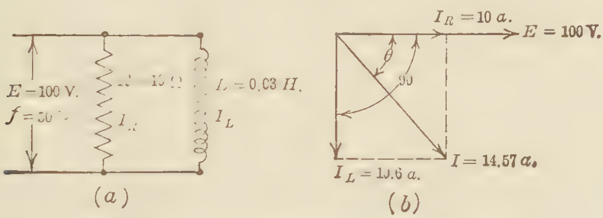


FIG. 65.—Resistance and inductance in parallel.

$$(c)\ \text{P.F.} = \frac{P}{EI} = \frac{1{,}000}{100 \times 14.57} = \frac{1{,}000}{1{,}457} = 0.686.$$

(*d*) An inspection of Fig. 65 (*b*) shows that

$$\tan\theta = \frac{10.6}{10} = 1.06$$

$$\theta = 46.7° \text{ (page 413).} \quad \textit{Ans.}$$

Also from (*c*), $\cos\theta = 0.686$

$$\theta = 46.7° \text{ (check.)}$$

45. Resistance and Capacitance in Parallel.—The solution of problems involving resistance and capacitance in parallel is illustrated by the following example:

Example.—A resistance of 40 ohms and a capacitance of 60μf. are connected in parallel across 100-volt, 60-cycle mains (Fig. 66). Find: (*a*) the current in the resistance; (*b*) the current taken by the capacitance; (*c*) the total current; (*d*) the total power; (*e*) the power-factor of the circuit; (*f*) the power-factor angle of the circuit.

(*a*) The current in the resistance

$$I_R = {}^{100}\!/\!_{40} = 2.50 \text{ amp.} \quad \textit{Ans.}$$

The voltage is common to both branches of this circuit, and is laid off horizontal (Fig. 66 (*b*)). The current I_R to scale is laid off in phase with the voltage.

(*b*) The condensive reactance from equation (14) (page 54)

$$X_C = \frac{10^6}{2\pi 60 \times 60} = 44.2 \text{ ohms.}$$

$$I_C = \frac{100}{44.2} = 2.26 \text{ amp.} \quad \textit{Ans.}$$

This current leads the voltage by 90°, as shown in the vector diagram (Fig. 66 (*b*)).

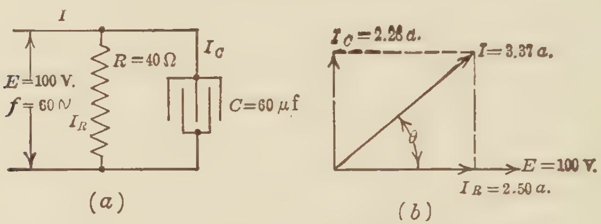


FIG. 66.—Resistance and capacitance in parallel.

(*c*) Since I_R and I_C are in quadrature, the resultant current I is obviously the square root of the sum of their squares.

$$I = \sqrt{(2.50)^2 + (2.26)^2} = \sqrt{6.25 + 5.11} = 3.37 \text{ amp.} \quad \textit{Ans.}$$

The resultant current I is shown in the vector diagram (Fig. 66 (*b*)).

(*d*) The average power taken by the capacitance is zero. Hence all the power taken by the circuit is accounted for in the resistance

The total power, therefore,

$$P = 100 \times 2.50 = 250 \text{ watts.} \quad \textit{Ans.}$$

$$(e) \quad \text{P.F.} = \frac{P}{EI} = \frac{250}{100 \times 3.37} = 0.742. \quad \textit{Ans.}$$

From Fig. 66 (*b*)

$$\text{Tan } \theta = \frac{I_C}{I_R} = \frac{2.26}{2.50} = 0.904.$$

$$\theta = 42.1° \text{ (page 412).} \quad \textit{Ans.}$$

Also, from (*e*) $\cos \theta = 0.742$

$$\theta = 42.1° \text{ (check.)}$$

46. Resistance, Inductance and Capacitance in Parallel.—With resistance, inductance, and capacitance in parallel, it is merely necessary to find the current in each branch of the circuit

and then to combine these currents vectorially to find the total current. This is illustrated by the following example:

Example.—A resistance of 40 ohms, an inductance of 0.08 henry and a capacitance of 50 μf. are connected in parallel (Fig. 67 (*a*)). Find: (*a*) the current in each branch of the circuit; (*b*) the total current; (*c*) the total power; (*d*) the power-factor of the circuit; (*e*) the power-factor angle of the circuit.

(*a*)

$$I_R = \frac{100}{40} = 2.50 \text{ amp.} \quad Ans.$$

$$X_L = 2\pi 60 \times 0.08 = 30.2 \text{ ohms.}$$

$$I_L = \frac{100}{30.2} = 3.31 \text{ amp.} \quad Ans.$$

$$X_C = \frac{10^6}{2\pi 60 \times 50} = 53.0 \text{ ohms.}$$

$$I_C = \frac{100}{53.0} = 1.885 \text{ amp.} \quad Ans.$$

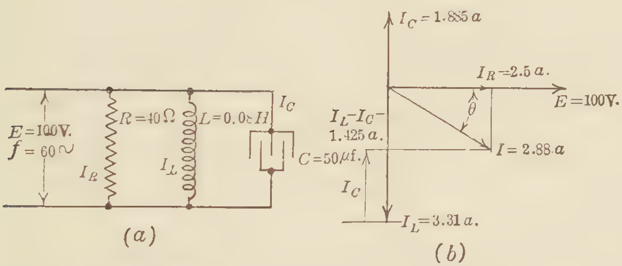


FIG. 67.—Resistance, inductance and capacitance in parallel.

These three currents are shown vectorially in Fig. 67 (*b*). I_R is in phase with the voltage E, I_L lags the voltage E by 90°, and I_C leads the voltage E by 90°.

(*b*) To find the total current I, it is merely necessary to find the vector sum of I_R, I_L, and I_C (Fig. 67 (*b*)). Since I_L and I_C are in opposition, their resultant is obviouly $3.31 - 1.885 = 1.425$ amp. This current is then combined in quadrature with I_R to find the total current I.

$$I = \sqrt{(2.50)^2 + (1.425)^2} = \sqrt{6.25 + 2.04} = \sqrt{8.29} = 2.88 \text{ amp.} \quad Ans.$$

(*c*) The inductance and capacitance consume no power. Hence all the power must be accounted for in the resistance.

$$P = EI_R = 100 \times 2.50 = 250 \text{ watts.} \quad Ans.$$

(*d*) $$\text{P.F.} = \frac{250}{100 \times 2.88} = 0.868. \quad Ans.$$

(*e*) From Fig. 67 (*b*),

$$\tan\theta = \frac{I_L - I_C}{I_R} = \frac{1.425}{2.50} = 0.570.$$

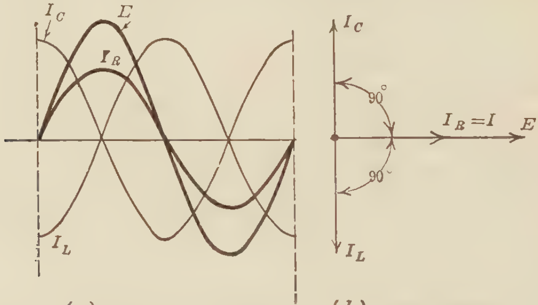
$$\theta = 29.7° \text{ (page 412).} \quad \textit{Ans.}$$

Also, from (*d*)

$$\cos\theta = 0.868, \ \theta = 29.8°. \quad \text{(check.)}$$

47. Resonance in Parallel Circuits.—Resonance occurs in series circuits when the voltage across the inductance is equal to the voltage across the capacitance. Under these conditions the current is in phase with the line voltage, and for a given value of resistance, the current in the circuit is a *maximum*.

Resonance may also occur in the parallel circuit. If in Fig. 67 the current in the inductance I_L is equal to the current in the capacitance I_C, the two cancel when the vector addition is made. The current in the resistance is the resultant current and is in



(*a*) Instantaneous Values (*b*) Vector Diagram

FIG. 68.—Resonance in a parallel circuit.

phase with the line voltage E. These conditions are illustrated in Fig. 68, which shows a parallel circuit in resonance. The instantaneous values of current and voltage are shown in (*a*). The voltage curve E is a sine curve. The current I_R in the resistance is in phase with the voltage E. The current I_L in the inductance lags the voltage by 90°. The current I_C in the capacitance leads the voltage by 90°. Since these last two currents are in opposition, they cancel at every instant. The current I_R in the resistance is therefore the resultant current. The vector diagram for this circuit is shown in Fig. 68 (*b*). The line voltage E is laid off horizontally, the current I_R in the resistance is in phase with E, the current I_L in the

inductance lags E by 90°, and the current I_C in the capacitance leads E by 90°. The current in the inductance, I_L, is equal numerically to the current in the capacitance I_C. Since these currents are in opposition, the line current I is equal to the current in the resistance I_R.

With pure inductance and pure capacitance only in parallel and adjusted for resonance, the line current is zero. This condition cannot be attained practically, although it may be approximated. That is, the circuit may be so tuned that but a small current flows from the source of supply.

With the series circuit having a fixed resistance the current is a *maximum* when the circuit is tuned for resonance. With the parallel circuit having a fixed resistance the current is a *minimum* when the circuit is tuned for resonance.

Example.—A circuit consisting of a resistance of 60 ohms and an inductance of 0.3 henry in parallel is connected across 100-volt, 25-cycle mains. (*a*) For what value of capacitance will the circuit be in resonance? (*b*) What is the line current under these conditions?

For resonance the current in the capacitance must be equal to the current in the inductance. Hence the condensive reactance must be equal to the inductive reactance.

The inductive reactance

$$X_L = 2\pi 25 \times 0.3 = 47.1 \text{ ohms.}$$

The condensive reactance

$$X_C = 47.1 \text{ ohms} = \frac{1}{2\pi 25 C}.$$

$$C = \frac{1}{157 \times 47.1} = 0.000135 \text{ farad}$$

$$= 135 \ \mu\text{f.} \quad \textit{Ans.}$$

From the foregoing problem it is obvious, that for resonance, $X_C = X_L$.

$$\frac{1}{2\pi f C} = 2\pi f L$$

$$f = \frac{1}{2\pi\sqrt{LC}}. \qquad (37)$$

This equation should be compared with equation (36) page 74, for the series circuit. Equation (37) holds only with pure inductance and pure capacitance. With resistance in series with the inductance or capacitance, equation (37) does not apply.

CHAPTER III

ALTERNATING-CURRENT INSTRUMENTS AND MEASUREMENTS

ELECTRODYNAMOMETER TYPE INSTRUMENTS

48. The Siemens Dynamometer.—Several types of alternating-current instruments operate on the electrodynamometer principle. The Siemens dynamometer (Fig. 69) is an example of this type of instrument in simple form. It consists primarily of two sets of coils. The coil F is fixed and the coil M, whose axis is at right angles to the axis of F, is free to turn through a small angle. M is suspended by a silk tread and its turning moment is opposed by a helical spring S. Current is led into the moving coil through two mercury cups C.

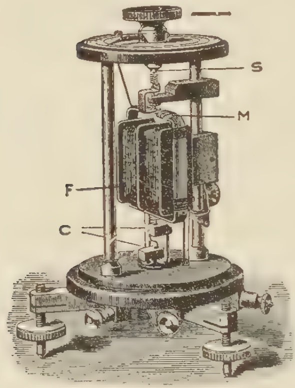


Fig. 69.—Siemens dynamometer.

When used as an ammeter, the two coils are wound with a few turns of coarse wire and are connected in series. When current flows through these coils, there is a tendency for the moving coil to swing into the plane of the fixed coil so that their magnetic fields act in conjunction (see Part I, page 12, Par. 16). When the current reverses, it reverses in the two coils simultaneously so that the torque is always in the same direction. The movable coil is not allowed to deflect, however, but is kept in its zero position by turning the knurled head H at the top of the instrument which acts on the coil through the spring. The angle by which it is necessary to turn this head is proportional to the turning moment of the coil. The turning moment is proportional to the product of the current in the fixed coil and the current in

the movable coil. When the two coils are in series the turning moment must be proportional to the current *squared*, and hence to the *heating effect* of the current (see page 21, Par. 13). The instrument is calibrated with a known direct current. An alternating current giving this same deflection has the same heating effect as the direct current. Hence the effective alternating-current amperes squared are equal to the direct-current amperes squared for equal settings of the torsion head. When direct current is used, it is advisable to reverse the direction of the current and average the set of readings. This eliminates the effect of stray fields.

This type of instrument is difficult to adjust and to manipulate, especially when the current fluctuates. It is not direct reading and because of its construction is adapted to laboratory work only.

If small wire be substituted for the coarse wire and an extension coil be connected in series, the instrument can be used as a voltmeter.

49. The Indicating Electrodynamometer.—As it is neither portable nor direct reading, the Siemens dynamometer itself is not adapted to portable and to switchboard instruments. However, many types of portable and switchboard instruments operate on the Siemens dynamometer principle. The general construction of a portable type of electrodynamometer instrument is shown in Fig. 70.

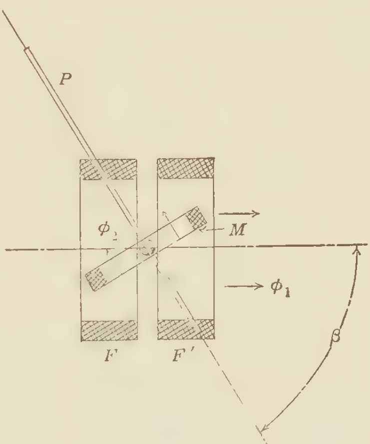


FIG. 70.—Principle of the electro dynamometer instrument.

Two fixed coils FF' are so connected that their magnetic fields act in conjunction. These coils may be considered as being two parts of a single coil, opened in the middle to allow the spindle of the moving coil to pass through.

M is a movable coil mounted on a vertical spindle. There is a hardened steel pivot at each end of the spindle, which turns in

jeweled bearings. Two spiral springs similar to those used in direct-current instruments (see Part I, page 120, Fig. 113) oppose the turning of coil M and at the same time carry the current into the coil. As the springs can carry but a very small current, the movable coil is wound with fine wire.

Assume that at some instant the direction of the magnetic field ϕ_1, which is due to the fixed coils, is from left to right. At the same instant the current in coil M produces a field ϕ_2 whose direction is along the axis of M. Coils tend to align themselves so that the number of magnetic linkages in the system is a maximum. Therefore, the moving coil M tends to turn in a clockwise direction so that its field will act in conjunction with ϕ_1. The turning of M is opposed by the control springs.

Obviously the torque developed is proportional to ϕ_1, ϕ_2 and $\sin \beta$, where β is the angle between the axis of coil M and the axis of coils FF'. As ϕ_1 and ϕ_2 are proportional to the currents in the coils FF' and M respectively, the torque is proportional to the product of the two currents and $\sin \beta$.

50. The Electrodynamometer Voltmeter.—Some types of alternating-current voltmeter operate on the electrodynamometer principle. The fixed coils FF', and the moving coil M are all wound with a comparatively large number of turns and are connected in series. As the instrument itself would take too large a current when connected across the ordinary circuit, a comparatively high resistance is connected in series.

The current flowing through the dynamometer is therefore proportional to the line voltage and causes coil M to turn, and the pointer attached to it moves over a scale graduated in volts. The scale will not be divided uniformly like that of the direct-current voltmeter, since the deflections are very nearly proportional to the *square* of the voltage. The divisions at the lower part of the scale are so small that poor precision is obtained with small readings.

This dynamometer-type of voltmeter takes about five times as much current as a direct-current voltmeter of the same rating and consumes an appreciable amount of power. As the moving coil operates in a comparatively weak field, this type of instrument is very susceptible to stray fields, and should not be used

too near inductive apparatus, wires carrying even moderate currents, and other stray fields.

This instrument may be used for direct current as well as for alternating current. Reversed direct-current readings should be taken in order to eliminate the effect of the earth's field and of any other stray fields.

Electrodynamometer ammeters are made only for very special purposes, since, with alternating current, shunt adjustments are not simple.

51. The Wattmeter.—Alternating-current power is equal to the product of the effective current and the effective voltage only when the power-factor is unity. Therefore, the ammeter and voltmeter method, as used with direct currents, can seldom be used to measure alternating-current power. Consequently, a *wattmeter* is necessary for measuring alternating-current power.

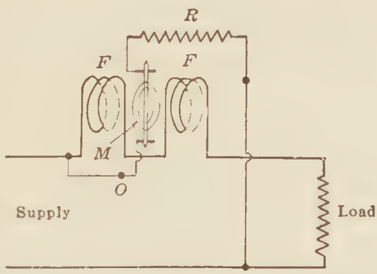


FIG. 71.—Connections for a wattmeter.

The wattmeter shown in Fig. 71 operates on the electrodynamometer principle. M is a moving coil wound with fine wire and is practically identical with the moving coil of the dynamometer voltmeter (Fig. 70). It is connected across the line in series with a high resistance R. The current is led into this coil through springs. The two fixed coils FF are wound with few turns of heavy wire, capable of carrying the load current. As there is no iron present, the field due to the current coils FF is proportional to the load current at every instant. The current in the moving coil M is proportional to the voltage at every instant. Therefore, for any given position of the moving coil, the torque is proportional at every instant to the product of the current and voltage or to the instantaneous power of the circuit. If the power-factor is other than unity, there is negative torque for part of the cycle. That is, during the periods when there are negative loops in the power curve (Fig. 54, page 62) the current in the fixed coil and the current in the moving coil reverse their directions with respect to each other, and so produce a negative torque.

The moving coil takes a position corresponding to the *average* torque.

As the torque acting on the moving coil varies from instant to instant, having a frequency twice that of either the current or the voltage, the coil tends to change its position to correspond with these variations of torque. If the moving system had little inertia, the needle would vibrate so that it would be impossible to obtain a reading. Because of the relatively large moment of inertia of the moving system, the needle assumes a steady deflection for constant values of average power. The position taken by the coil corresponds to the *average* value of the power, which is the result desired.

It should be noted (Fig. 71) that the voltage terminal marked "*O*" is connected directly to one end of the moving coil. This terminal is ordinarily connected to that side of the line to which the current-coil is connected (also see Part I, page 148, Par. 124).

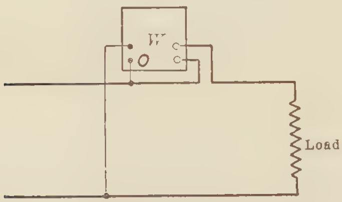


FIG. 72.—Instrument measures power consumed by its own current-coil.

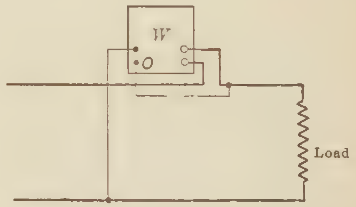


FIG. 73.—Instrument measures power consumed by its own potential-circuit.

52. Wattmeter Connections.—In Fig. 72, wattmeter *W* is shown measuring the power taken by a certain load. In order to measure this power correctly, the wattmeter current-coil should carry the *load* current, and the wattmeter voltage-coil, in series with its resistance, should be connected directly across the *load*.

The current in the wattmeter current-coil is the same as the load current, but the wattmeter potential-circuit is not connected directly across the load, but is measuring a potential in excess of the load potential by the amount of the impedance drop in the wattmeter current-coil. Therefore, the wattmeter reads too

high by the amount of power consumed in its own current-coil. Under these conditions the true power

$$P = P' - I^2R_c$$

where P' is the power indicated by the wattmeter, I is the current in the wattmeter current-coil, and R_c is the resistance of this coil. This loss is ordinarily of the magnitude of 1 or 2 watts at the rated current of the instrument, and may usually be neglected.

If the wattmeter be connected as shown in Fig. 73, the wattmeter potential-circuit is connected directly across the load, but the wattmeter current-coil carries the potential-coil current in addition to the load current. In fact, the wattmeter potential-circuit may be considered as being a small load connected in parallel with the actual load whose power is to be measured. Therefore, the power consumed by this potential-circuit must be deducted from the wattmeter reading. The true power taken by the load,

$$P = P' - \frac{E^2}{R_p}$$

where P' is the wattmeter reading, E the load voltage and R_p the resistance of the wattmeter potential-coil circuit.

An idea of the magnitude of this correction may be obtained from the following example.

Example.—A certain wattmeter indicates 157 watts when it is connected in the manner shown in Fig. 73. The line voltage is 120 volts and the resistance of the wattmeter potential-circuit is 2,000 ohms. How much power is taken by the load?

$$P = 157 - \frac{\overline{120}^2}{2,000} = 157 - 7.2 = 149.8 \text{ watts.}$$

It will be observed that a considerable percentage error would result in this case if the wattmeter loss were neglected.

The current- and potential-circuits of a wattmeter must each have a rating corresponding to the current and voltage of the circuit to which the wattmeter is connected. A wattmeter is rated in amperes and volts, rather than in watts, because the indicated watts show neither the amperes in the current-coil nor the voltage across the potential-circuit.

If the current in an ammeter or the voltage across a voltmeter exceed the rating of the instrument, the pointer goes off scale

and so warns the user. A wattmeter may be considerably overloaded and yet the load power-factor be so low that the needle is well on the scale. For this reason a voltmeter and an ammeter

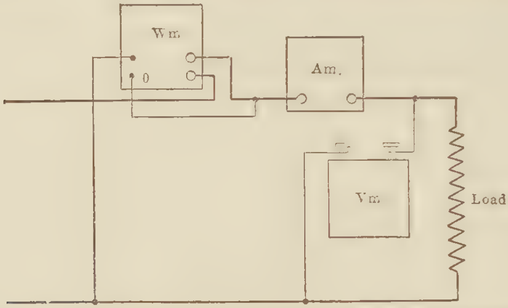


Fig. 74.—Wattmeter, ammeter and voltmeter connections for measuring power

should ordinarily be used in conjunction with a wattmeter so that it is possible to determine whether either the voltage or the current exceeds the wattmeter rating.

Figure 74 shows the connections which may be used to measure the power taken by a single-phase load. In addition to the load power, the wattmeter measures the power taken by its own potential-circuit, the power taken by the voltmeter, and the power taken by the ammeter. Correction for the first two may be necessary, although with steady conditions, the voltmeter may be disconnected when reading the wattmeter and the ammeter.

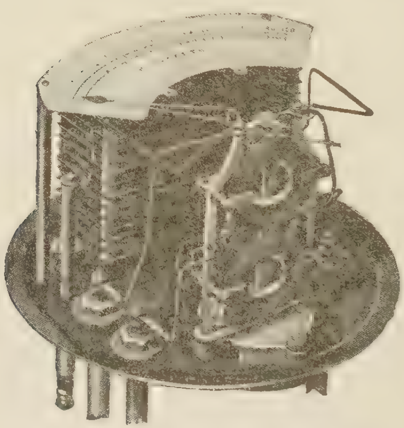


Fig. 75.—Interior view, Weston polyphase wattmeter.

53. Polyphase Wattmeter.—Ordinarily, it requires two or more wattmeters to measure the total power of a two-phase or of a three-phase circuit. If the load fluctuates, it is difficult to obtain accurate simultaneous readings of two wattmeters. At

power-factors less than 0.5, in a three-phase circuit, one of the wattmeters reverses its reading (see page 121, Par. 75). This necessitates reversing the connections of one of the instruments, which is often inconvenient. If both wattmeters be combined in one, that is, if both moving coils be mounted on the same

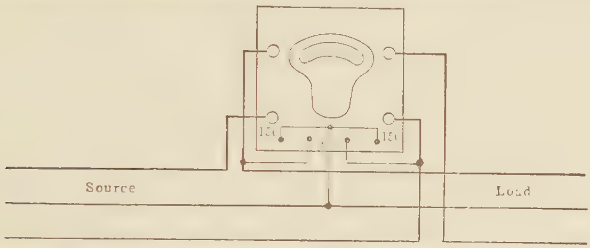


FIG. 76.—Connections for polyphase wattmeter on 3-phase circuit.

spindle, the turning moments for each element add or subtract automatically, and the total power is read on a single scale.

Figure 75 shows the construction of a Weston polyphase wattmeter in which the two elements are clearly shown. Figure 76 shows one method for connecting a polyphase wattmeter in a three-phase circuit.

IRON-VANE INSTRUMENTS

54. Iron-vane Instruments.—The most common types of alternating-current voltmeters and ammeters depend for their operation on the magnetization of a light iron vane mounted on the spindle. Such instruments are simple, rugged, and their cost is low. Although they may not be adapted to measurements requiring the highest precision, their precision is higher than is required for most measurements.

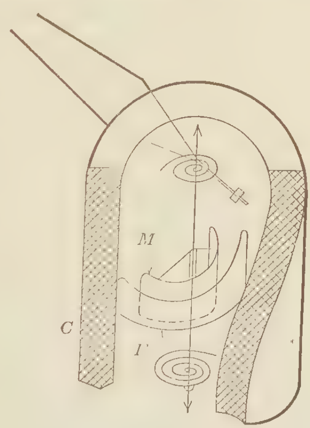


FIG. 77.—Weston iron-vane type of instrument.

One such type of instrument, manufactured by the Weston Electrical Instrument Company, is shown in Fig. 77.

A small strip of soft iron *M* bent into cylindrical form, is

mounted axially on a spindle which is free to turn. Another similar strip F which is more or less wedge-shaped, and with a larger radius than M, is fixed inside a cylindrical coil. In the voltmeter the cylindrical coil is wound with fine wire and is connected in series with a high resistance. When connected across the line, the current through the instrument is subtantially proportional to the circuit voltage. When current flows through this exciting coil, both iron vanes become magnetized. The upper edges of the two strips will always have the same magnetic polarity, and the lower edges will always have the same magnetic polarity, but when the upper edges are north poles, the lower edges are south poles. Therefore, there will always be a repulsion between the two upper edges, and also between the two lower edges of the iron strips. This repulsion tends to move the spindle against the action of two springs. A pointer mounted on the spindle moves over a graduated scale and indicates the voltage. Air damping is obtained by the use of a light aluminum vane moving in a restricted space.

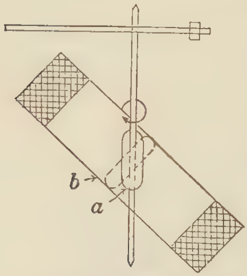


FIG. 78.—Inclined coil, iron-vane type of instrument.

Another method of using the iron-vane principle is shown in Fig. 78. A small iron vane is mounted obliquely on the spindle. When the pointer is at zero, this vane lies at an angle to the coil axis, as at *a* (Fig. 78). When current flows in the coil, the vane attempts to take such a position that the direction of its axis shall coincide with that of the magnetic field, which acts along the coil axis. This position is shown at *b* (Fig. 78). The vane in seeking this position turns the spindle which carries the pointer. The turning moment is opposed by springs. In the later models, the coils of these instruments are surrounded by iron laminations which shield them from stray fields. In the cheaper models, air damping is used, being obtained by a light aluminum vane attached to the moving element. The more expensive models employ magnetic damping, such as is used with watthour meters, a light aluminum vane moving between the poles of permanent magnets.

These two types of iron-vane instruments may be used for ammeters by winding the coils with a comparatively few turns of wire whose cross-section is sufficient to carry the current. In these iron-vane types of instruments, no current is led to the moving element.

55. Hot-wire Instruments.—This type of instrument, described in Part I, Chap. VII, page 126, reads equally well on both direct- and alternating-current circuits. As its deflection depends upon the square of the current (I^2R loss) the hot-wire instrument can be used as a transfer from alternating to direct current and *vice versa.*

56. Alternating-current Watthour Meter.—The direct-current watthour meter can be used with alternating current, as the

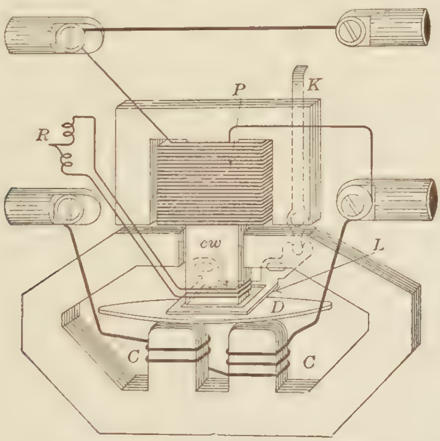


FIG. 79.—Diagram of induction watthour meter.

reversal of line voltage reverses both its armature and its field current simultaneously and the direction of the torque remains unchanged. At low power-factors, however, considerable error may be introduced by the inductance of the armature circuit which causes the armature current to lag the line voltage by a small angle. This error may be compensated by shunting the current-coils of the meter with a low non-inductive resistance.

The induction watthour meter is so much cheaper and so superior to the direct-current type that there is little necessity for using the direct-current type on alternating-current circuits.

A rear view of one type of induction meter is shown in Fig. 79. P is a potential-coil which is highly inductive and is placed on one lug of the laminated magnetic circuit, this lug being over the disc D. CC' are two series or current-coils placed on two projecting lugs beneath the disc. These coils are so wound that if one tends to send flux upward the other tends to send it downward. cw is a small auxiliary or compensating winding placed on the potential-lug and its ends are connected to the resistance R. In order that the meter may register correctly, the potential-coil flux must lag the line voltage by 90°. As it is impossible to make the resistance of the potential-coil zero, its current will lag by an angle less than 90°. At low power-factors this introduces considerable error in the meter registration. However, by properly adjusting the resistance R, the potential-coil *flux* may be brought into the 90° relation and the meter will register substantially correctly at all power-factors. To adjust the compensation, the meter is made correct at unity power-factor and then the power-factor is dropped to some low value, as 0.5. If the registration is now in error, it is due to improper compensation. The meter is again made to register correctly by changing the resistance R, the two small wires of this resistance being either twisted or untwisted and then soldered. If the meter underregisters when the load current lags, the resistance R should be *decreased;* if the meter overregisters with lagging current the resistance R should be *increased*. The reverse is true with leading current.

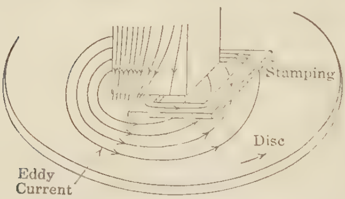


Fig. 80.—Shaded-pole principle of the light-load adjustment.

L is a small metallic stamping placed under the potential lug and can be moved laterally by means of the lever K. Its function is to provide the small torque just necessary to overcome the friction of the meter. The operation of this adjustment is as follows:

Figure 80 shows the stamping under the lug, set off center. When the flux starts to pass down through the lug a current is immediately induced in the short-circuited stamping. This current, by Lenz's law, opposes the flux entering the stamping so that the flux is crowded to the left-hand side of the lug as shown.

When the flux starts to decrease, the current in the short-circuited stamping tends to oppose the decrease in the flux. This retards the time-phase of the flux in the right-hand side of the lug with respect to that in the left-hand side of the lug. The result is a sweeping of the flux from left to right across the lug. This sliding flux cuts the disc and sets up eddy currents in it. These currents, reacting with the flux, produce a torque tending to drive the disc in the direction in which the stamping is displaced from its position of symmetry. This is the "shaded-pole" principle which is also used to start small single-phase induction motors (see page 269).

The driving torque of the meter is produced by currents induced in the disc, these currents reacting with an alternating magnetic field. For example, the alternating flux produced by the current lugs *CC* cuts the disc and induces eddy currents in it. These currents find themselves in an alternating magnetic field of the same frequency, produced by the potential lug. Motor action results and the disc tends to rotate. (The currents induced in the disc by the potential lug may also be considered as reacting with the magnetic field produced by the current lugs *CC*.) It may be shown that with proper compensation, the driving torque at each instant is proportional to the power at that instant.

In order to register watthours, a retarding torque proportional to the angular velocity of the disc is necessary. As with the direct-current watthour meter this retarding torque is obtained by the disc cutting a field of constant strength produced by *permanent* magnets. This causes a *retarding* torque which is proportional to the speed of the disc. Therefore, both the *driving* torque (motor action) and the *retarding* torque (generator action) are produced on the same disc.

57. Calibration and Adjustment of the Induction Watthour Meter.—The induction watthour meter is calibrated in much the same manner as the direct-current watthour meter. A standard indicating wattmeter is used to measure the average power over a stated interval and the revolutions of the disc of the watthour meter are counted with the aid of a stop watch. The average meter watts are calculated by means of the equation

$$W = \frac{K \times N \times 3{,}600}{t} \qquad (38)$$

where K is the meter constant, N the revolutions of the disc and t the time in seconds.

As a rule, an ammeter and a voltmeter are used in connection with such a test, as shown in Fig. 81, in order to determine the power-factor. Instrument losses should be carefully investigated and corrections made if necessary. In Fig. 81, the wattmeter is connected so that it measures the correct voltage output of the meter, but it does not measure the correct current output, since some of the current coming through the meter flows through the wattmeter potential-circuit. Therefore to obtain the correct output of the watthour meter with this connection of the wattmeter, the loss in its potential-circuit must be *added* to the wattmeter reading. No correction for the ammeter and voltmeter power is necessary since they form a part of the load.

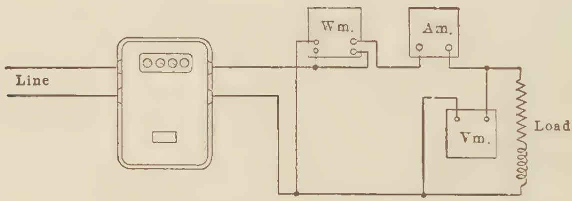


FIG. 81.—Connections for testing alternating-current watthour meter.

After the meter is adjusted at full load and unity power-factor by means of the retarding magnets, it is adjusted at light load by means of the light-load adjustment. The power-factor is then lowered. Any error occurring now must be due to improper lagging. The registration is then made correct by adjusting the resistance R (Fig. 79) which is in series with the lagging coil. If the meter registers low with lagging current, the resistance R should be decreased; if it registers high the resistance R should be increased. With leading current these operations should be reversed.

The induction watthour meter has certain advantages over the direct-current meter. As there is no coil-wound armature, the rotating element of the induction meter is much lighter than that of the direct-current meter. Moreover, it has no commutator or delicate brushes, which are frequent sources of trouble with the direct-current meter.

The induction meter is also made in the polyphase type. Two single-phase elements act on a common spindle. There are two sets of damping magnets.[1]

58. Frequency Indicators.—Frequency indicators are based on two principles, that of electrical resonance and that of mechanical resonance. The latter type is the more common and is simpler in operation. A number of steel reeds, each having a white index on its end, are clamped between two metal strips. Each reed has its own mechanical frequency of vibration. Behind this bank of reeds there is an electromagnet, the coil of which is excited by the circuit whose frequency it is desired to measure. The reed whose frequency is that of the circuit will vibrate with the greatest amplitude (Fig. 82). With the exception of one or two reeds near this one, none of the others will be

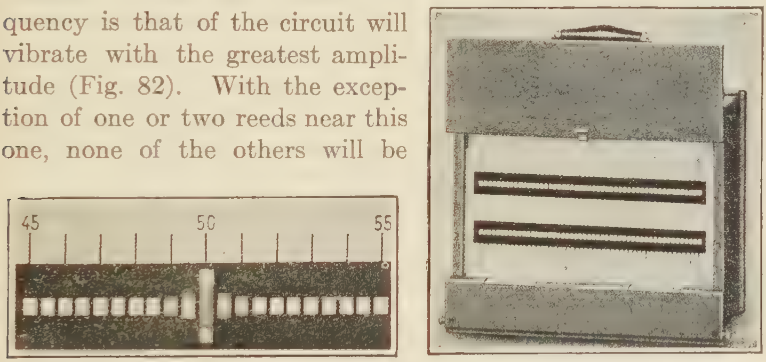


FIG. 82.—Frahm vibrating-reed frequency meter.

affected. Therefore the frequency is determined by noting the scale reading opposite this reed. Were the reeds unpolarized, they would be attracted equally well by either a north or a south pole. An adjacent permanent magnet keeps the reeds polarized, so that the reed of a particular mechanical frequency will respond to the same electrical frequency. Figure 82 shows the Frahm type of indicator, as manufactured by Hartmann and Braun.

59. Synchroscope.—Before connecting an alternator to the bus-bars and in parallel with other alternators, it is necessary not only that its voltage be the same as that of the bus-bars but that it be in phase opposition as well. This corresponds to having direct-current generators of the same polarity before connecting them in parallel.

[1] For a more detailed analysis, see F. A. Laws, "Electrical Measurements."

A synchroscope is an instrument for indicating when machines are in the proper phase relation for connecting in parallel, and at the same time for showing whether the incoming machine is running fast or slow. Figure 83 shows such a synchroscope. If the incoming machine has the same frequency as the bus-bars, the pointer remains stationary. When the machines are in the proper phase relation for closing the switch, the pointer is over an index on the dial, this position being shown in Fig. 83. The direction of rotation of the pointer shows whether the incoming machine is fast or slow. The generator switch is usually thrown when the pointer is rotating slowly in the "fast" direction and is approaching the index.

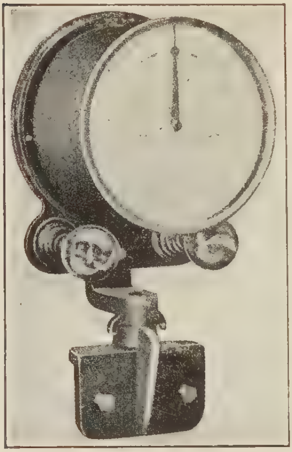


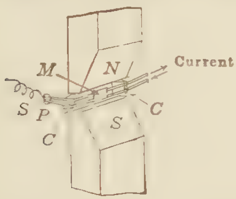
FIG. 83.—Synchronism indicator on swinging bracket.

60. The Oscillograph.—It is often desired to investigate transient conditions in electrical circuits, such, for example, as the current and voltage relations during the blowing of a fuse, or during the short-circuit of an alternator, or in oscillations produced by switching, etc. Further, it is desirable to have apparatus which will show the current and the voltage waves in alternating-current circuits during steady conditions. The oscillograph is an instrument which is capable of meeting these requirements.

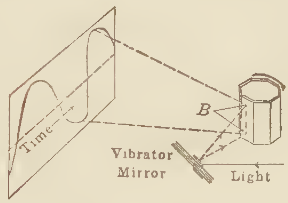
Its principle is quite simple, being that of a D'Arsonval galvanometer (Part I, Chap. VII, page 115), as shown in Fig. 84 (a). A small phosphor-bronze strip or filament is stretched over two clefts CC around a small pulley P and back again. The spring S acting on the pulley keeps the two lengths of the strip in tension. This filament is placed between the poles of a strong electromagnet. When a current flows through the filament, one length of the filament moves outwards and the other inwards. A very small mirror M is cemented across the two lengths of the filament and is given a rocking motion by this movement of the filament. If a beam of light be reflected from this mirror, it

will be drawn out into a straight line by the mirror vibration. If this beam of light be made to strike a rotating mirror, in the manner shown in Fig. 84 (*b*), the rotation of the mirror introduces a time element and the wave is drawn out so that its characteristics are shown.

The instrument is merely a galvanometer having a single turn and a very light moving element. This makes the moment of inertia very small. Also, the filament is under considerable tension, so that its natural frequency of vibration is very high, being from 3,000 to 10,000 cycles per second. These characteristics are necessary in order that the filament may respond accurately to the comparatively high-frequency variations which



Vibrating element of oscillograph.



Method of drawing out vibrating beam into a wave.

FIG. 84.

it is called upon to follow. The moving element is usually immersed in oil so that its movement is properly damped and the filament is kept cool.

Figure 85 shows the general arrangement of a laboratory type of oscillograph.

The light from the arc lamp strikes the two total-reflecting prisms by means of which the beam is turned at right angles and directed upon the vibrator mirrors at *V*. These mirrors reflect the light back through the cylindrical lens, which concentrates the beam. A plane mirror *M* reflects the light down to a rotating mirror which in turn reflects it, drawn out as a wave, on the viewing screen. It is often desired to obtain a photographic record of the phenomena which occur. For this purpose a sensitive photographic film is wound on the film drum, which is driven by a motor. The mirror *M* is then pulled up out of the way and a mechanism causes the shutter to open and close during

one revolution of the drum. In this case the time axis is furnished by the movement of the film.

The oscillograph vibrators are connected into the circuit in the same manner as direct-current ammeter and voltmeter coils are connected (Fig. 86). As the current vibrator can carry but a small current, about 0.1 amp., it is connected in parallel

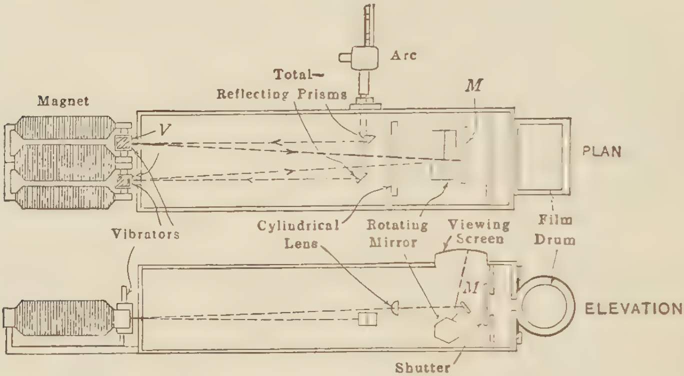


Fig. 85.—Typical oscillograph.

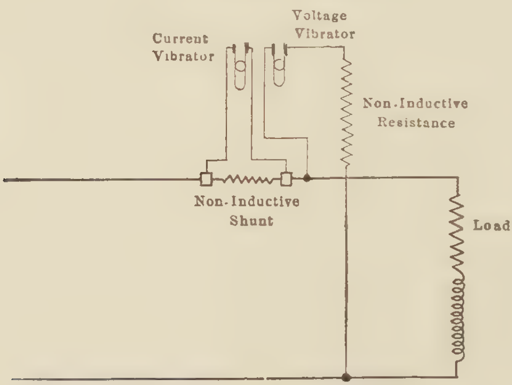


Fig. 86.—Method of connecting oscillograph vibrators in circuit.

with a non-inductive shunt which is in series with the line. The voltage vibrator is connected across the line in series with a high non-inductive resistance. The current vibrator will then vibrate with an amplitude proportional to the circuit current and in phase with it. The current through the voltage vibrator will be proportional to the circuit voltage and in phase with it.

CHAPTER IV

POLYPHASE SYSTEMS

With the possible exception of the comparatively few power plants which supply energy to single-phase electric railways, practically all alternating-current power is generated as polyphase power. Aside from single-phase railway motors, nearly all power apparatus rated at 10 hp. and over operates polyphase and, takes power, therefore, from polyphase systems. Hence, an understanding of polyphase systems is essential to the study of the industrial applications of alternating currents. It will be shown that polyphase systems are merely the proper combination of two or more single-phase systems, and when so considered, their analysis is comparatively simple.

61. Reasons for the Use of Polyphase Currents.—In many industrial applications of alternating currents, there are objections to the use of single-phase power.

In a single-phase circuit, the power delivered is pulsating. Even when the current and voltage are in phase, the power is zero twice in each cycle, as shown in Fig. 49 (page 56). When the power-factor is less than unity, not only is the power zero four times in each cycle, but it is also *negative* during two periods in each cycle (see Fig. 54, page 62). This means that the circuit returns energy to the generator for a part of the time and is analogous to a single-cylinder gasoline engine in which the flywheel returns energy to the cylinder during the compression part of the cycle. Over the complete cycle, both the single-phase circuit and the flywheel receive an excess of energy over that which they return to the source. The pulsating character of the power in single-phase circuits makes such circuits objectionable in many instances.

A polyphase circuit is somewhat like a multicylinder gasoline engine. With the engine, the power delivered to the flywheel is practically steady, as one or more cylinders are firing when the

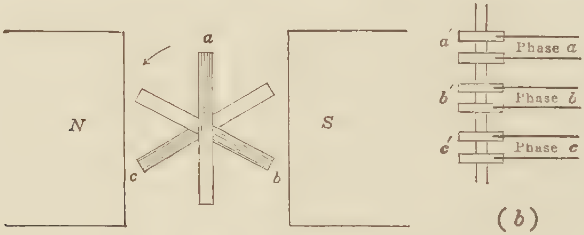
others are compressing. This same condition exists in polyphase electrical systems. Although the power of any one phase is pulsating and may be negative at times, the *total power* is constant if the loads are balanced. This makes polyphase systems highly desirable for power purposes.

The rating of a given motor, or generator, increases with the number of phases, an important consideration. Below are the approximate ratings of a given machine for different numbers of phases, assuming the single-phase rating as 100.

Single-phase	100
Two-phase	140
Three-phase	148
Six-phase	148
Direct-current	154

The same machine operating 3- or 6-phase has about 50 per cent greater rating than when operating single-phase. This is a very important consideration in favor of generating and utilizing polyphase power.

62. Generation of a 3-phase Current.—As the three-phase system is now the most common of the polyphase systems, it



(*a*) (*b*)

FIG. 87.—Generation of 3-phase current.

will be considered first. Figure 87 (*a*) shows three simple coils *a*, *b*, and *c*, which are all alike. They are fastened rigidly together and are mounted 120° apart on an axis which can be rotated. This combination of coils and axis constitutes an elementary 3-phase armature. This armature is shown rotating in a counter-clockwise direction in a uniform magnetic field. The current can be conducted from each of these three coils by means of slip-

rings, as shown in Fig. 87 (*b*). The terminals of coil *a* are connected to rings *a*′, those of *b* to rings *b*′, etc., making six slip-rings in all.

When the armature is in the position shown (Fig. 87 (*a*)), the coil *a* is in the magnetic neutral plane and its induced e.m.f. is zero (see page 10). This is shown in Fig. 88 (*a*) where the induced e.m.f. E_{oa} starts at its zero value and is increasing in a positive direction. When the armature has rotated through 120 space-degrees, the e.m.f. in coil *b* will be zero and increasing in a positive direction. This is shown by the e.m.f. curve E_{ob} (Fig. 88 (*a*)). That is, e.m.f. E_{ob} lags e.m.f. E_{oa} by 120 time-degrees, since this is a 2-pole alternator.

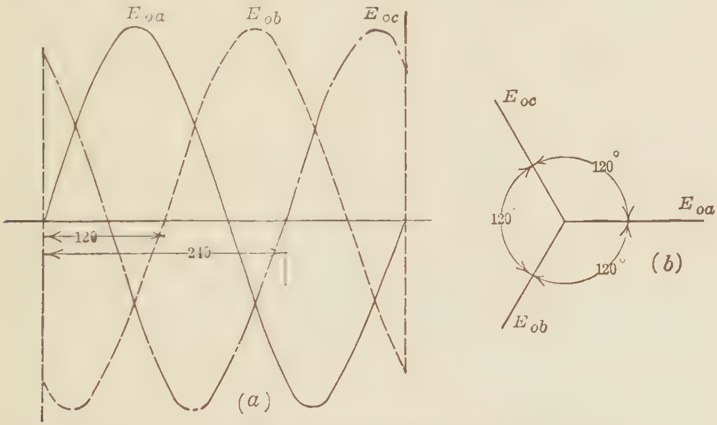


Fig. 88.—Three-phase voltage waves and vector diagram.

When the armature has rotated through 240 space-degrees, the e.m.f. in coil *c* will be zero and increasing in a positive direction. This is shown by the e.m.f. curve E_{oc} (Fig. 88 (*a*)). That is, e.m.f. E_{oc} lags E_{oa} by 240 time-degrees, and lags E_{ob} by 120 time-degrees. The sequence of phase rotation is E_{oa}, E_{ob}, and E_{oc}. These three e.m.fs. are equal, since the three coils are alike and all rotate at the same velocity in the same magnetic field.

Although commercial 3-phase alternators differ in construction from the elementary alternator shown in Fig. 87 (*a*),[1] the relations existing among the e.m.fs. are identical in the two types of machine. That is, the coils on the stationary armature of the

[1] See Chap. V, p. 145.

commercial alternator are so placed that the e.m.fs. generated in the three phases are equal and differ in phase by 120°.

The three e.m.fs., E_{oa}, E_{ob}, and E_{oc} are shown vectorially in Fig. 88 (*b*), the lengths of the vectors representing *effective* values.

It will be observed in Fig. 88 (*a*) that at any one instant the algebraic sum of the e.m.fs. is zero. That is, if the three coils were connected in series and in sequence such as *a-b-c*, the e.m.f. across the three would be zero (see page 113). For example, when E_{oa} is zero and increasing positively, E_{oc} is positive and equal to 86.6 per cent of its maximum value and E_{ob} is negative and equal to 86.6 per cent of its maximum value. Hence at this instant the sum of the three e.m.fs. is zero. When E_{oa} is positive maximum, E_{ob} and E_{oc} are negative and each is equal to half its maximum value. Hence at this instant also the sum of the three e.m.fs. is zero. Similarly it may be shown that the sum of the three e.m.fs. is zero at *every* instant of time.

Each of the coils of Fig. 87 (*a*) can be connected through its two slip-rings to a single-phase circuit. This gives six slip-rings and three independent single-phase circuits, phase *a*, phase *b*, and phase *c*. With a rotating field and stationary armature, which is the most common type of generator met in practice, the six slip-rings would not be necessary, but six leads would be taken directly from the armature. In practice, however, a machine seldom supplies three independent circuits by the use of six wires.

63. Symbolic Notation.—The solution of problems involving circuits and systems containing a number of currents and voltages is simplified and is less susceptible to error if the current and voltage vectors are designated by some systematic notation, of which the following is one type. If a voltage is acting to send current from point *a* to point *b* (Fig. 89 (*a*)) it shall be denoted by E_{ab}. On the other hand, if the voltage tends to send current from *b* to *a* it shall be denoted by E_{ba}. Obviously, $E_{ab} = -E_{ba}$. It may seem as if alternating currents cannot be considered as having direction since they are undergoing continual reversal in direction. The assumed direction of a current, however, is determined by the actual direction of the flow of *energy*. In an alternator, the energy comes *out* of the armature and the current is considered as flowing *out* of the armature, even although it is actually flowing *into* the armature for half the time.

Corresponding to the voltage E_{ab} (Fig. 89 (*a*)) the current I_{ab} flows from *a* to *b* in virtue of this voltage. The current flowing from *b* to *a* must be opposite in direction to that flowing from *a* to *b*. Therefore, $I_{ba} = -I_{ab}$. This relation is illustrated in Fig. 89 (*b*), in which I_{ab} differs in phase from I_{ba} by 180°. E_{ba} is 180° from E_{ab}.

In Fig. 89 (*a*), the voltage acting from *a* to *c* is equal to the voltage acting from *a* to *b* plus the voltage acting from *b* to *c*,

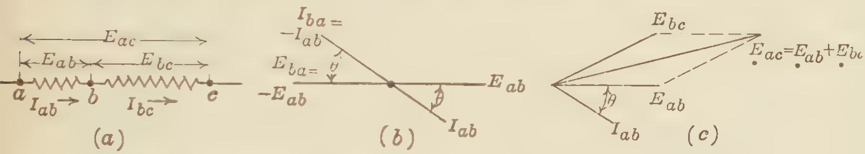


FIG. 89.—Symbolic notation applied to voltage and current vectors.

since the whole is equal to the sum of its parts. That is, $\underset{\cdot}{E}_{ac} = \underset{\cdot}{E}_{ab} + \underset{\cdot}{E}_{bc}$.

The order of the subscripts should be noted. The first and last subscripts on the left-hand side of the equation are the same and in the same order as the first and last on the right-hand side. The adjacent subscripts on the right-hand side must always be the same, as for example, the *b*'s.

In Fig. 89 (*c*), the voltage E_{ab} and E_{bc} are shown vectorially. Their vector sum E_{ac} is obtained by adding E_{ab} and E_{bc} vectorially.

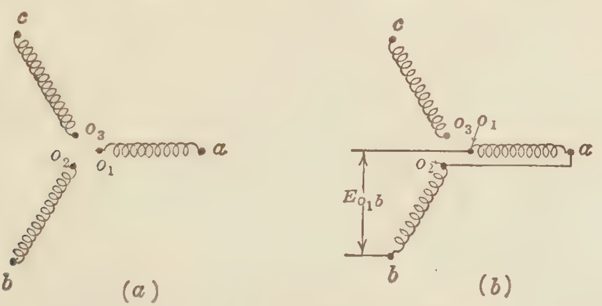


FIG. 90.—Addition of voltages differing in time-phase by 120°.

64. Combining 3-phase Voltages with 120° Phase Difference. The three coils, forming the armature of the 3-phase alternator (Fig. 87) may be shown diagrammatically by three coils,

o_1a, o_2b, and o_3c (Fig. 90 (a)) these coils representing coils a, b, and c, respectively of Fig. 87.

The voltage tending to send current (or energy) from o_1 to a is represented by the vector E_{o_1a} (Fig. 91); the voltage tending to send current from o_2 to b is represented by the vector, E_{o_2b}; and the voltage tending to send current from o_3 to c is represented by the vector E_{o_3c}. Let it be required to determine the voltage between coil ends o_1 and b when o_2 is connected to a (Fig. 90 (b)).

The two voltages E_{o_1a} and E_{o_2b} differ by 120° in phase as is shown by the vectors (Fig. 91 (a)). The two coils are connected in series, since the method of connection is from the inner terminal o_1 to the outer terminal a, thence to the inner terminal o_2 and to the outer terminal b. Hence the voltage E_{o_1b} across o_1b is the vector sum of E_{o_1a} and E_{o_2b}. This vector addition is shown in

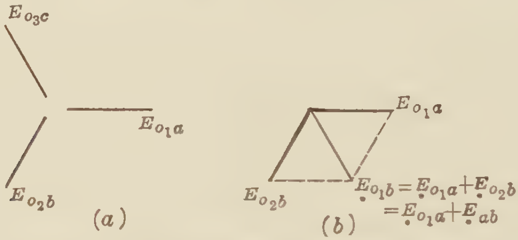


FIG. 91.—Vector addition of 3-phase voltages.

Fig. 91 (b). The resultant voltage E_{o_1b} is equal numerically to each of the component voltages.

The points a and o_2 now coincide, and $\dot{E}_{o_1b} = \dot{E}_{o_1a} + \dot{E}_{ab}$. It is to be noted that the symbolic notation described in Par. 63 is followed. The first and last subscripts on the left-hand side of the equation are the same as the first and last subscripts on the right-hand side of the equation. The last subscript a of the first term of the right-hand side of the equation is the same as the first subscript of the second term.

65. Combining 3-phase Voltages with 60° Phase Difference.—In Fig. 90, the *outer* end of coil o_1a was connected to the inner end of coil o_2b and the resultant voltage across the open ends was found. Let it be required to find the resultant voltage across these same two coils when the connections to coil o_1a are reversed. The *inner* end o_1 of coil o_1a is now connected to the inner end o_2

of coil o_2b, Fig. 92 (a). It is desired to obtain the voltage E_{ab} across their open ends.

Since the connection of the coil o_1a has been reversed with respect to o_2b, the vector E_{o_1a} representing the voltage of the coil o_1a, a must be reversed with respect to E_{o_2b}. This is done in Fig. 92 (b) where E_{ao_1} is E_{o_1a} reversed. The resultant voltage is found by adding E_{ao_1} and E_{o_2b} (Fig. 92 (b)). That is,

$$\dot{E}_{ab} = \dot{E}_{ao_1} + \dot{E}_{o_2b}$$

Since o_1 and o_2 are now connected together, these points are at the same potential. Hence, $E_{o_2b} = E_{o_1b}$. Therefore, $\dot{E}_{ab} = \dot{E}_{ao_1} + \dot{E}_{o_1b}$. The relation of subscripts should be noted. The resultant voltage represented by the vector E_{ab} is greater in magnitude than either of the component voltages. In fact it is equal

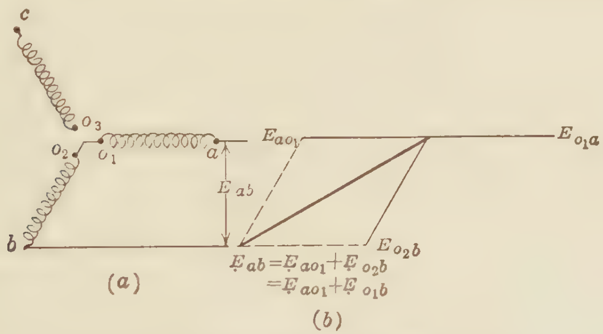


Fig. 92.—Vector addition of 3-phase voltage.

numerically to either voltage multiplied by $\sqrt{3} = 1.732$ as will be shown later. It leads E_{ao_1} by 30° and lags E_{o_2b} by 30°.

The foregoing method of using letter subscripts with the voltage (or current) symbols eliminates the necessity for arrowheads with the vectors and by maintaining the relations of the subscripts in the manner just described, confusion as to the direction of a vector is avoided.

It is to be noted that if two 3-phase alternator coils be connected from outer to inner end, the voltage across the open ends is equal numerically to either coil voltage. If the inner ends (or the outer either) be connected together, the resultant voltage across the open ends is equal numerically to $\sqrt{3}$ times the voltage of either coil.

66. Y-connection.—If the inner ends o_1, o_2, and o_3 of the three coils (Fig. 92 (*a*)) be connected together at the common point *o* (Fig. 93 (*a*)), the coils are said to be Y-connected. Three lines *aa'*, *bb'*, and *cc'* from the outer ends carry the energy to the

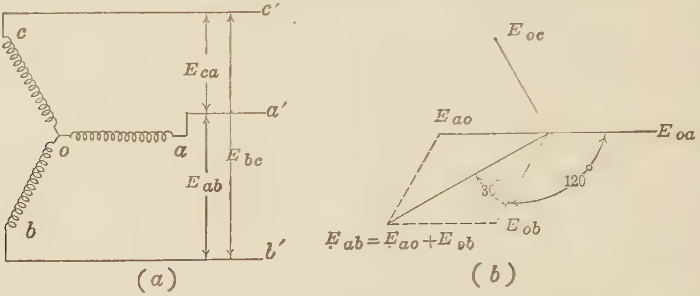


FIG. 93.—Y-connection and corresponding voltage vector diagram.

external circuit. A fourth wire from the neutral *o* is sometimes employed, giving a 3-phase, 4-wire system (see page 309). Let the three coil-voltage vectors be E_{oa}, E_{ob}, and E_{oc}. These three coil voltages are all equal and differ in phase from one another by 120°, as shown in the vector diagram (Fig. 93 (*b*)). Let it be required to find the three line voltages E_{ab}, E_{bc}, and E_{ca}.

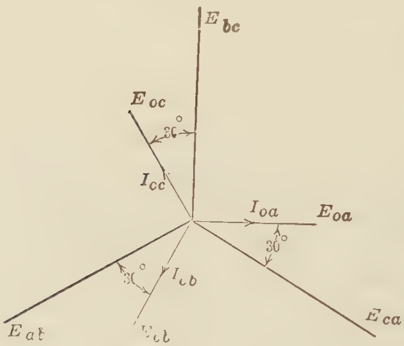


FIG. 94.—Relation of line to coil voltages and currents in a Y-system, unity power-factor.

From the sequence of the subscript notation and also from consideration of Fig. 92, the voltage $\dot{E}_{ab} = \dot{E}_{ao} + \dot{E}_{ob}$.

The voltage E_{ob} is given in the vector diagram (Fig. 93 (*b*)). The voltage E_{ao} is not given, but the voltage E_{oa} is given. E_{ao} may be obtained by reversing E_{oa} as shown. The voltage E_{ab} is then found by adding vectorially E_{ao} and E_{ob}. As in Fig. 92 (*b*), the voltage E_{ab} has a magnitude equal to the $\sqrt{3}$ or 1.732 times the coil voltage and it differs in phase from the coil voltage by 30°.

In a similar manner the other two line voltages, E_{bc} and E_{ca} may be obtained. That is, $\dot{E}_{bc} = \dot{E}_{bo} + \dot{E}_{oc}$ and $\dot{E}_{ca} = \dot{E}_{co} + \dot{E}_{oa}$. The three coil voltages and the three resulting line voltages, E_{ab}, E_{bc}, and E_{ca} are shown in Fig. 94. (Notice the sequence of subscripts for the line voltages, E_{ab}, E_{bc}, and E_{ca}.) *In a balanced 3-phase system, the line voltage is equal to the coil voltage multiplied by* $\sqrt{3}$. *The three line voltages are all equal and differ in phase by* 120°. *There is a phase difference of* 30° *between the line and the coil voltage.*

Example.—Each of the three coil-circuits of a 3-phase, Y-connected alternator generates an e.m.f. of 220 volts. What is the e.m.f. across each of the three line terminals?

The three line voltages,

$$E_{line} = \sqrt{3} \times 220 = 381 \text{ volts.} \quad \textit{Ans.}$$

67. Currents in the Y-system.—Since each of the line wires, aa', bb', and cc' (Fig. 93 (a)) is in series with one of the alternator coil-circuits, the current in each coil-circuit must be equal to the current in the line to which it is connected or otherwise current would enter or leave at one of the junctions, as at a. *In a Y-system, therefore, the line currents and their respective coil currents must be equal.*

It follows from Kirchhoff's first law that the sum of the three coil currents in a Y-system having no neutral wire must be zero. The fact that this is true for a balanced load may be seen by studying Fig. 88 (a), as the three curves may also represent the three Y-currents, rather than voltages, since the currents are equal and differ in phase from one another by 120°. It has already been shown that at any instant the algebraic sum of the three curves (Fig. 88 (a)) is zero. Hence the algebraic sum of the three currents at every instant is zero.

Figure 94 shows vectorially the three coil currents, each in phase with its coil voltage. Since the three currents are equal and differ in phase by 120°, their vector sum is zero.

With unbalanced loads, that is with unequal currents not necessarily having phase differences of 120°, the algebraic sum of the currents at any instant and the vector sum of the currents must also be zero.

With unity power-factor, the coil currents are in phase with their respective coil terminal voltages. The currents under these condi-

tions are *not* in phase with the line voltages. This is illustrated in Fig. 94. The current I_{oa} is in phase with its coil voltage E_{oa}, the current I_{ob} is in phase with its coil voltage E_{ob}, etc. The current I_{oa} leads the line voltage E_{ca} by 30°. The coil current I_{oa} is the line current $I_{aa'}$. Hence, with unity power-factor in a 3-phase system, the line current differs in phase by 30° from the line voltage.

If the power-factor is other than unity, the coil current lags (or leads) the *coil* voltage by an angle θ, as shown in Fig. 95. For example, the current I_{oa} in coil *oa* lags the *coil* voltage E_{oa} by the angle θ. The current I_{oa} lags the *line* voltage E_{ca} by the angle $\theta - 30°$).

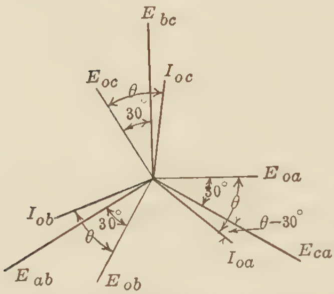


Fig. 95.—Relation of line to coil voltages and currents in a Y-system, power-factor = cos θ.

The current I_{oa} is the line current $I_{aa'}$. Hence, when the coil current differs in phase from the coil voltage by the angle θ, the line current differs in phase from the line voltage by the angle $\theta - 30°$ (or $30° - \theta$).

An inspection of Fig. 95 shows that the current in the other two coils also lags its respective coil voltage by the angle θ, but lags the line voltage by the angle $\theta - 30°$. It will be shown that the power-factor of such a 3-phase system is *cos* θ, and *not* cos $(\theta - 30°)$. In other words the power-factor is determined by the phase angle between *the coil voltage* and *the coil current.*

68. Power in a Y-system.—The power developed in each of the three coils (Fig. 93 (*a*)) must be independent of the system to which the coil is connected. If, for example, E_{oa} is the voltage across coil *oa* and a current I_{oa} flows in coil *oa*, and the voltage and current are in phase (Fig. 94), the power in coil *oa* must be equal to $E_{oa}I_{oa}$. That is, the power delivered by each coil is

$$P' = E_{oa}I_{oa} \text{ (unity power-factor).}$$

A balanced 3-phase system is assumed. Hence the coil currents are equal numerically, and each is in phase with its respective coil voltage.

The total power of the system is, therefore,

$$P = 3E_{coil}I_{coil}. \qquad \text{(I)}$$

In the Y-system the coil current equals the line current, and the coil voltage

$$E_{coil} = \frac{E_{line}}{\sqrt{3}}.$$

Substituting in (I) $E_{line}/\sqrt{3}$ for the value of E_{coil}

$$P = \frac{3}{\sqrt{3}} E_{line}I_{coil} = \sqrt{3}\, E_{line}I_{line}. \qquad (39)$$

In a balanced 3-phase system, the line power at unity power-factor is equal to $\sqrt{3}$ times the line voltage multiplied by the line current.

Example.—A 3-phase, Y-connected generator delivers 80 amp. to each line and the terminal voltage (between lines) is 2,300 volts. The coil voltage and the coil current are in phase with each other, making the power-factor of the system unity. (*a*) What is the coil voltage? (*b*) What power is delivered by each coil? (*c*) What is the system power?

(*a*) The coil voltage,

$$E_{coil} = \frac{2{,}300}{\sqrt{3}} = 1{,}328 \text{ volts.} \quad \textit{Ans.}$$

(*b*) $P' = 1{,}328 \times 80 = 106{,}200$ watts $= 106.2$ kw. *Ans.*

(*c*) $P = 106.2 \times 3 = 318.6$ kw. *Ans.*

Also using equation (39)

$P = \sqrt{3} \times 2{,}300 \times 80 = 318.7$ kw. (check).

If each coil current lags (or leads) its coil terminal voltage by the angle θ (Fig. 95) the power per coil (from equation (25), page 66) is

$$P' = E_{coil}\, I_{coil} \cos \theta_{coil}.$$

The total power of the system

$$P = 3E_{coil}\, I_{coil} \cos \theta_{coil}.$$

Substituting, $E_{coil} = E_{line}/\sqrt{3}$ and $I_{coil} = I_{line}$.

The system power becomes

$$P = \sqrt{3}\, E_{line}I_{line} \cos \theta_{coil} \qquad (40)$$

and the system power in kilowatts is equal to

$$\frac{\sqrt{3}}{1{,}000} E_{line}\, I_{line} \cos \theta_{coil}. \qquad (41)$$

It is clear from equations (39) and (40) that $\sqrt{3}\ E_{line} I_{line}$ gives the *volt-amperes* of the system. The power is equal to the product of the volt-amperes and the power-factor (see page 63). The *kilovolt-amperes*

$$\text{Kv-a.} = \frac{\sqrt{3}}{1{,}000} E_{line} I_{line}. \tag{42}$$

The system power-factor, which is the coil power-factor, is from (41)

$$\text{P.F.} = \cos \theta_{coil} = \frac{P}{\sqrt{3}\ E_{line} I_{line}} \tag{43}$$

where P is the total power of the system.

Therefore, in a balanced 3-phase system, the system power-factor is the cosine of the angle between the coil current and the coil terminal voltage.

If the system is unbalanced, that is, if the currents or voltages are not equal or are not 120° apart, the question arises as to just what the system power-factor is under these conditions. Where such unbalancing is not very great, equation (43) is used, the line currents and line voltages being averaged. The system power-factor has little significance when the unbalancing is considerable.

Example.—A 3-phase, Y-connected alternator has three coils, each of which delivers 150 amp. at 1,328 volts, and the phase difference between the coil voltage and coil current is 40°, the current lagging. (*a*) What is the voltage rating of the alternator? (*b*) What is the kilovolt-ampere rating of the alternator? (*c*) At what power-factor is it operating? (*d*) What power in kilowatts is it delivering?

(*a*) $1{,}328\sqrt{3} = 2{,}300$ volts. *Ans.*

(*b*) From equation (42)

$$\text{Kv-a.} = \frac{\sqrt{3}}{1{,}000}\ 2{,}300 \times 150 = 598. \quad \textit{Ans.}$$

(*c*) $\text{Cos}\ \theta_{coil} = \cos 40° = 0.766.$ *Ans.*

(*d*) From equation (41).

$$P = \frac{\sqrt{3}}{1{,}000}\ 2{,}300 \times 150 \times 0.766 = 458 \text{ kw.} \quad \textit{Ans.}$$

Also from (*b*)

$$P = 598 \times 0.766 = 458 \text{ kw.} \quad \text{(check.)}$$

69. Delta-connection.—Figure 90 (*b*) (page 105) shows the outer end *a* of one of the three coils or coil-circuits of a three-

phase alternator connected to the inner end o_2 of the next coil. The outer end b of coil o_2b may be connected to the inner end o_3 of coil o_3c; and likewise the outer end c of coil o_3c may be connected to inner end o_1 of coil o_1a forming a closed circuit.

This connection of the three coils is shown in Fig. 96 (a). It will be observed that points o_2 and a, o_3 and b, and o_1 and c,

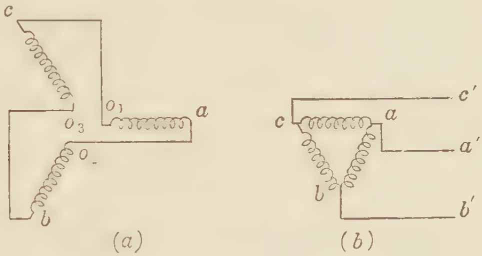


FIG. 96.—The delta-connection of alternator coils.

are common. Hence the o's may be eliminated. This is done in Fig. 96 (b) which shows the same three coils arranged to form the sides of an equilateral triangle.

The voltage E_{ca} is now the same as E_{o_1a}, E_{ab} the same as E_{o_2b}, and E_{bc} the same as E_{o_3c} (Fig. 91). These three voltages with the new subscripts are shown in Fig. 97.

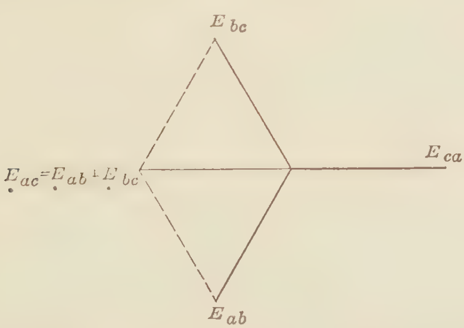


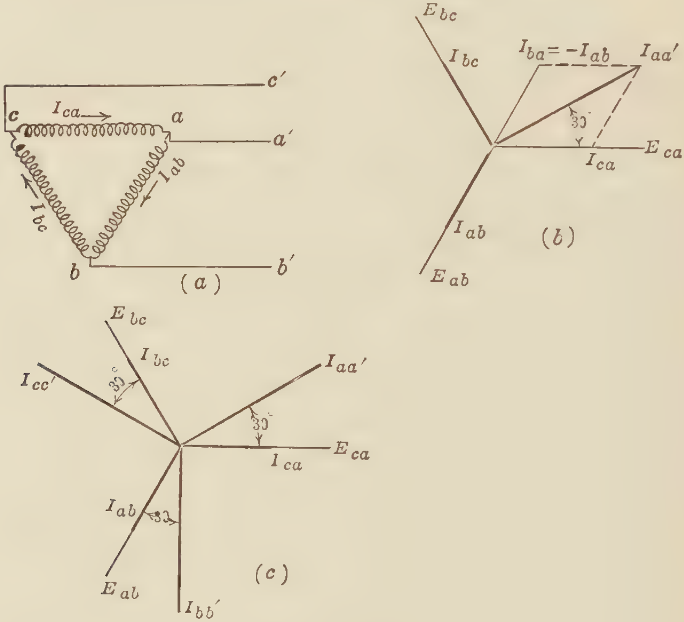
FIG. 97.—Vector sum of 3 delta voltages equals zero.

This method of connecting the three elements or coil-circuits of a 3-phase system is called the *delta-connection.*

Under balanced conditions no current circulates around the delta, even though it is a closed circuit. The total voltage acting

around the delta is $\dot{E} = \dot{E}_{ab} + \dot{E}_{bc} + \dot{E}_{ca}$. The sum of three vectors all equal in magnitude and 120° apart is zero, as a study of Fig. 97 will show. If, for example, E_{ab} and E_{bc} are added vectorially, their sum E_{ac} is equal and opposite to E_{ca}. This sum, when added to E_{ca} gives a resultant zero.

The three line connections are made at the junctions of the three coils at a, b, and c (Fig. 96 (b)). It is clear that the three line voltages are the same as the three coil voltages. *Therefore, in a delta-system the line voltages and the coil voltages are equal.*



(c)

FIG. 98.—Currents and voltages in a delta-system

70. Currents in a Delta-system.—Figure 98 (a) shows the delta-system of Fig. 96 with the three coil currents I_{ab}, I_{bc}, and I_{ca} flowing from a to b, from b to c, and from c to a, respectively. Assume balanced conditions and also assume that each current is in phase with its coil voltage as shown by the vector diagram (Fig. 98 (b)). That is, I_{ab} is in phase with E_{ab}, etc.

Let it be required to find the value of the line current $I_{aa'}$. At junction a, I_{ca} flows toward the junction, I_{ab} flows away from the junction and I_{aa}' flows away from the junction.

By Kirchhoff's first law, the current

$$I_{aa}' = I_{ca} - I_{ab}.$$

To subtract one vector from another, it is reversed and added (see page 28).

In the vector diagram (Fig. 98 (b)) the current $(-I_{ab})$ is found by reversing I_{ab} giving I_{ba}. I_{ba} or $(-I_{ab})$ is then added to I_{ca} to give $I_{aa'}$. Since $(-I_{ab})$ and I_{ca} are 60° apart and are equal numerically, their vector sum $I_{aa'} = \sqrt{3}\, I_{ca}$ or $1.732\, I_{ca}$.

If $I_{bb'}$ is found in a similar manner by subtracting I_{bc} from I_{ab}, and $I_{cc'}$ by subtracting I_{ca} from I_{bc}, the three line currents will have the magnitudes and directions shown in Fig. 98 (c). It is to be noted that the three line currents differ in phase from their respective coil currents by 30°. Also when the coil currents are in phase with their respective coil voltages, the line currents differ in phase from the line voltages by 30°. This is also true of the Y-system (see Fig. 94). The foregoing relations between line currents and coil *currents* hold, whether or not the coil currents are in phase with their respective coil voltages.

Therefore, in a balanced delta-system, the line currents are equal to $\sqrt{3}$ times the coil currents.

Example.—The three coils of the alternator (page 109) are connected in delta, and each delivers 90 amp. at 220 volts. What is the current in each line?

$$I_{line} = \sqrt{3} \times 90 = 1.732 \times 90 = 156 \text{ amp.} \quad \textit{Ans.}$$

There is obviously 220 volts across each line.

71. Power in the Delta-system.—As in the Y-system, the power developed in any one of the coils of a delta-system must be independent of the system to which the coil is connected.

That is the power in each coil,

$$P' = E_{coil} I_{coil} \cos \theta_{coil}.$$

Where θ_{coil} is the phase angle between coil voltage and coil current. The total power developed by the three coils,

$$P = 3E_{coil} I_{coil} \cos \theta_{coil}.$$

In the delta-system, $E_{coil} = E_{line}$, and $I_{coil} = I_{line}/\sqrt{3}$.

Hence,

$$P = \frac{3}{\sqrt{3}} E_{line} I_{line} \cos \theta_{coil}.$$
$$= \sqrt{3} E_{line} I_{line} \cos \theta_{coil}, \tag{44}$$

as in the Y-system.

The kilowatts,

$$\text{Kw.} = \frac{\sqrt{3}}{1{,}000} E_{line}\, I_{line} \cos \theta_{coil}. \tag{45}$$

Obviously, the kilovolt-amperes,

$$\text{Kv-a.} = \frac{\sqrt{3}}{1{,}000} E_{line} I_{line}. \tag{46}$$

The power-factor in the delta-system is the *coil* power-factor and is equal to the cosine of the angle between the coil terminal voltage and the coil current. Hence, from (44),

$$\text{P.F.} = \cos \theta_{coil} = \frac{P}{\sqrt{3}\, E_{line}\, I_{line}} = \frac{\text{kw.}}{\text{kv-a.}}.$$

Example.—Consider the 3-phase alternator of page 112, but with its three coils, each delivering 150 amp. at 1,328 volts, connected in delta. The phase difference between coil voltage and current is 40°. (*a*) What is the voltage rating of the alternator? (*b*) What is its ampere rating? (*c*) What is its kilovolt-ampere rating? (*d*) At what power-factor is it operating? (*e*) What power is it delivering?

(*a*) Since the line voltage and coil voltage are equal, the voltage rating must be 1,328 volts. *Ans.*

(*b*) $I = \sqrt{3} \times 150 = 260$ amp. *Ans.*

(*c*) From equation (46)

$$\text{Kv-a.} = \frac{\sqrt{3}}{1{,}000} 1{,}328 \times 260 = 598. \quad \textit{Ans.}$$

(*d*) P.F. $= \cos 40° = 0.766$.

(*e*) $P = \frac{\sqrt{3}}{1{,}000} 1{,}328 \times 260 \times 0.766 = 458$ kw. *Ans.*

It will be observed that the kilovolt-ampere rating and also the kilowatt rating of an alternator do not change when the connection is altered from Y to delta.

72. Y- and Delta-connected Loads.—Not only may sources of energy, such as alternator coils, be connected in either Y or delta, but loads, such as resistances, transformer primaries, coils of 3-phase motors, etc. may be connected in either Y or delta. This is illustrated by the following examples.

Example.—Three resistances, each of 10 ohms are connected in Y across 100-volt, 60-cycle mains (Fig. 99). (*a*) How much current does each resistance take? (*b*) What current flows in the line? (*c*) What is the power of the system?

(*a*) The voltage across each resistance

$$E_R = \frac{100}{\sqrt{3}} = 57.7 \text{ volts.}$$

$$I = \frac{E}{R} = \frac{57.7}{10} = 5.77 \text{ amp.} \quad \textit{Ans.}$$

(*b*) Since the resistances are connected in Y, the line current must be the same as the current in the resistances, or 5.77 amp. *Ans.*

(*c*) The power taken by each resistance

$$P_R = 57.7 \times 5.77 = 333 \text{ watts.}$$

The total power,

$$P = 3 \times 333 = 1{,}000 \text{ watts.} \quad \textit{Ans.}$$

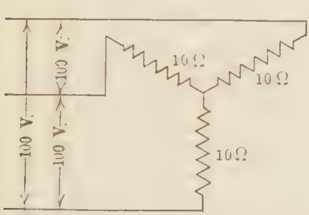


FIG. 99.—Resistances connected in Y.

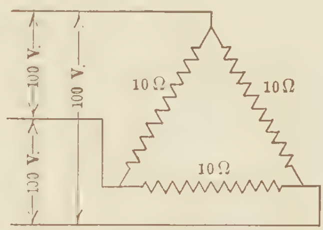


FIG. 100.—Resistances connected in delta.

Using equation (39) (page 111)

$$P = \sqrt{3} \times 100 \times 5.77 = 1{,}000 \text{ watts.} \quad \textit{Ans.}$$

Example.—Repeat the foregoing problem with the three resistances connected in delta across the same system (Fig. 100).

(*a*) Since each resistance is across two conductors of the 3-phase system, the voltage across each is 100 volts.

The current in each

$$I_R = \frac{100}{10} = 10 \text{ amp.} \quad \textit{Ans.}$$

(*b*) The line current

$$I = 10\sqrt{3} = 17.32 \text{ amp.} \quad \textit{Ans.}$$

(*c*) The power taken by each resistance,

$$P_R = 100 \times 10 = 1{,}000 \text{ watts.}$$

The total power,

$$P = 3 \times 1{,}000 = 3{,}000 \text{ watts.} \quad \textit{Ans.}$$

Using equation (44)

$$P = \sqrt{3} \times 100 \times 17.32 = 3{,}000 \text{ watts.} \quad \textit{Ans.}$$

It will be observed that three resistances connected in delta across a given 3-phase system, take three times the power that they take when connected in Y.

Example.—Each of three lamp-banks operates at 115 volts and at this voltage takes 10 amp. (Fig. 101). If these lamp-banks are connected in Y and to neutral across a 3-phase, 4-wire system with 115 volts to neutral, determine: (*a*) the line current; (*b*) the line voltage; (*c*) the current in the neutral; (*d*) the total power.

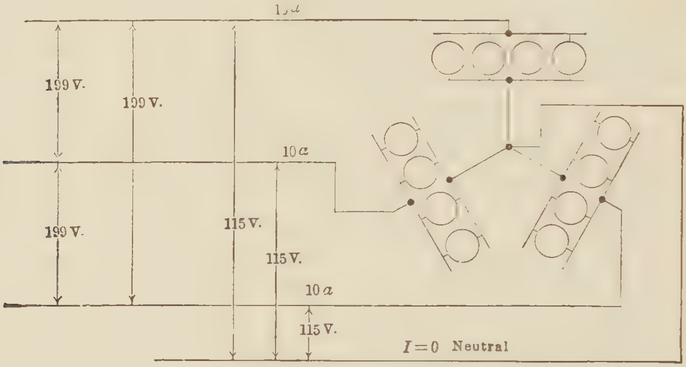


FIG. 101.—115-Volt lamp banks connected in Y.

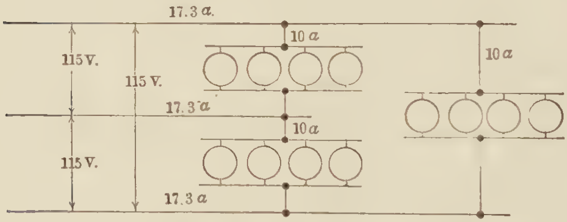


FIG. 102.—115-Volt lamp banks connected in delta.

(*a*) The line current must equal the current taken by each lamp-bank and is, therefore, 10 amp. *Ans.*

(*b*) The line voltage

$$E = \sqrt{3} \times 115 = 199 \text{ volts.} \quad \textit{Ans.}$$

(*c*) Since the three currents are all equal and differ in phase by 120°, their vector sum must be zero. Hence, the neutral current is zero.

(*d*) The power,

$$P = \sqrt{3} \times 199 \times 10 = 3{,}450 \text{ watts.} \quad \textit{Ans.}$$

Example.—If the three lamp-banks of the foregoing problem are connected in delta and operate at the same voltage, Fig. (102) determine: (*a*) the current in each line conductor; (*b*) the total power.

(*a*) Since the lamp-banks are now in delta, the line current,

$$I = \sqrt{3} \times 10 = 17.32 \text{ amp.}$$

(*b*) The power

$$P = \sqrt{3} \times 115 \times 17.32 = 3{,}450 \text{ watts.} \quad \textit{Ans.}$$

In this case the power does not change when the lamp-banks are changed from the Y- to the delta-connection, since the voltage is simultaneously reduced to $1/\sqrt{3}$ times its former value.[1]

73. Uses of Y- and Delta-connections.—In many instances it is immaterial whether the Y- or the delta-connection is used. For example, small alternators operate satisfactorily whether connected in Y or in delta. With units of large output, circulatory currents would probably flow around the delta, if this connection were used. These circulatory currents are due to the wave-form being non-sinusoidal. With the Y-connection, such circulatory currents cannot exist. At the same time the wave-form of the terminal voltage is improved by the use of the Y-connection. The Y-connection gives a neutral with which the system may be grounded. With motors, such as induction motors, both connections give equally good results. In fact some manufacturers connect a motor in delta for 220 volts, for example, and connect it in Y for 440 volts. Thus one line of motors can be readily adapted to two different voltages.[2]

METHODS OF MEASURING POWER IN 3-PHASE SYSTEM

74. Three-wattmeter Method.—Let (1), (2), and (3) (Fig. 103 (*a*)) be the three coils of either a Y-connected alternator or of a Y-connected load. (The ordinary wattmeter reads up scale when connected as shown, if the coils shown diagrammatically represent a source of energy. If these coils represent loads, the current-coil connections of the wattmeter should be reversed (see Fig. 104).) If the neutral of the Y is accessible, it is possible to measure the power of each phase by connecting the current-coil of a wattmeter in series with the phase and by connecting

[1] See p. 157 for the method of phasing coils in Y and in delta.

[2] For the discussion of the delta- and Y-connection for transformers, see p. 217.

the wattmeter potential-coil across the phase, as shown in Fig. 103 (*a*). Therefore, W_1, W_2, and W_3 measure the power in loads 1, 2, and 3 respectively, regardless of power-factor, degree of balance, etc.

The total power

$$P = W_1 + W_2 + W_3.$$

If the loads are balanced,

$$W_1 = W_2 = W_3.$$

If the potential-circuits of the three wattmeters have equal resistances, these three potential-circuits constitute a balanced Y-load, having a neutral O'. As coils 1, 2, and 3 and these 3-wattmeter potential-circuits are both symmetrical systems, O' must be at the same potential as O. Therefore, no current flows

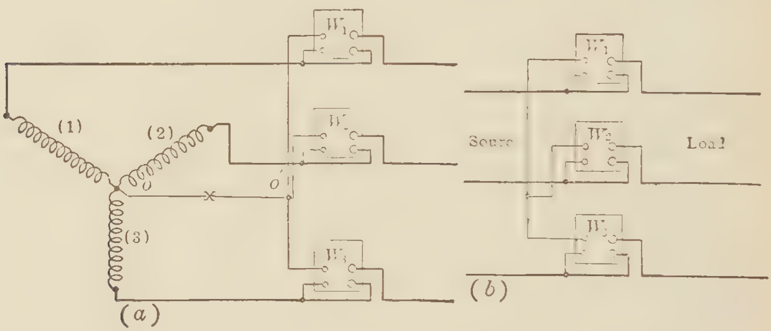


FIG. 103.—The 3-wattmeter method of measuring 3-phase power.

between O and O' and the line can be cut at X without changing existing conditions. Figure 103 (*b*) shows the 3-wattmeter connection for a 3-phase system. It can be shown that the total power is the sum of the wattmeter readings even though the wattmeter potential-circuits have different resistances. Under these conditions, however, the wattmeters may not all have the same reading, even with balanced loads.

The 3-wattmeter method is well adapted to measuring power in a system where the power-factor is continually changing, as in obtaining the phase characteristics of a synchronous motor. If the three instruments have equal potential-circuit resistances, they read alike regardless of power-factor, if the loads are bal-

anced. The 3-wattmeter method is necessary in a 3-phase, 4-wire system, as a system of n wires ordinarily requires $n - 1$ wattmeters in order to measure the power correctly.

75. Two-wattmeter Method.—The power in a 3-phase, 3-wire system can be measured by means of two wattmeters connected as shown in Fig. 104. The current-coils of the two wattmeters are connected in series with any two lines such as $a'a$ and $b'b$ (Fig. 104). The potential-circuit of each instrument is connected across its respective line and the third line. For example, the potential-circuit of W_1 is connected from line $a'a$ to line $c'c$; the potential-circuit of W_2 is connected from line $b'b$ to line $c'c$.

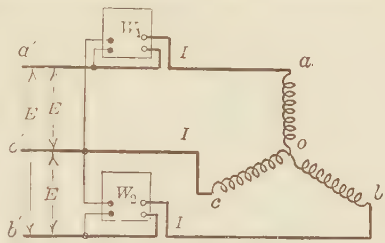


FIG. 104.—The 2-wattmeter method of measuring 3-phase power.

The total power taken by this load is given by

$$P = W_1 \pm W_2 \tag{47}$$

regardless of power-factor, unbalancing in the system, etc. The load need not necessarily be Y-connected as shown, but may, for example be delta-connected.

If the system is balanced, it can be shown that the wattmeter readings are:

$$W_1 = EI \cos (30° - \theta) \tag{47a}$$

$$W_2 = EI \cos (30° + \theta) \tag{47b}$$

where E is the line voltage, I the line current, and θ the power-factor angle of the system, that is the angle of phase difference between *coil* voltage and *coil* current.

If the power-factor is unity, that is if $\theta = 0$, both wattmeters read the same. Both W_1 and W_2 are positive and the total power is given by the sum of their readings. (The wattmeters also read the same when $\theta = 90°$, in which case the reading of W_2 is negative, when $\theta = 180°$, both readings then being negative.

If $\theta = 60°$, the power-factor is 0.5, since $\cos 60° = 0.5$. Under these conditions, W_1 still reads positive since the $\cos(-30°) = \cos(+30°)$ (see page 407) and this is positive. W_2 must read zero, since $\cos 90°$ is zero. That is, when the power-factor is 0.5, W_1 reads *all* the power of the system.

With power-factors less than 0.5, W_1 still reads positive, but W_2 reads negative, since the cosine of angles between 90 and 120° is negative (see page 407). If the wattmeters are connected symmetrically as shown in Fig. 104, and both read up scale when the power-factor exceeds 0.5, W_2 will tend to read backwards when the power-factor drops below 0.5. The connections of the current-coil of W_2 must therefore be reversed in order that the pointer may read up scale. The reading of W_2 is then *subtracted* from W_1 in order to obtain the total power of the system. That is, the minus sign in equation (47) is now used.

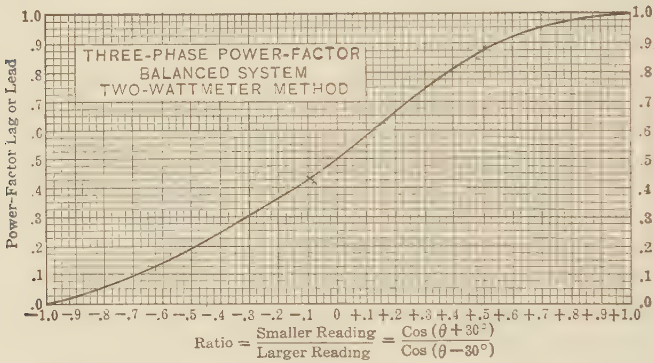


Fig. 105.—Power-factor diagram, 2-wattmeter method.

By dividing equation (47*b*) by equation (47*a*) and substituting various values of θ, it is possible to obtain the curve (Fig. 105) which shows power-factor ($\cos\theta$) plotted as ordinates, and the ratio of the smaller to the larger wattmeter reading $\left(\frac{W_2}{W_1}\right)$ as abscissas. Knowing the two wattmeter readings, it is possible to obtain the system power-factor without knowing the volt-amperes.

Example.—The power input to an induction motor is measured by the 2-wattmeter method. When the motor is near its rated load, one wattmeter reads + 5,800 watts and the other reads + 2,500 watts. (*a*) What is the total power taken by the motor? (*b*) At what power-factor is it operating?

(*a*) $P = 5{,}800 + 2{,}500 = 8{,}300$ watts. *Ans.*

(*b*) $\frac{W_2}{W_1} = \frac{+2{,}500}{+5{,}800} = +0.431.$

From the curve (Fig. 105) the power-factor is 0.83. *Ans.*

Example.—When the motor in the foregoing example is operating at light load, the two wattmeters read +2,400 watts and −1,100 watts. (It is necessary to reverse the current-coil connections of one wattmeter in order that it may read up scale. Its reading of 1,100 watts must therefore be preceded by a minus sign.) (*a*) What is the total power taken by the motor under these conditions? (*b*) At what power-factor is the motor operating?

(*a*) $P = 2{,}400 - 1{,}100 = 1{,}300$ watts. *Ans.*

(*b*) $\frac{W_2}{W_1} = \frac{-1{,}100}{2{,}400} = -0.458.$

From the curve (Fig. 105) P.F. = 0.21. *Ans.*

The polyphase wattmeter, and its connections for measuring 3-phase power are described on page 90.

The 2-wattmeter method cannot be used to measure power with a 3-phase, 4-wire system unless the current in the neutral wire is zero. The *3-wattmeter* method may be used under these conditions.

OTHER POLYPHASE SYSTEMS

76. Two-phase Systems.—Although 3-phase systems are superseding other polyphase systems, there are still many 2-phase or quarter-phase systems in existence. The 2-phase system is rarely used for transmission, but is used for distribution and in some instances there are special advantages in using 2-phase machines.

Two-phase current is generated in the elementary generator (Fig. 106 (*a*)) by two coils A and B, 90° apart. Figure 106 (*b*) shows the e.m.f. waves generated by these coils. The voltage of A leads that of B by 90°. When one voltage is a maximum the other is zero. Figure 106 (*c*) shows these 2-phase voltages represented vectorially.

The two phases may be carried along, insulated from each other, to supply two separate single-phase circuits, or they may supply a common load such as an induction motor (Fig. 107). The two phases are entirely insulated from each other (Fig. 107), and no single load can be supplied between the two phases. Moreover, only one value of voltage is obtainable, since the voltages of the two phases are equal.

If, however, the generator-coils be connected at their neutral points and a neutral conductor carried along with the other

conductors, a quarter-phase, 5-wire system results (Fig. 108 (*a*)). The coils are then said to be *star-connected.* Moreover, three different voltages are available. If the voltages between the

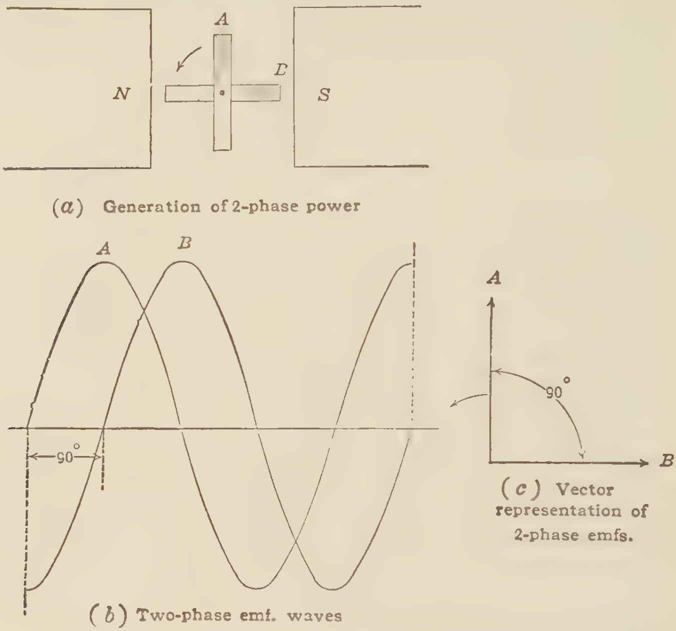


Fig. 106.—Phase relations of 2-phase electromotive forces.

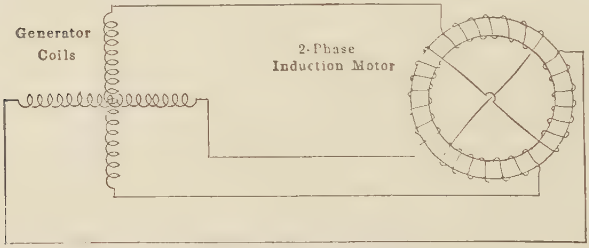


Fig. 107.—Two-phase circuit in which two phases are insulated.

outer wires of each phase be 200 volts, 200 volts is available between wires *AA* and also between wires *BB.* These two voltages differ in phase by 90° or are in quadrature with each other. A voltage of 100 volts exists between each outer wire and neutral.

Since the voltages from A to O, and from B to O, are each equal to 100 volts, the voltage between the wires A and B is $100\sqrt{2}$ or 141 volts.

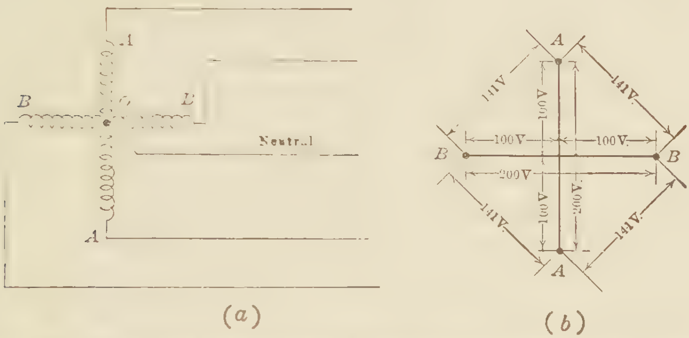


FIG. 108.—Two-phase inter-connected system giving 4-phase, 5-wire system.

This system is not so readily balanced as the 3-phase system, which is an objection to its use. Another objection is the greater number of wires.

If one end of the coil A be connected to one end of the coil B, a 3-wire, 2-phase system results (Fig. 109). This gives an

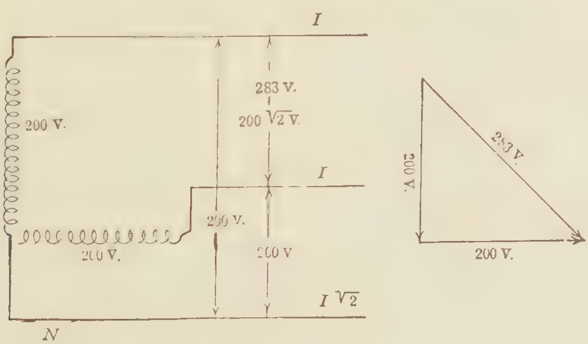


FIG. 109.—Two-phase, 3-wire system.

unsymmetrical polyphase system. Since the voltage across each coil is 200 volts, and these voltages are in quadrature, the voltage across their open ends is $200\sqrt{2} = 283$ volts, as shown by the vector diagram.

This system is little used because of its dissymmetry and the considerable amount of voltage unbalancing which results, even when moderate loads are applied. When currents of I amp. each and in quadrature flow between the outer conductors and the common return N the current in the common return is $I\sqrt{2}$ amp.

Therefore, the common return must ordinarily have greater cross-section than that of the two outers.

A 2-phase alternator may be mesh-connected in the manner shown in Fig. 110. With a closed 2-pole winding, four equidis-

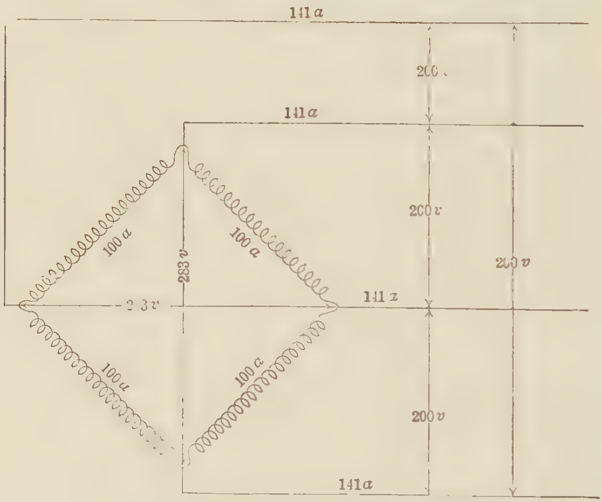


FIG. 110.—Mesh-connected, 2-phase winding

tant taps are brought out as shown. This is called the *mesh-* or *ring-connection.* The coils constituting the winding must be properly connected for otherwise the winding will be short-circuited on itself. If each coil generates 200 volts, the voltage across adjacent line wires is 200 volts and that across line wires diametrically spaced is $200\sqrt{2}$ or 283 volts, since the voltages across adjacent coils are in quadrature.

If each coil delivers 100 amp., the line currents must be $100\sqrt{2}$ or 141 amp., since the currents in two adjacent coils are in quadrature and combine to flow in a common wire. Since each

coil delivers 200 amp. at 200 volts, the output of the system is

$$\frac{4 \times 200 \times 100}{1,000} = 80 \text{ kv-a.}$$

It will be seen that the two-phase mesh- or ring-connection corresponds to the 3-phase delta-connection.

77. Measurement of Power in 2-phase Systems.—A 2-phase insulated system consists merely of two separate single-phase

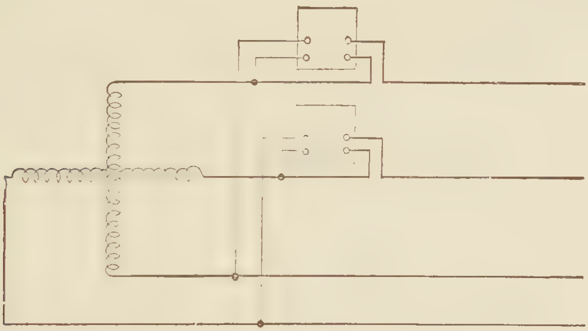


FIG. 111.—Measurement of power in an insulated 2-phase, 4-wire system.

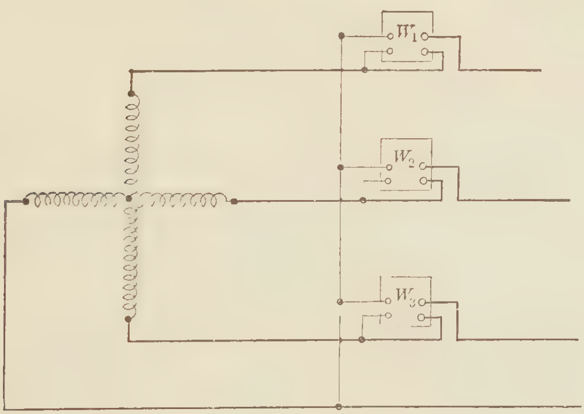


FIG. 112.—Measurement of power in a 2-phase inter-connected system.

systems. Hence a wattmeter is connected in each of the two phases (Fig. 111) and the total power is given by their sum. If the system is in any manner interconnected (Fig. 112), at least three wattmeters are necessary. The total power is their sum.

If the loads on the two phases happen to be balanced, no current flows between phases at the connection point, and two wattmeters connected as in Fig. 111 may be used to measure the total power.

In a 4-phase, 5-wire system (Fig. 108), four wattmeters are necessary. They are preferably connected with a current-coil in each of the outer wires and each potential-circuit is connected between its proper line wire and neutral.

A 2-phase, 3-wire system may be considered as two single-phase circuits connected at a common point. Hence, two wattmeters connected as in Fig. 113 may be used to measure the total power.

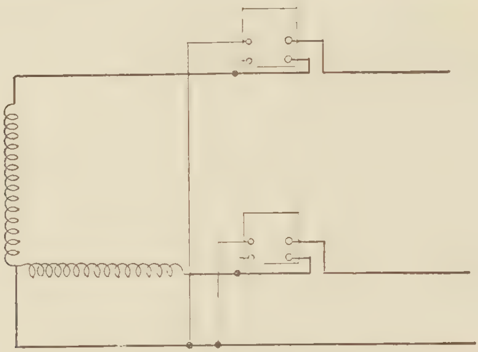


FIG. 113.—Measurement of power in a 3-wire, 2-phase system.

It will be seen that this connection is identical with the 2-wattmeter method (Fig. 104). It is not necessary that the wattmeters be connected in the two outer lines as shown. One current-coil may be connected in the common return and the common potential connection for both wattmeters must then be brought to that conductor in which no wattmeter current-coil is connected.

78. Six-phase Systems.—If six similar coils or groups of coils in an alternator winding 0-1, 0-2, . . . 0-6 (Fig. 114) be 60 electrical space-degrees apart, a 6-phase system may be obtained. If the six coils are connected symmetrically, each with one end to a common point 0, and the proper phase relations are observed, a 6-phase star-connection results. If each coil generates 100 volts, the six voltages 0-1 . . . 0-6 are each 100 volts. Also the voltage between each adjacent pair of terminals 1-2, 2-3, . . ., etc. is 100 volts. The diametrical voltages, between 1-4, 2-5, etc. are

each 200 volts. This 6-phase system may be considered as a double Y, of which coils 1, 3, and 5 form one Y since their voltages all differ in phase by 120°, and coils 2, 4, and 6 form the second Y, displaced 60° from the first Y. Therefore, the voltages across 1-3, 3-5, 5-1, 2-4, 4-6, and 6-2 are each $100\sqrt{3}$, or 173.2 volts.

The six coils of Fig. 114 may also be connected to form a 6-phase mesh- or ring-connection, as shown in Fig. 115. For example, the inner end of coil 1 may be connected to the outer

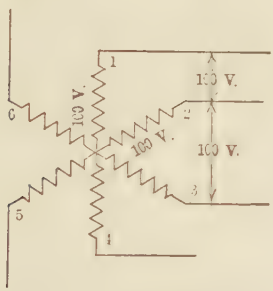


FIG. 114.—Six-phase star connection.

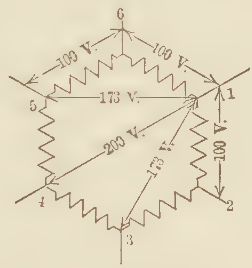


FIG. 115.—Six-phase ring or mesh-connection.

end of 2, etc. If the voltage of each coil is 100 volts, the six voltages between adjacent terminals 1-2, 2-3, etc., are each 100 volts as shown. The diametrical voltages 1-4, 2-5, and 3-6 are each 200 volts and the voltages 1-3, 2-4, etc. are each $100\sqrt{3}$, or 173.2 volts.

Six-phase connections are almost always used on the alternating-current side of synchronous converters, since the rating of the converter is increased considerably by their use (see pages 293 and 295).

CHAPTER V

THE ALTERNATOR

It has been shown in Chaps. I and III that alternating voltage can be generated by simple coils rotating in magnetic fields. These coils are provided with the necessary slip-rings for conducting the current to an external circuit where it can be utilized. Although these types of alternators demonstrate clearly the fundamental principles underlying the generation of alternating current, they are not practicable for generating electrical energy on a commercial basis, since the small power output for their weight makes them uneconomical. However, the principles involved in the generation of alternating current in these simple alternators and the principles involved in the generation of alternating current in the large commercial types of alternator are *identical*. The commercial types differ from the simple types in that the arrangement of copper and iron is such that much larger outputs are obtained per pound of material. In this chapter the construction and winding of commercial types of alternators will be described.

79. Rotating-field Type.—The generation of an e.m.f. in a conductor may take place with the magnetic field stationary and the conductor moving through this field, as in a direct-current generator, or with the conductor stationary and the field moving past the conductor. It is merely necessary that there be *relative* motion between the conductor and the field. In direct-current machines, the commutator makes it necessary that either the armature be the rotating member, or that the brushes revolve with the field.

As alternators have no commutator, it is not necessary that the armature be the rotating member. Most commercial alternators have stationary armatures, inside of which the field poles rotate, as shown in Figs. 117, 120, etc. This construction has two distinct advantages. A rotating armature requires two

or more slip-rings for carrying the current from the armature to the external circuit. Such rings must be more or less exposed, and are difficult to insulate, particularly for the higher voltages of 6,600 and 13,200 volts at which alternators are commonly operated. These rings may become a frequent source of trouble, due to arc-overs, short-circuits, etc. A stationary armature requires no slip-rings, and the armature leads can be continuously insulated conductors from the armature coils to the bus-bars. It is more difficult to insulate the conductors in a rotating armature than in a stationary one, because of centrifugal force and the vibration resulting from rotation.

(a) (b)

Fig. 116.—Effect of slot depth on the width of tooth necks in a rotor and in a stator.

When the field is the rotating member, the field current must be conducted to the field winding through slip-rings. As the field voltage seldom exceeds 250 volts and the amount of power is small, no particular difficulty is encountered in the operation of such slip-rings.

Usually it is difficult to get sufficient copper on the surface of a rotating armature. This is particularly true with high-speed, high-voltage machines having armatures of small diameter. Increased space for copper may be obtained by deepening the slots. If the armature be the rotating member, the deepening of the slots is limited by the contraction of the tooth necks, as shown in Fig. 116 (*a*). No such difficulty is encountered if the

armature be stationary, since the tooth necks increase in width with the deepening of the slots (Fig. 116 (*b*)).

80. Induced Electromotive Force with Rotating-field Structure.—The method by which an alternating e.m.f. is induced in the armature of the rotating-field type of alternator is illustrated in Fig. 117, which shows two poles of the rotating-field structure and a single coil placed on the stationary armature. The field structure is shown rotating in a clockwise direction. At the instant shown, the center of the north pole is directly beneath coil-side *a* of the coil *ab* and the center of the south pole is directly beneath coil-side *b*.

The direction of the induced e.m.f. may be determined by Fleming's right-hand rule. Fleming's right-hand rule applies to a *moving* conductor and a *stationary* field. Hence, assume (Fig. 117) that the field structure is stationary and that the armature rotates. Since the field is shown rotating in a clockwise direction, the armature must now be assumed to rotate in a counter-clockwise direction in order that the *relative* motion between armature and field shall remain unchanged. Applying Fleming's right-hand rule to conductor *a*, the direction of the lines of magnetism from the north pole being upward and the motion of the conductor being to the left, the direction of the induced e.m.f. is *inward* in conductor *a*. The direction of induced e.m.f. must, therefore, be outward in conductor *b*.

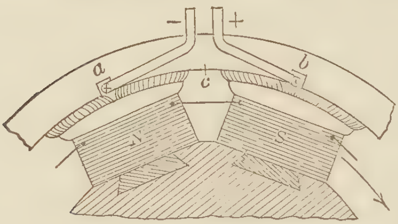


Fig. 117.—Induced e.m.f. with a rotating-field structure.

Since the direction, as well as the magnitude of an induced e.m.f. depends on the *relative* motion of conductor and flux, the direction of the e.m.f. in *a* will still be inward in conductor *a* and outwards in conductor *b*, if the armature is stationary and the field rotates in the clockwise direction as shown.

At the instant shown, conductors *a* and *b* are directly opposite the pole-centers, and, therefore, are being cut by the flux at the maximum rate. Hence, the induced e.m.f. in each is a maximum. When the center of the north pole is directly under the coil axis

c, conductors *a* and *b* are in the center of the interpolar region and no flux is cutting them, so their induced e.m.fs. are zero. When a south pole moves under conductor *a*, the direction of the induced e.m.f. in *a* is outward and the left-hand terminal of the coil becomes positive. Hence, an alternating voltage is induced in every armature conductor. (Also see Par. 11, page 17.)

The induced e.m.f. in coil *ab* may also be considered as being due to change in the flux *linking the coil* (see Part I, page 167, equation (64)). At the instant shown in Fig. 117 *no* flux links the coil. When the center of the north pole is directly opposite *c all the north-pole* flux links the coil; when the center of the north pole is directly opposite conductor *b*, again *no* flux links the coil; when the center of a south pole is directly opposite *c* all the south-pole flux links the coil. Hence, the flux *linking* the coil is alternately changing from north to zero and from zero to south, thereby inducing an alternating e.m.f. in the coil.

In the actual alternator, several coils lying in adjacent slots are connected in series to form one phase of the circuit. This utilizes more of the winding space on the armature and gives greater output for a given weight of machine. Alternator windings must be designed so that not only are e.m.fs. induced in them in the manner just described, but that the desired number of phases is obtained and the output per unit area of armature surface is large.

ALTERNATOR WINDINGS

81. General Principles.—If the principles underlying the generation of alternating e.m.fs. be kept in mind, it is not difficult to design complete windings to give any required number of phases and voltages and at the same time make the winding economical so that the maximum output is obtained with the minimum amount of copper and iron.

The usual direct-current armature generates alternating current, and if provided with properly connected slip-rings, alternating current may be obtained from it. On the other hand, only certain types of alternator windings can be used for direct-current armatures. The ordinary direct-current winding is a *closed winding*, that is, the winding forms a continuously closed, conducting circuit. Alternator windings may be either open or

closed. For example, the delta-connected winding is a *closed* winding, whereas the Y-connected winding is an *open* winding.

The general principles which govern direct-current windings hold also for windings of alternators. The span of each coil must be approximately one pole-pitch; that is, the two sides of any coil must lie under adjacent poles. The coils must be so connected that their e.m.fs. add.

There are several types of alternator windings. The more usual types will be described. Knowing the principles involved in the usual windings, it is not difficult to apply them to the special windings which are only occasionally encountered.

82. Single-phase Windings.—At the present time alternators are seldom wound with single-phase windings. This type of winding is special, and a machine so wound can seldom be used for purposes other than those for which it is designed. Single-phase windings, however, are used extensively for single-phase motors.

When single phase is desired, it is common to utilize a 3-phase, Y-connected alternator and to obtain the single-phase from two of the Y-connected terminals (Fig. 118). This gives a spare phase, in case of injury to either of the other two. A standard 3-phase generator is thus utilized and if necessary it can be used for 3-phase service. The alternators in the Cos Cob Station which supplies 25-cycle, single-phase current for the New York, New Haven, and Hartford R.R. electrification, are Y-connected and operate connected as shown in Fig. 118.

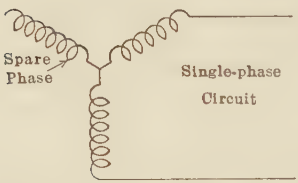


Fig. 118.—Single phase obtained from Y-connected alternator.

A knowledge of single-phase windings makes it easier to understand polyphase windings, since polyphase windings are merely two or more single-phase windings having definite geometrical relations to one another. Two simple single-phase windings for 6-pole machines are shown in Fig. 119 (*a*) and (*b*). Both windings are only partially distributed since all the armature winding space is not utilized. If conductors were placed in the blank spaces their e.m.fs. would be so far out of phase with the resultant

e.m.fs. of the system that they would contribute but little to the total e.m.f. It is, therefore, wasteful of copper to utilize these winding spaces.

In (*a*) a lap-winding is shown. All the adjacent coils constituting each of the various belts are first connected in series. These belts are then connected in series. There are three such belts in Fig. 119 (*a*). In (*b*) a wave-winding is shown. The first coil of a belt is connected in series with the first coil of the next belt, etc., until all the first coils are so connected. The connection is then continued to the second coil of the first belt, which is connected in series with the second coil of the various belts in

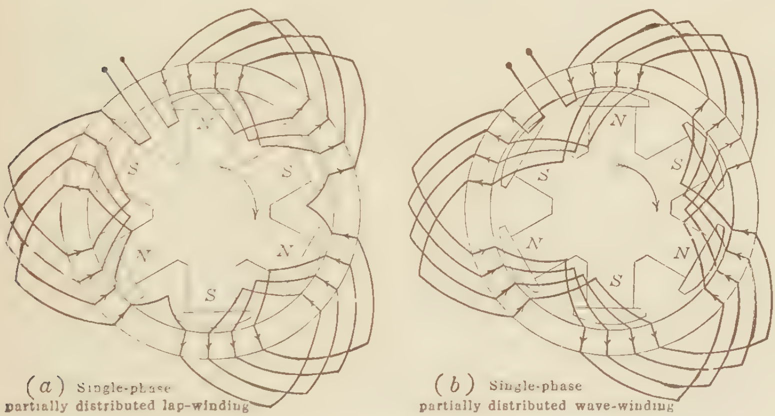
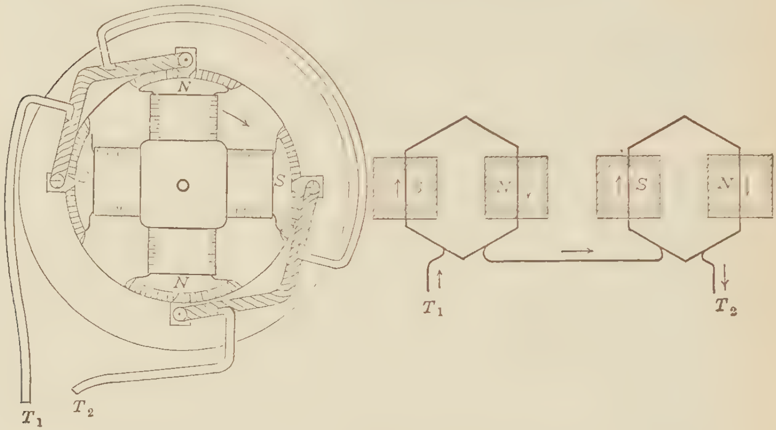


Fig. 119.—Single-phase lap- and wave-windings.

order. This method of connection is continued until every coil in every belt is included. It will be noted that in both (*a*) and (*b*) there are the same number of series-connected conductors between the machine terminals. Therefore, both windings give the same voltage. This is not true with lap- and wave-windings in direct-current machines (see Part I, page 223).

A simple type of single-phase winding for a 4-pole machine is shown in Fig. 120. It consists of but two coils placed in four slots and connected in series. Since there is but one coil or coil-group per pair of poles there is but a half-coil or coil-group per pole. The winding also consists of but one layer. Hence, this is called a single-layer, half-coil winding, having one slot per pole.

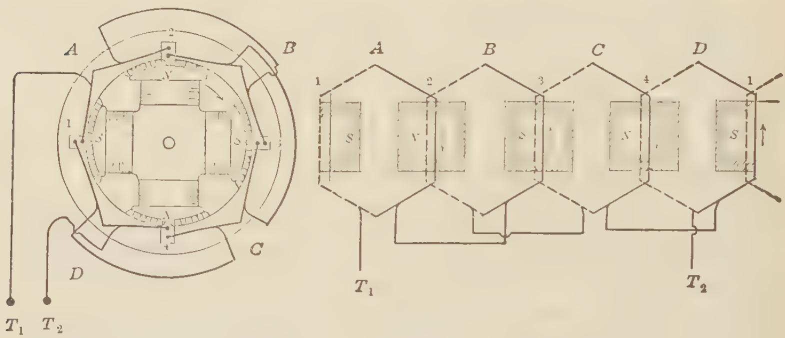
(The windings of Fig. 119 are also single-layer, half-coil windings having four slots per pole.) The connections of the winding are shown diagrammatically in (*b*).



(*a*) (*b*)

Fig. 120.—Single-layer, half-coil winding, one slot per pole.

If two additional coils be placed on the armature of Fig. 120 in such a manner that they span the pairs of poles not spanned by the coils of Fig. 120, a *whole-coil* winding is obtained. That is,



(*a*) (*b*)

Fig. 121.—Two-layer, whole-coil winding, one slot per pole.

there is now one whole coil or coil-group per pole. These additional coils may be placed in the same slots with the other coil, thus giving a 2-layer winding (Fig. 121). The whole-coil wind-

ing of Fig. 121 may be made a half-coil winding by swinging coil B so that it spans the same pair of poles as coil A and by swinging coil D so that it spans the same pair of poles as coil C. The reversed connection of the alternate coils B and D should be noted. This is necessary in order that the e.m.fs. of all the coils should be *additive*, as is shown by study of the direction of the arrows. The connections of the winding are shown diagrammatically in (*b*).

Windings utilizing but one slot per pole are not ordinarily used in practice, since but a fraction of the armature surface is utilized. Figure 122 shows a development of the winding of Fig. 121, but with two slots per pole. The two coils of each group

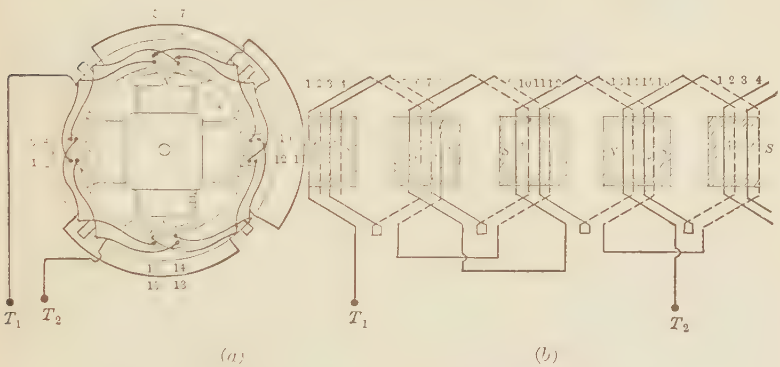


FIG. 122.—Whole-coil, 2-layer winding, two slots per pole.

are first connected in series, and the groups are then connected in series as in of Fig. 121, the reversed connections to alternate coil-groups being made. The connections of the winding are shown in (*b*), the coil-sides having numbers corresponding to those in (*a*). This type of winding, from the nature of the end-connections, is called the *barrel type*.

It is obvious that this type of winding may be extended to three or more slots per pole.

A single-phase *spiral* winding is shown in Fig. 123. Instead of the coils lapping one another, each coil-group is made up of separate coils of different breadths, and placed on the armature in the manner shown in Fig. 123 (*a*). The coils of the group are connected in series. The winding derives its name from the

spiral manner in which the coils are connected in series as is indicated by the wiring diagram (Fig. 123). A third coil of very small pitch shown dotted could be added to this winding, being placed in the slots marked *a*. This third coil has so small a pitch that the e.m.fs. induced in its two sides are almost in phase opposition, and therefore would contribute very little to the induced e.m.f. of the winding as a whole. Hence, the third coil is omitted.

Since the ends of the coils are bent so that they all lie in the same plane, perpendicular to the shaft of the machine, it is called

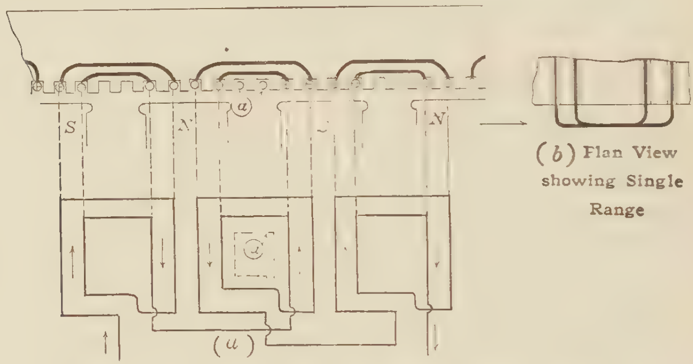


FIG. 123.—Single-phase, single-range, spiral winding.

a *single-range* winding. Two- and three-range windings are used with polyphase machines having spiral windings.

83. Two-phase Windings.—It was shown in Chap. IV, page 124, Fig. 106, that 2-phase e.m.fs. are obtained from two simple coils fastened together with their planes at 90° to each other. The e.m.f. induced by one coil lags that induced in the other coil by 90 time-degrees. In commercial machines, the same principle of spacing the windings of the two phases 90 electrical space-degrees apart is used, the only difference being that the armature is stationary and the winding must ordinarily be adapted for a considerable number of poles.

Figure 124 shows a simple type of 2-phase winding, having but one slot per pole per phase. Each phase of this winding is similar to the single-phase winding of Fig. 121. To obtain this winding, it is only necessary to add a duplicate winding (Fig. 124) placed

in slots half way between those shown. In Fig. 124, the *A* phase is shown with heavy lines and the *B* phase is shown with light

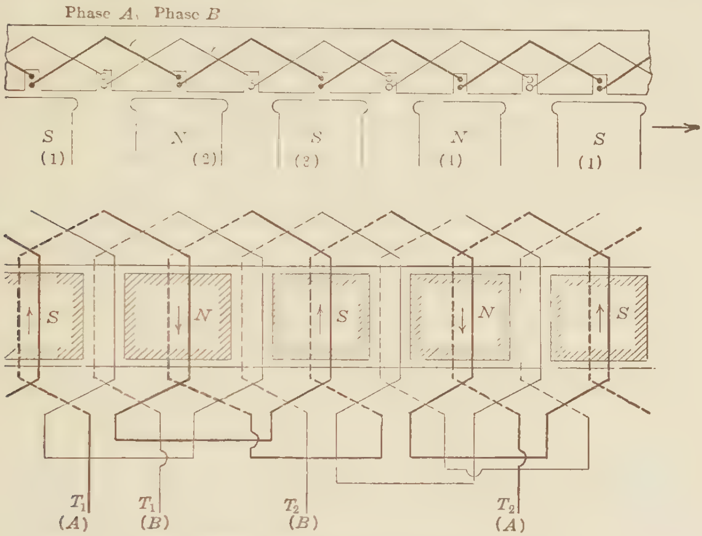


FIG. 124.—Two-phase, 2-layer winding, one slot per pole per phase.

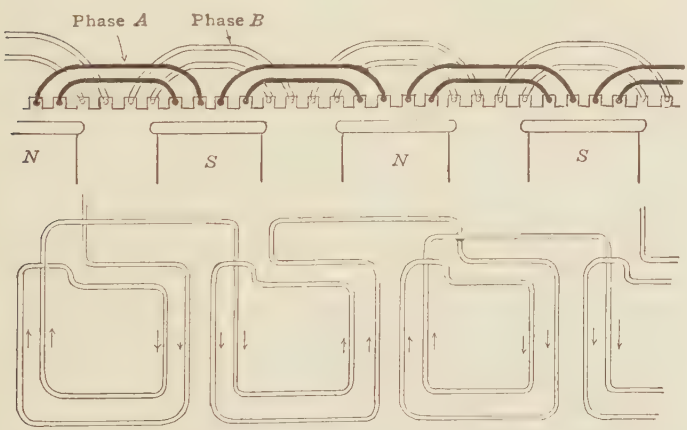


FIG. 125.—Two-phase chain winding, 2-range.

lines. The sides of the coils which lap under are shown dotted. The terminals T_1 and T_2 of each phase are brought out and may be connected in the manner described in Chap. IV (page 123).

Figure 125 shows the spiral winding (Fig. 123) adapted to a 2-phase machine. A second winding, similar to that in Fig. 123 and symmetrically spaced is added. In order that the coil-ends of phase A may pass those of phase B, the bent-up coil-ends must lie in at least two planes perpendicular to the shaft of the machine. This makes such a winding a 2-*range* winding.

84. Two-phase Lap-winding.—The lap-winding is the most common type of alternator winding. With it, there are very few limitations in the choice of number of slots, pitch, etc. The coils are all alike, requiring the minimum number of spares, and the winding is very flexible in the matter of connections. For example, with a lap-winding it is a simple matter to change a 440-volt winding to one of 220 volts, by paralleling.

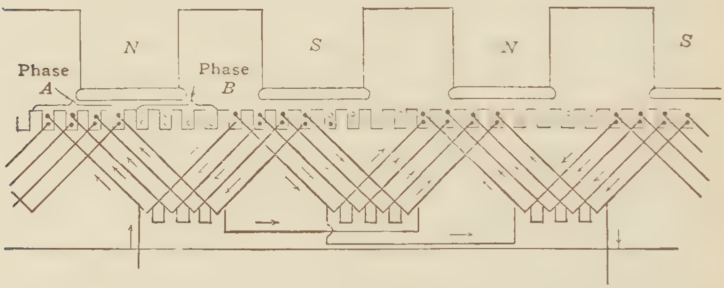


FIG. 126.—Two-phase, full-pitch, lap-winding, four slots per pole per phase.

To obtain a 2-phase lap-winding it is merely necessary to add slots and coils to the 2-phase winding shown in Fig. 124. The connections of the coils of any one phase are almost identical with those in the direct-current windings described in Part I, Chap. X. Direct-current lap-windings may be used for single-phase and for polyphase voltages by taps at suitable points with connections to slip-rings, as is done in the synchronous converter (see page 292, Par. 165). Such windings give mesh- or delta-connected windings. The phases of the ordinary lap-winding may be connected either in delta or in Y.

Figure 126 shows a 2-phase lap-winding, in which there are eight slots per pole, making four slots per pole per phase. This is a full-pitch winding, the coil-pitch being eight slots, which is the number of armature slots per pole. The connections of the

coils of phase *A* should be noted. The coils of each coil-group are all connected in series and the group is then treated as a unit. The *A*-phase coil-groups are then connected in series as shown. The reversed connection to alternate coil-groups should also be noted. The arrows show the direction of the induced e.m.f. at one instant. The connections of phase *B* are omitted for the sake of clearness as they are identical with those of phase *A*. It will be observed that in this full-pitch winding, the coil-sides in any one slot are both of the same phase. This is not true with fractional-pitch windings.

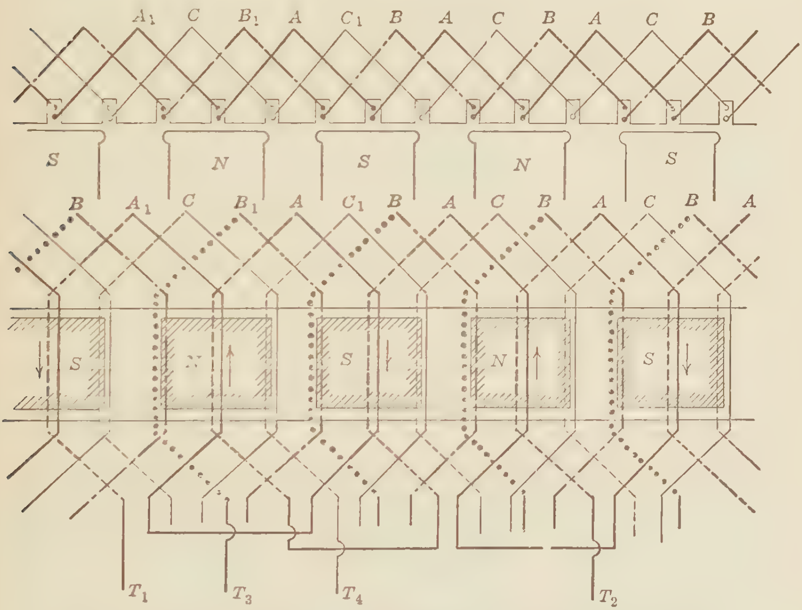


FIG. 127.—Three-phase, 2-layer winding, one slot per pole per phase.

85. Three-phase Windings.—The difference between 2- and 3-phase windings is merely in the number of phase-belts per pole. Figure 127 shows the simple winding of Fig. 121 adapted for three phase. For clearness the end connections of phase *A* alone are shown. It is necessary merely to add two more windings, equally spaced, between those of Fig. 121 in order to obtain a 3-phase winding having one slot per pole per phase.

The phase rotation given in Fig. 127 is in the sequence A-B-C, and not A-C-B as might be inferred from the order of the lettering. Note that coil B_1 is 120 electrical space-degrees to the right of A_1, remembering that the distance between the centers of adjacent poles is 180 electrical space-degrees. Coil C_1 is 120 electrical space-degrees to the right of B_1. Therefore, the e.m.fs. in these three coils differ in phase by 120° and with the rotating field moving from left to right the phase sequence is A-B-C. If, for example the machine were to be Y-connected the terminal T_1 of the A phase would be connected to terminal T_3 of the B phase and to the terminal T_4 of the C phase to form the common neutral, since the active conductors connecting to these terminals are 120 electrical space-degrees apart. The coil C between A_1 and B_1 is but 60 electrical space-degrees to the right of A_1, hence, its e.m.f. lags that of A_1 by 60 time-degrees. Its e.m.f. must be made to lag that of A_1 by 240 time-degrees, hence its connections are reversed. If the winding were completed it would be found that this coil was one of the alternate ones whose terminals are reversed.

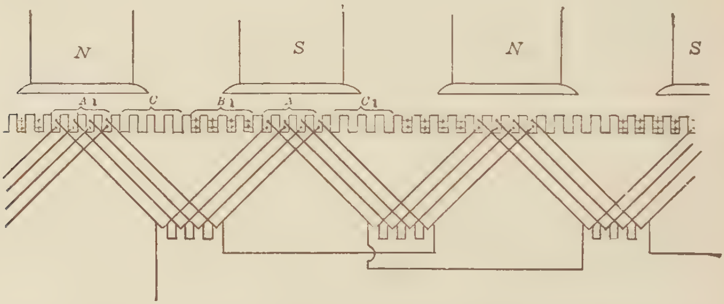


FIG. 128.—Three-phase, full-pitch, 2-layer, lap-winding.

86. Three-phase, Full-pitch Lap-winding.—Figure 128 shows a 3-phase, full-pitch, 2-layer, lap-winding having 12 slots per pole. This winding differs from that of Fig. 127 only in having four slots per pole for each phase rather than one. The four coils are connected in series and this group is connected in the same manner as the single coils of Fig. 127. Note that the phase sequence as before is A-B-C, the letters A_1, B_1, and C_1 showing the corresponding coil-side belts. B_1 is 120° to the right of A_1, and C_1 is 120° to the right of B_1. The belt C is only 60°

to the right of A_1 and its connections must be reversed in order that its e.m.f. may lag that of A_1 by 240°.

87. Three-phase, Fractional-pitch Winding.—Figure 129 shows two poles and a portion of the armature of an alternator having 12 slots per pole. If a full-pitch winding is to be placed on this armature, the two sides of every coil must have a span

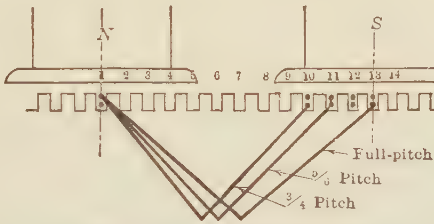


FIG. 129.—Full- and fractional-pitch coils.

corresponding to one pole-pitch, or 12 slots. Therefore, if one side of the coil is in the top of slot 1, its other side must lie in the bottom of slot 13. If, instead of the coil span being 12 slots, it is only 10 slots, its other side will be in slot 11. When the span of coils is shortened in this manner a *fractional-pitch* winding results. The foregoing coil-span is 10 slots, giving a $^{10}\!/_{12}$- or

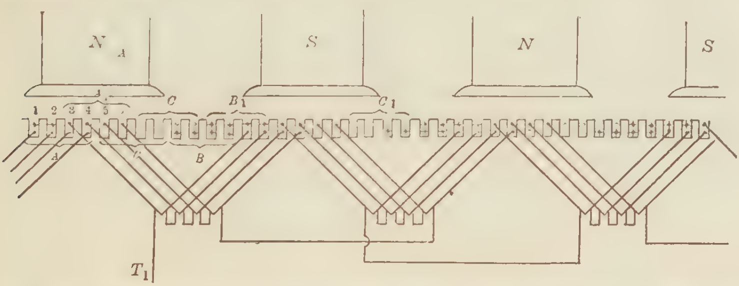


FIG. 130.—Three-phase, ⅚-pitch, 2-layer lap-winding, four slots per pole phase.

⅚-pitch winding. If the coil-span is but 9 slots, a $^{9}\!/_{12}$ or ¾-pitch winding results (Fig. 129).

Fractional-pitch windings are common, since they save copper in the end-connections. Also in alternators the wave-form of the e.m.f. is improved. Owing to the fact that the two coil-sides are not under corresponding parts of the poles simultane-

ously, a phase difference exists in their e.m.fs. Hence, the e.m.f. of the coil is reduced. With a $\frac{5}{6}$ pitch, the e.m.f. is 0.966 of the value for a full-pitch coil, so that the reduction in e.m.f. is slight for pitches of the order of $\frac{5}{6}$.

Figure 130 shows a 3-phase, $\frac{5}{6}$-pitch lap-winding. It is identical to that of Fig. 128 except that the bottom layer is moved two slots to the left. The top layer remains unchanged.

It will also be observed that in the $\frac{5}{6}$-pitch winding of Fig. 130, only two of the slots of each phase under any one pole

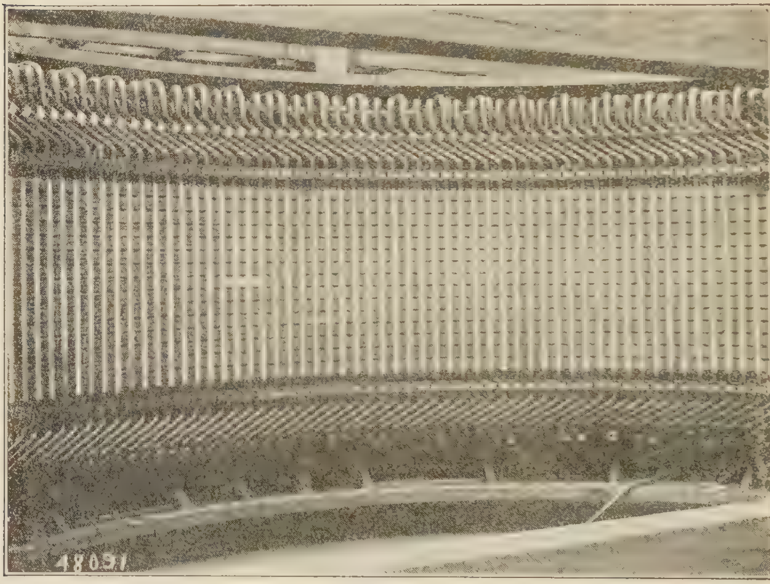


FIG. 131.—Showing winding and end-connections of an alternator armature.

contain conductors of that phase only. The four adjacent slots contain both conductors of this phase and conductors of the other two phases. For example, slots 1 and 2 contain both phase-A and phase-B conductors; slots 3 and 4 contain phase-A conductors only; and slots 5 and 6 contain both phase-A and phase-C conductors. Of this group, slots 3 and 4 contain phase-A conductors only. The fact that certain slots contain conductors of different phases reduces slightly the inductance of the winding

A portion of the finished armature winding of a slow-speed generator is shown in Fig. 131. The wooden-slot wedges, and the ventilating ducts are clearly shown. The method of making the end-connections and the method of binding the ends down to strengthen them against mechanical stresses should be noted.

ALTERNATOR CONSTRUCTION

88. Stator or Armature.—The stator or stationary member of the alternator is almost always the armature, the field structure

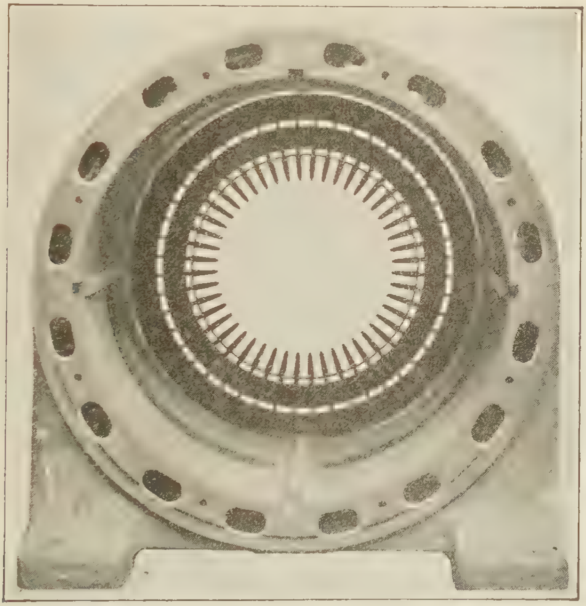


Fig. 132.—Punching and frame of a turbine-driven alternator.

being the rotating member or rotor. When the machine is in operation, the armature iron is continuously cut by the flux of the rotating field and must, therefore, be laminated in order to reduce eddy-current losses. In machines of small diameter, each lamination is a single circular punching.

High-speed turbo-alternators having armatures of small diameter are usually built up of single circular stampings, as shown in Fig. 132. In the larger sizes it is necessary to build

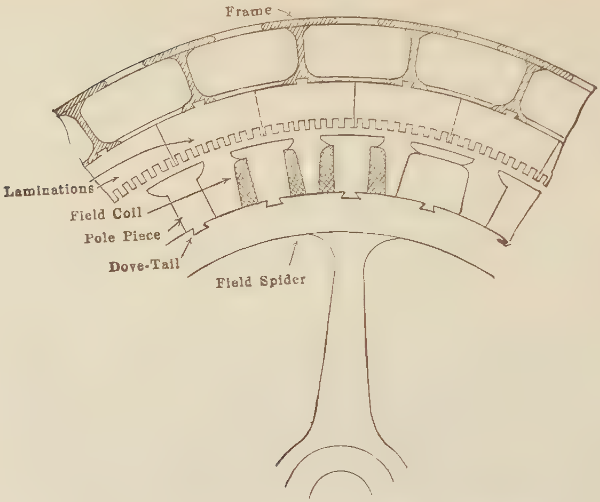


FIG. 133.—Cross-section of engine-driven alternator.

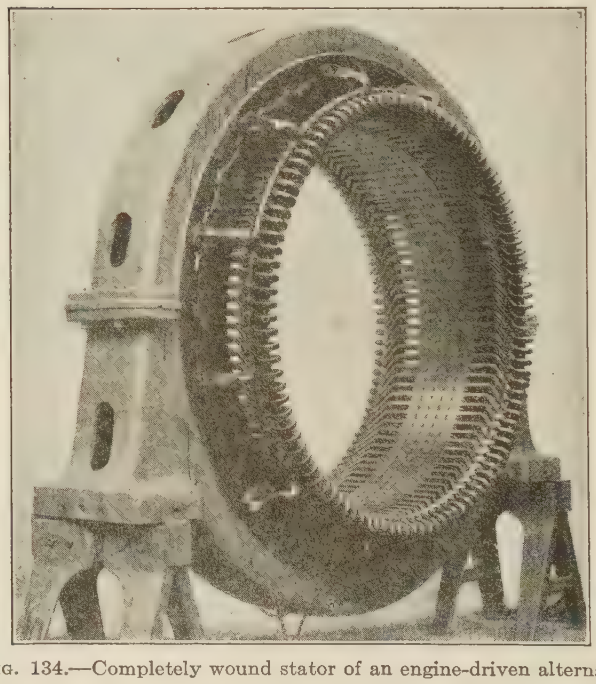


FIG. 134.—Completely wound stator of an engine-driven alternator.

the armature of sectors which alternately butt and lap. It is difficult to ventilate the armatures of turbo-alternators owing to their small diameters. The two rows of perforations in the stator back of the armature teeth are ventilating ducts which assist in carrying away the heat developed in the stator teeth and in the winding. Engine-driven alternators must rotate at compara-

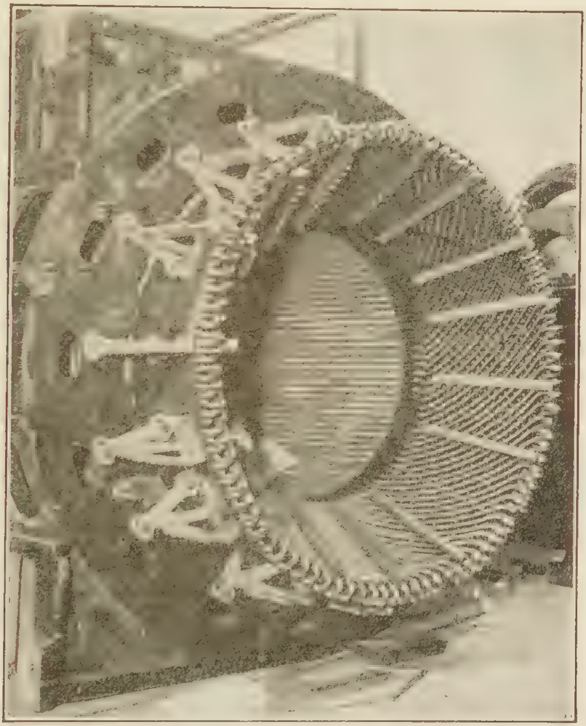


Fig. 135.—Bracing of end connections of a turbo-alternator to withstand short-circuit stresses.

tively slow speeds and hence must have a large number of poles in order to give a sufficiently high frequency and output. Therefore, the armatures must have a comparatively large diameter. The pole-pieces are made up of laminations riveted together and are dovetailed to the field spider (Fig. 133). The armature is built up of small overlapping segments, dovetailed to the frame

of the machine. Figure 133 shows the general construction of such an alternator. The frame itself is usually a hollow-box casting. This gives the necessary mechanical stiffness, with the minimum weight, and the space within the frame allows a free circulation of air for ventilating purposes. Figure 134 shows the complete stator of an engine-driven alternator. The ventilating ducts through the laminations and the bracing of the coil-ends should be particularly noted. The frame is in two sections bolted together. This permits separation for shipment.

Large units must be so designed that their armature coils can withstand not only the stresses incident to normal operation, but also the enormous mechanical stresses which occur at short-circuit, due to the attraction and repulsion of the armature currents. The coil-ends, unless well supported, are likely to be dragged out of position by electromagnetic stresses produced by the short-circuit currents. This is particularly true of turbo-alternators, whose internal reactance is comparatively low, and whose short-circuit currents, therefore, may be of considerable magnitude. Figure 135 illustrates the care taken in bracing the coil-ends in one of the largest types of turbo-alternator. The end-connections are clamped solidly between insulated strips which are bolted together and to the metal brackets, which are in turn bolted to the frame of the machine.

89. Slots.—Alternator slots are divided into two general classes, the open slot and the semiclosed slot. The open slot, shown in Fig. 136 (*a*), is the more common because the coils can be form-wound and insulated prior to being placed in the slots, giving the least expensive and most satisfactory method of winding.

The semiclosed or overhung type of slot (Fig. 136 (*b*)), is seldom used in alternators but is usually necessary in induction motors. The larger area of tooth face reduces the air-gap reluctance and also reduces the tufting of the flux which tends to produce ripples in the e.m.f. wave. It is usually necessary to place the conductors in the slot one at a time, which is expensive and uneconomical of slot space. It is also difficult to apply insulation.

In both types of slot the conductors are usually held in the slot by wooden or fiber wedges, as shown in the figures. The effect of the semiclosed slot may be obtained by the use of open slots

and magnetic wedges. These wedges are only partly of iron so that the slot is not entirely closed.

The internal temperatures of modern turbo-alternators are so high that built-up mica is found to be the insulation best able

FIG. 136.—Open and semiclosed slots.

to withstand the high temperatures and high-voltage stresses simultaneously. Such mica is pressed around the active part of the conductor, forming a solid, homogeneous mass. Molded forms of this built-up mica line the slots.

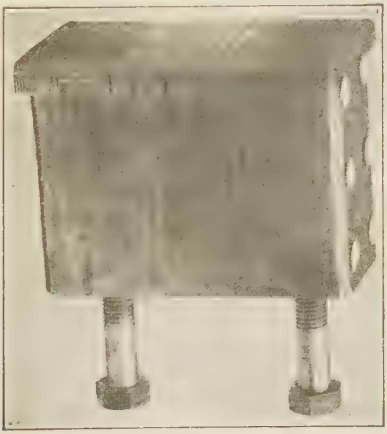


FIG. 137.—Pole piece of a 65 kv-a., 50-cycle, 377 r.p.m. alternator.

90. Rotating-field Structure.—From the standpoint of their field construction, alternators may be divided into three classes: the very slow-speed, engine-driven alternator (75 to 100 r.p.m.); the medium-speed, belt-driven and water-wheel-driven type

(100 to 750 r.p.m.); and the high-speed turbo-alternator (750 to 3,600 r.p.m.).

The poles of practically all salient-pole generators have cores made up of laminations (Fig. 137) in order to reduce pole-face

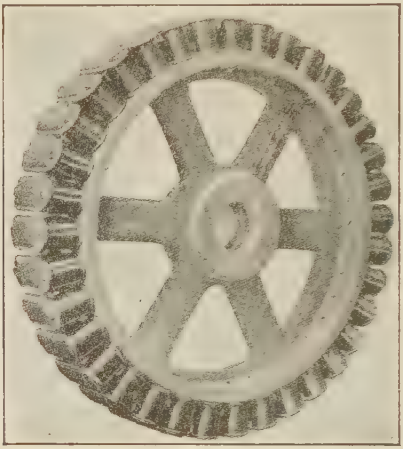


Fig. 138.—36-pole rotor with strip-wound coils.

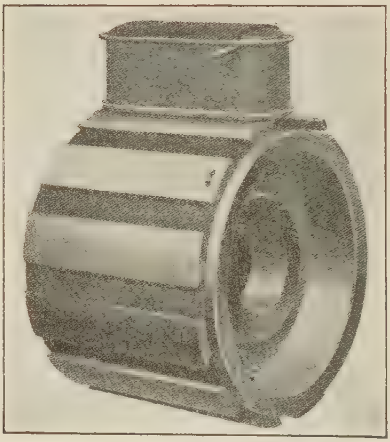


Fig. 139.—Revolving field spider with dove-tailed poles.

losses. In slow-speed machines, the poles are often bolted to a cast-iron spider (Fig. 138) or they may be dovetailed to the spider in the manner indicated in Figs. 133 and 139.

At higher speeds, centrifugal forces require that the poles be dovetailed to the spider. In small machines, the spider may be of solid steel, as shown in Fig. 139. The pole-pieces dovetail to this spider and are wedged in by keys driven one from each end.

In the larger types of generator, the spider is made of steel plates riveted together, as shown in Fig. 140. The poles are dovetailed to the spider in the manner indicated. The slots in the pole faces of this rotor should be noted. Damper or *amortisseur* windings are placed in these slots as will be described later (see page 287, Fig. 251).

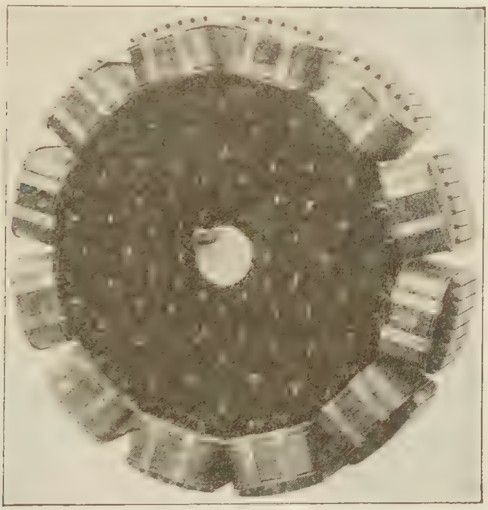


Fig. 140.—Twelve-pole, 600-r.p.m. rotor.

Salient poles cannot be used for high-speed turbo-generators, owing to the large centrifugal forces developed and to the excessive windage. Therefore, a *non-salient-pole* rotor is used. There are two common types of such a rotor, the parallel-slot type (Fig. 141) and the radial-slot type (Fig. 142).

The winding in the parallel-slot type is of strip copper, wound by hand in the slots. The wires are held in the slots by means of non-magnetic metallic wedges. There is not sufficient space to run the shaft through the center of the rotor, so it is bolted to the ends by phosphor-bronze flanges (Fig. 141). These flanges

must be non-magnetic or they would short-circuit the magnetic poles. This construction gives a smooth rotor with little wind-

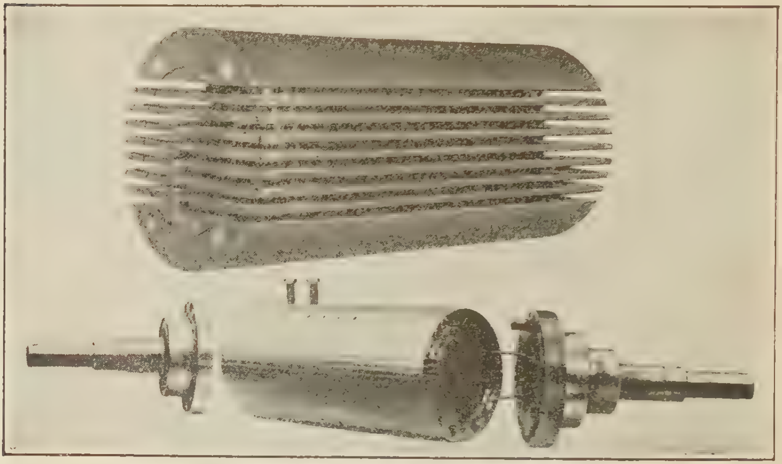


Fig. 141.—Parallel-slot, 2-pole rotor for a turbo-alternator.

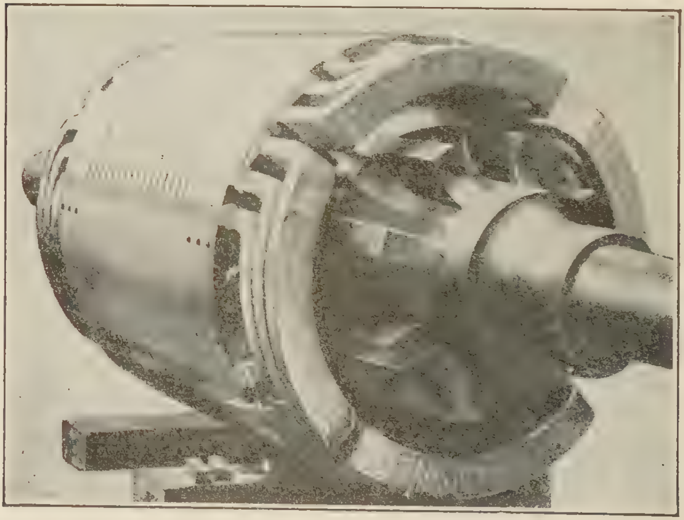


Fig. 142.—Radial-slot type of rotor, having four poles.

age loss and strong mechanically, especially as regards the support of the coil-ends. Parallel-slot rotors are seldom used except for

2-pole rotors in small machines. The metal back of the slots becomes too small in cross-section to withstand the centrifugal forces, when attempt is made to adapt this type of rotor to more than two poles.

Figure 142 shows a 4-pole, radial-slot rotor. Although the coil-ends are not held as strongly in this type of rotor as they are in the parallel-slot type, it is better adapted to rotors having more than two poles, because there is not the reduction of iron section back of the slots with increase in the number of poles, such as occurs in the parallel-slot type.

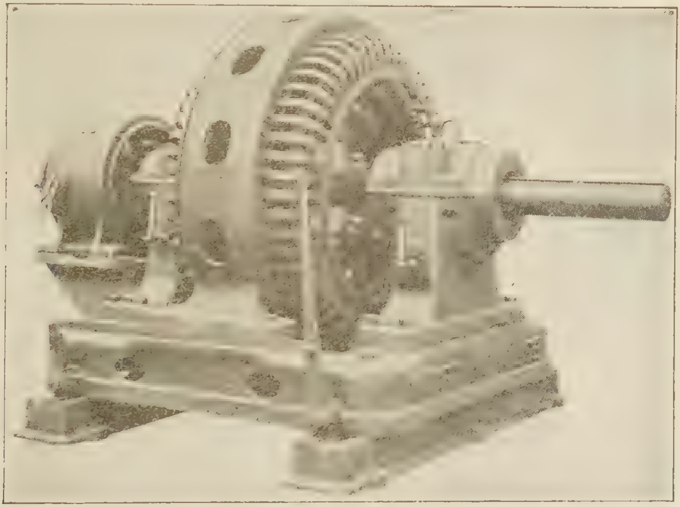


FIG. 143.—150 kv-a., 900 r.p.m., 2,400-volt, 60-cycle alternator, with direct-connected exciter.

Machines having these types of field are *non-salient-pole* alternators and the windings are called *distributed* field windings (see Par. 91).

The field connections are usually carried out through the center of the shaft to slip-rings. Two or more carbon brushes resting on the slip-rings carry the current to the winding. The excitation voltage is usually 120 or 250 volts and in the larger stations is supplied by bus-bars devoted to excitation only. In smaller installations, the exciter may be mounted directly on the alternator shaft (Fig. 143). When directly connected to the alternator

the cost of the exciter is much greater than when belt driven, because of the slow speed. Large central stations have an exciter bus supplied by separate motor- or steam-driven exciters. To increase reliability, a storage battery is often floated across this bus. Steam-driven exciters are also held in readiness for emergencies.

91. Distributed Field Winding.—In salient-pole alternators the turns for exciting each field pole are concentrated in a single coil on each pole. In the non-salient-pole alternator the exciting turns are distributed over the surface of the rotor, which is more or less smooth. Figure 144 (*a*) shows a distributed winding for one pole. This winding is a spiral winding (see page 138, Fig. 123) similar to that shown in Fig. 142. A lap-winding may be used, however. Figure 144 (*b*) shows the flux produced by the

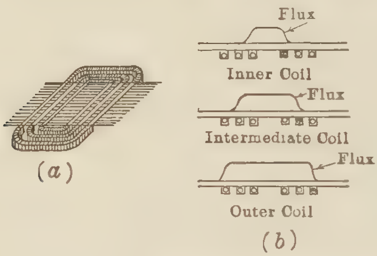


Fig. 144.—Distributed field winding and the resulting flux.

inner, the intermediate, and the outer coil as if each acted alone. By the corkscrew rule the direction of the flux is upwards. Figure 144 (*c*) shows the total flux due to the combined action of the three coils. The flux distribution is more or less sinusoidal and the e.m.f. wave of alternators having this type of field winding is ordinarily more nearly sinusoidal than that of alternators having salient poles.

92. Ventilation.—One of the most difficult factors in the design of high-speed, high-capacity alternators is the obtaining of sufficient ventilation. For example, a 20,000-kw. unit, having an efficiency of 96.5 per cent, requires 700 kw. to be dissipated. The considerable depth of iron back of the slots (Fig. 132), the fact that the armature is long and its diameter small, that the rotor must ordinarily be a solid-steel forging and thus cannot have air-ducts (Fig. 141), all require that special means be taken to carr-

away the heat developed in the armature and field copper and in the iron. Since the rotor is smooth, it produces little fan action as compared with salient-pole rotors (Figs. 138 and 140) and air must be forced through the machine from external sources.

The foregoing 20,000-kw. unit might require from 60,000 to 70,000 cu. ft. of air per minute. This air is usually supplied by separate blowers, and to remove the dirt and increase the cooling properties of the air, it is usually passed through an air washer consisting of a curtain of water. Otherwise dirt would soon clog

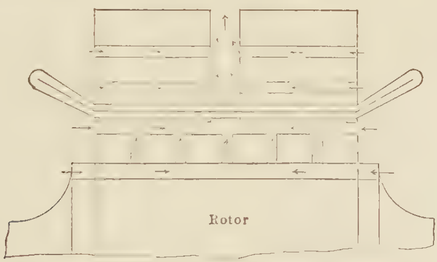


FIG. 145.—Passage of ventilating air through the ducts of a turbo-alternator.

the ventilating ducts and the machine would overheat. The air is ordinarily supplied by an outside blower.

Figure 145 shows the passage of the air as it is forced through the axial ducts back of the laminations (see Fig. 132) and out through a center radial duct, in a turbo-alternator.

ALTERNATOR ELECTROMOTIVE FORCE

93. Induced Electromotive Force.—It will be remembered that the e.m.f. in a single conductor l cm. long, cutting a field, whose flux density is B gausses, at a velocity of v cm. per second is

$$e = Blv10^{-8} \text{ volts}$$

when B, l, and v are mutually perpendicular (see Part I, page 207). The total flux is proportional to B and l. The velocity is proportional to the frequency. It can, therefore, be shown that the voltage per phase is

$$E = 2.22Z\phi f10^{-8} \text{ volts per phase} \tag{48}$$

where Z is the total number of series-connected conductors per phase, ϕ is the total flux in maxwells entering the armature from

one north pole (assumed to be sinusoidally distributed), and f is the frequency.

Example.—The armature of a 6-pole, 3-phase, 60-cycle alternator has 18 slots and there are 16 conductors per slot. There are 3,000,000 magnetic lines or maxwells entering the armature from one north pole, and this flux is sinusoidally distributed along the air-gap. What is the induced e.m.f. per phase?

The total surface conductors

$$= 18 \times 16 = 288.$$

The conductors per phase

$$Z = \frac{288}{3} = 96.$$

Using equation (48)

$$E = 2.22 \times 96 \times 3 \times 10^6 \times 60 \times 10^{-8}$$
$$= 384 \text{ volts. } \textit{Ans.}$$

If the *turns* per phase N are used rather than the number of *conductors* Z, (48) becomes

$$E = 4.44 N\phi f 10^{-8} \text{ volts per phase.} \qquad (49)$$

Equations (48) and (49) assume that there is but one slot per pole per phase as in the armature of Fig. 127 (page 141). Therefore, the e.m.fs. induced in all conductors under the same pole are in phase with one another. Ordinarily there is more than one slot per pole per phase. For example, in Fig. 128, there are 12 slots per pole and therefore four slots per pole per phase. Figure 146 (a) shows a sketch of such a winding. Since one pole-pitch represents 180° and there are 12 slots per pole, there must be $^{180}/_{12}$ or 15 electrical space-degrees between adjacent slots. If the field pole moves from left to right, coil 2 is cut by the center of the pole 15 electrical time-degrees later than coil 1. Therefore, the e.m.f. generated in coil 2 will lag that generated in coil 1 by 15 time-degrees as shown in (b). Likewise the e.m.f. generated in coil 3 will lag that generated in 2 by 15°, etc. Therefore, all four e.m.fs. differ in phase by 15° and their resultant e.m.f. E is *less* than it would be if the foregoing phase differences did not exist. Hence a factor k_b called the *breadth factor*, which is less than unity, must be introduced into equations (48) and (49). With a 3-phase winding having four slots per pole per phase, $k_b = 0.958$.

If a fractional-pitch winding is used, the e.m.f. is still further reduced as was stated in Par. 87. That is equations (48) and (49) must be multiplied by the pitch-factor k_p, which is 0.966 for a $\frac{5}{6}$-pitch winding.

Hence, (48) becomes

$$E = 2.22\, k_b k_p Z \phi f 10^{-8} \text{ volts per phase.} \qquad (50)$$

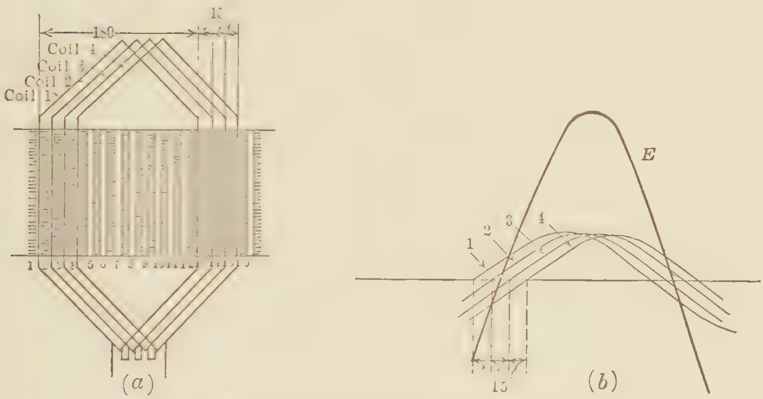


FIG. 146.—Effect of distributed winding on resultant e.m.f.

Example.—(*a*) Find the e.m.f. in the example, page 156, assuming five-sixths pitch and the winding distributed in 72 slots rather than in 18 slots. There are now four slots per pole per phase and four conductors per slot. The number of conductors Z per phase remains unchanged.

(*b*) If this alternator is Y-connected, what is its open-circuit terminal voltage:

(*a*) $E = 2.22 \times 0.958 \times 0.966 \times 96 \times 3 \times 10^6 \times 60 \times 10^{-8}$

$= 355$ volts. *Ans.*

(*b*) $E_{line} = \sqrt{3} \times 355 = 615$ volts. *Ans.*

94. Phasing Alternator Windings.—Three-phase alternator windings may be connected either in Y or in delta. Instances often occur in practice where six leads come from the machine, these leads being the three pairs of terminals from the three phases. The proper phase relations must be observed in making the connections, whether they are to be connected in Y or in delta.

Let aa', bb', and cc' (Fig. 147) be the three coil windings of a 3-phase machine.

Assume first that these three windings are to be connected in Y. First connect ends a and b together. Measure $E_{a'b'}$, the voltage across their open ends. This should equal $\sqrt{3}$ times the coil voltage. It may be equal to the coil voltage, in which case one coil should be reversed. Next tie the end c of coil cc' to point ab. The voltages $E_{b'c'}$ and $E_{a'c'}$ should each be $\sqrt{3}$ times the coil voltage. If not, the coil cc' should be reversed.

If it be desired to connect the coils in delta, the ends a and b' (Fig. 148 (a)) should first be connected. The voltage $E_{a'b}$, across their open ends, should now be equal to the coil voltage. If not, one of the two coils should be reversed. End c' of coil cc' should then be connected to b. The voltage $E_{ca'}$, across the open ends should be zero,

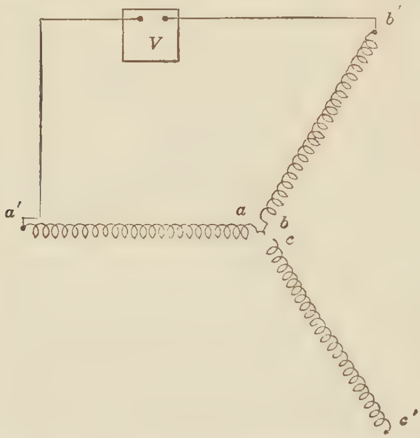


FIG. 147.—Connecting alternator coils in Y.

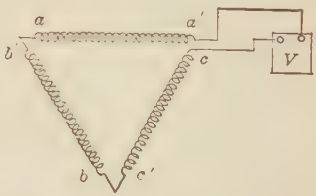


FIG. 148.—Connecting alternator coils in delta.

since the vector sum of the three voltages around a delta must be zero (see Par. 69, page 112). If this voltage is practically zero, the two ends c and a' may then be closed. The voltage $E_{ca'}$ may be twice the coil voltage. If this is found to be the case, coil cc' should be reversed. Coil cc' is phased with a voltage equal to and in phase with its own voltage. Hence, the voltage across the open ends $a'c$ must be either zero or twice the coil voltage.

95. Rating of Alternators.—The rating of electric machinery is determined in general by its temperature rise. This temperature rise is caused by the losses in the machine. The I^2R loss in the armature, due to the load current, limits the output of a

machine. This loss depends upon the *value* of the armature current and is independent of power-factor. For example, 100 amp. in a single-phase, 200-volt generator will produce the same I^2R loss whether the load power-factor be unity, 0.4, or any other value. The output in *kilowatts*, however, is proportional to the power-factor. If the above generator is limited to 100 amp., its output will be 20 kw. at unity power-factor but only 8 kw. at 0.4 power-factor. The rating is 20 kv-a. (kilovolt-amperes) regardless of power-factor.

For the above reasons, alternators are ordinarily rated in kilovolt-amperes (kv-a.). If a machine is rated in kilowatts, unity power-factor is assumed, unless otherwise specified. In stating the output of a machine it is always well to state the power-factor.

The rating of the prime mover driving an alternator is independent of the alternator power-factor. The same turbine could be used to drive a 200-kv-a. machine operating at 0.5 power-factor or a 100-kv-a. machine operating at unity power-factor, although the first alternator would have double the kilovolt-ampere rating of the second.

Example.—A 3-phase, 60-cycle, 6,600-volt, turbine-driven alternator is rated at 6,250 kw. at 0.7 power-factor and has a full-load efficiency of 0.95 excluding the field loss. (*a*) What is the kilovolt-ampere rating of the alternator? (*b*) What is its current rating? (*c*) What horsepower does the turbine deliver at full load? (*d*) If the alternator were loaded to its full rating at unity power-factor what horsepower would be required to drive it, assuming that the efficiency remained unchanged.

(*a*) $\text{Kv-a.} = \dfrac{6{,}250}{0.7} = 8{,}930.$ *Ans.*

(*b*) $I = \dfrac{8{,}930{,}000}{\sqrt{3} \times 6{,}600} = 781 \text{ amp.}$ *Ans.*

(*c*) $\text{Kw. input} = \dfrac{6{,}250}{0.95} = 6{,}580.$

$\text{Hp. input} = \dfrac{6{,}580}{0.746} = 8{,}830.$ *Ans.*

(*d*) $\text{Kw. input} = \dfrac{8{,}930}{0.95} = 9{,}400.$

$\text{Hp. input} = \dfrac{9{,}400}{0.746} = 12{,}600.$ *Ans.*

CHAPTER VI

ALTERNATOR CHARACTERISTICS AND OPERATION

Alternator Regulation.—When load is applied to an alternator, the terminal voltage drops unless the load happens to require a leading current. With a leading current the voltage tends to rise when load is applied, as will be demonstrated later (see page 173, Fig. 157). When load is applied to a direct-current shunt generator, the terminal voltage drops as is shown in Part I, Chap. XI. This drop in voltage is due to three causes: the I_aR_a drop in the armature, armature reaction, and the drop in field current which results from the decrease in terminal volts. As commercial alternators are excited from a separate source, there is no decrease of field current due to the drop in the alternator terminal voltage. Therefore, the alternator is more nearly comparable with the separately excited, direct-current generator (see Part I, page 249). Both the I_aR_a drop in the alternator armature and armature reaction ordinarily cause a drop of terminal voltage as load is applied. Another factor which causes the alternator voltage to drop with application of load is the *reactance* of the alternator armature. This will be discussed later.

With fixed conditions of field current, speed, etc. the terminal voltage of the separately excited, direct-current generator depends on the *magnitude* of the load current. Under similar conditions the terminal voltage of alternators depends not only on the magnitude of the load current but also on the *power-factor* of the load. A knowledge of the variation of voltage with current and power-factor is usually essential, since the amount by which the voltage varies with various conditions of load has an important bearing on the operation of the system as a whole. If the machine supplies incandescent lamps, its voltage must remain very nearly constant or else special regulators are necessary on the lighting circuits. Moreover, the voltage of an alter-

nator may remain within a few per cent. of being constant from no load to full load if the power-factor is *unity*, while at *low power-factors* the voltage may drop 30 or 40 per cent. from no-load to full-load *current* (not kilowatts).

The variation of voltage with load for alternators is therefore very much larger than with direct-current generators. For example, with shunt generators of commercial size the variation of voltage from no load to full load is small, and it is usually possible to compound a shunt generator so that its terminal voltage is practically constant at all loads. In the alternator, the armature reactance-drop, which is not present in the direct-current generator, and the greater effect of armature reaction, result in much larger percentage drops in voltage with increase in load. In addition, alternators cannot be readily compounded.

In the larger types of alternator, the large values of current which result from short-circuit may cause serious damage to the machine and to the system. The value of this short-circuit current is closely related to the voltage characteristics of the alternators connected to the system, so that a knowledge of the voltage characteristics of alternators is helpful in designing the circuit breakers, switches, power-limiting reactances, etc.

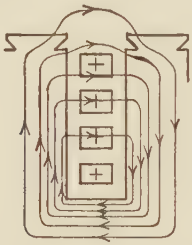
From the operating point of view, therefore, a study of the load-voltage characteristics of alternators is even more essential than the study of the characteristics of direct-current generators. Moreover, it is not only desirable to know the characteristics themselves, but it is even more important to know the factors which underlie these characteristics. Knowing these factors and their relative magnitudes, it is not difficult to understand *why* alternators have characteristics similar to those shown in Figs. 156 to 159 inclusive.

The three factors which cause change of terminal voltage with load are armature *reactance*, armature *resistance*, and armature *reaction*.

96. Armature Reactance.—Since the armature conductors of alternators are embedded in lots, hence are nearly surrounded by iron, considerable flux must link the armature coils when current flows. For example, Fig. 149 (*a*) shows four coil-sides embedded in a single alternator slot. The insulation and wedge are omitted. It is assumed at the instant shown that the current flows inwards

in all four conductors. By the corkscrew rule, the flux must have a clockwise direction about the four conductors. It crosses the slot and tooth tips, and completes its circuit through the armature iron. It will be observed (Fig. 149 (*a*)) that all four conductors are linked by some of this flux, but more flux links the bottom conductor than the other conductors.

Figure 149 (*b*) shows, to a smaller scale, a complete armature coil embedded in two slots. It will be observed that the flux which links the coil-sides in the slots links the entire coil as well. Flux also links the coil-ends, but since the coil-ends are not embedded in iron, this flux, although not negligible is much smaller in value than the flux linking the portions of the conductor which



(*a*) **Flux-slot linkages**

(*b*) **Flux-coil linkages**

Fig. 149.—Inductance of armature coil.

lie in the slots. This flux which links the armature coils, but not the field coils, is called the *armature-leakage flux*.

When flux links a current, *self-inductance* results (see Part I, page 164). Since the current in alternator coils is alternating, these coils must have *reactance*. If the inductance per *phase* of the armature is L_a, the reactance per phase is

$$X_a = 2\pi f L_a$$

where f is the frequency.

It is obvious that the alternator having the deeper and narrower slots will have the higher self-inductance, other factors being equal. Also the alternator operating at the higher frequency has the greater armature reactance, other factors being equal.

97. Armature Resistance.—The armature iron forms a considerable portion of the path of the flux which links the armature conductors (Fig. 149 (*a*)). Since this flux is alternating, it is

accompanied by hysteresis and eddy-current losses, which occur in the iron immediately surrounding the slots. The power required to supply these losses must be supplied by the armature current, since this leakage flux linking the armature conductors is produced by the armature current. The combined hysteresis and eddy-current losses vary very nearly as the current *squared.*

The slot-leakage flux (Fig. 149 (a)) tends to crowd the current toward the tops of the conductors. If the conductors have large cross-sections, this same flux induces eddy currents in the conductors themselves.

All these factors tend to make the apparent copper loss with alternating current considerably greater than it would be with an equal value of direct current. With direct current the armature copper loss is given by I^2R_0 where R_0 is the direct-current or ohmic resistance. To obtain the total loss associated with the flow of alternating current, R_0 must be *increased* to a value R, where R is the *effective* resistance of the armature. The ratio of R to R_0 varies in different alternators, but it is of the order of 1.2 to 1.6, the larger values applying to 60-cycle machines. It is not easy to determine R accurately. Ordinarily R is found by increasing R_0, the ohmic resistance, by 50 per cent, R_0 being determined by direct-current measurement.

Example.—The resistance between two terminals of a 3-phase, Y-connected alternator is measured with direct current and found to be 0.32 ohm. The ratio of effective to ohmic resistance is assumed to be 1.5. What is the effective resistance per phase of this alternator?

Since two phases of the machine are connected in series between each pair of machine terminals, the ohmic resistance of each phase must be 0.32/2 or 0.16 ohm.

The effective resistance,

$$R = 1.5 \times 0.16 = 0.24 \text{ ohm.} \quad \textit{Ans.}$$

98. Armature Reaction.—In direct-current machines, the armature ampere-turns act on the magnetic circuit of the machine in such a way as to change both the direction and the magnitude of the air-gap flux. For a given armature current, the direction of this armature reaction depends on the position of the brushes. In an alternator with a given armature current, the direction and magnitude of the air-gap flux obviously cannot depend on brush position, but they do depend on the phase relation existing

between the current and the voltage and, hence, on the power-factor of the load.

Figure 150 shows a portion of the field and armature of an alternator, the direction of rotation of the armature being from left to right. An armature coil is shown in (*a*) at the instant when its sides are directly under the pole centers and the coil is

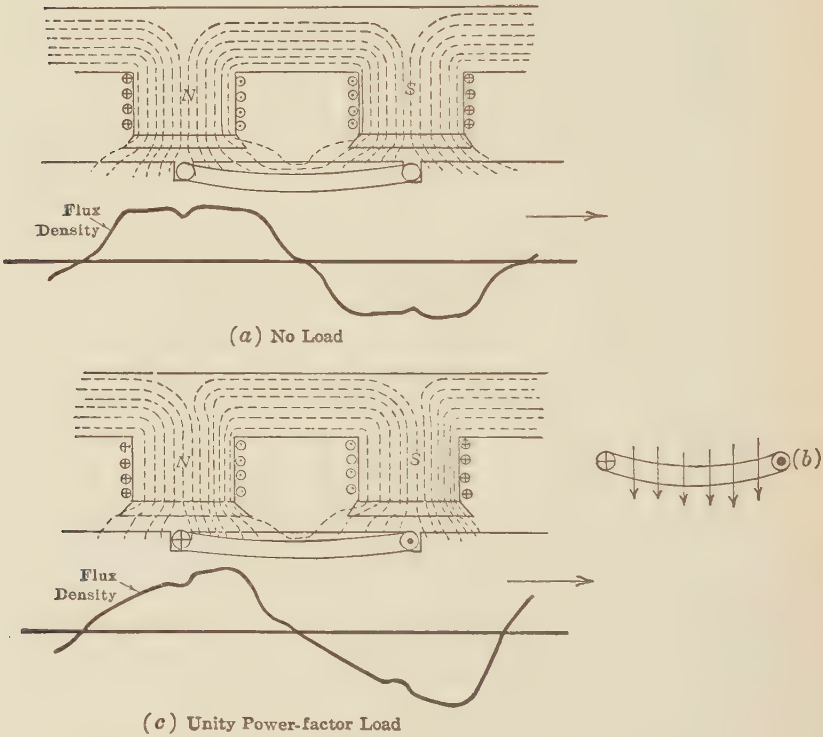


Fig. 150.—Flux distribution in the air-gap of a salient-pole alternator.

carrying no current. Since its m.m.f. is zero it has no effect on the flux distribution. Directly beneath is shown the flux distribution curve, whose ordinates are flux density. The curve is practically flat on top and the flux under each pole is distributed symmetrically.

Figure 150 (*c*) shows the coil when it carries a current in phase with the induced e.m.f. At the instant shown the induced e.m.f.

in the coil is at its maximum value. If the current is in phase with this induced voltage, corresponding approximately to a load of unity power-factor, the current is at its maximum value at this same instant. The direction of the induced e.m.f. and, hence, of the current in the coil, which may be determined by Fleming's right-hand rule, is inward in the left-hand side of the coil and outward in the right-hand side, as shown in (*b*) and (*c*). The coil m.m.f. at this instant, as determined by the corkscrew rule, acts downward as shown in (*b*). The effect of this coil on the magnetic circuit, that is, the effect of armature reaction, is shown in (*c*). The m.m.f. of the coil being downward strengthens the flux in the right-hand side of the north pole and weakens it in the left-hand side of the south pole. This effect is further illustrated

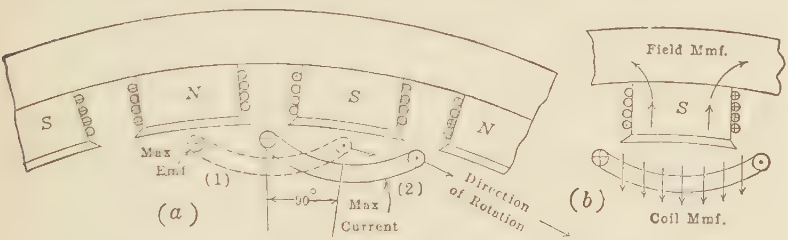


FIG. 151.—Armature reaction due to current lagging 90°.

by the flux-distribution curve. The curve is peaked on the right-hand side and depressed on the left-hand side. The area under the curve, hence, the value of the flux, is changed but little.

This same effect occurs in direct-current machines when the brushes are in the geometrical neutral (see Part I, page 237).

In an alternator, therefore, it may be said that with unity power-factor loads the effect of armature reaction is to distort the flux in the direction of rotation of the armature,[1] the total value of flux being but little changed.

Figure 151 (*a*) shows an alternator, similar to the one shown in Fig. 150, with an armature coil rotating in a clockwise direction. In Fig. 151, however, the current is assumed to *lag the induced e.m.f. by* 90°. When the coil is in position (1) its induced e.m.f. must be a maximum, since the coil-sides are directly under the

[1] With rotating fields, the flux is distorted in a direction opposite to that of rotation.

centers of the poles. At this instant, however, the current must be zero (see page 132, Fig. 117). The current does not reach its maximum value until the coil has traveled 90 electrical space-degrees further, and has reached position (2). When in position (1) the induced e.m.f. acts *inward* in the left-hand side of the coil. The current which lags 90°, therefore, must act inward in the left-hand side of the coil 90° later when the coil reaches position (2). When in position (2) the coil center is directly under the center of a south pole as shown in (*b*). By the corkscrew rule, its m.m.f. is downward, and, therefore, it acts in direct *opposition* to the south pole. *When the armature current lags the induced e.m.f. by* 90°, *its m.m.f. acts in direct opposition to the main field.* As a result, the field is materially weakened by a lagging current and this is accompanied by a reduction of the induced e.m.f.

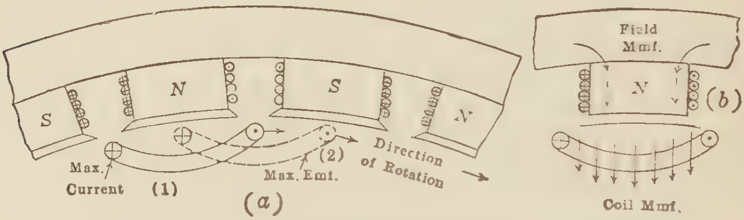


FIG. 152.—Armature reaction due to current leading 90°.

This result is similar to that of moving the brushes forward 90° in a direct-current generator. All the armature ampere-turns are then demagnetizing, tending to weaken the field.

Figure 152 shows the conditions existing when the current leads the induced e.m.f. by 90°. As before, the e.m.f. reaches its maximum value when the coil sides are directly under the pole-centers, position (2), (Fig. 152 (*a*)). The current, however, reaches its maximum value 90 electrical space-degrees *ahead* of this position, or at (1). Since, by Fleming's right-hand rule, the direction of the induced e.m.f. is inward in the left-hand side of the coil when it is at (2), the direction of the current, which reaches its maximum value 90 electrical time-degrees earlier, must also act inward in the left-hand side when the coil is at (1). When at (1) the coil is directly beneath a north pole, and by the corkscrew rule its m.m.f. acts downwards as shown in (*b*). The m.m.f. or

the ampere-turns of the coil, therefore, now *assist or strengthen* the main field, as they are acting directly in conjunction with it. *When the armature current leads the induced e.m.f. by 90° its m.m.f. acts in conjunction with the main field.* Accordingly, a leading current tends to *strengthen* the field, and therefore increase the induced e.m.f.

A given generator with a fixed field current and a fixed armature circuit will have the greatest value of flux, and, hence, the highest induced e.m.f. when the current leads, and the lowest values of flux and induced e.m.f. when the current lags. At unity power-factor the values of flux and induced e.m.f. will be intermediate.

99. Armature Impedance-drop.—In the direct-current generator the terminal voltage differs from the e.m.f. induced in the armature by the resistance-drop in the armature. That is the induced e.m.f. may be found by *adding* the armature resistance-drop to the terminal voltage (see Part I, page 250).

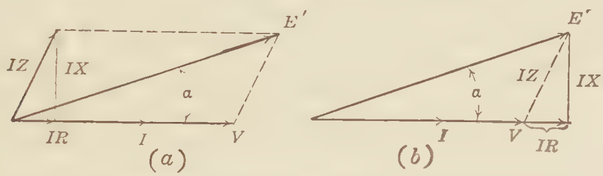


FIG. 153.—Alternator vector diagram for unity power-factor.

In the alternator there is not only a resistance-drop in the armature but also a reactance-drop due to the self-inductance of the armature. The combined resistance- and reactance-drop give an armature *impedance-drop*. To determine the induced e.m.f. in the alternator armature, the armature *impedance-drop* must be added vectorially to the terminal voltage. This is illustrated by the following examples.

Current in Phase with Terminal Voltage.—Let V be the terminal voltage of the alternator (Fig. 153 (a)), when it delivers current I at unity power-factor. As the power-factor is unity, I is in phase with V. The armature itself may be treated like a simple series-circuit having resistance and reactance (see Fig. 56, page 64). The resistance-drop IR is in phase with the current. The reactance-drop IX leads the current by 90°. The vector sum of these two drops gives the armature impedance-drop IZ. The e.m.f. induced in the armature is the vector sum of the impedance-drop IZ,

and the terminal voltage V. By completing the parallelogram (Fig. 153 (*a*)) having V and IZ as two of its sides, the diagonal E' is their vector sum and, hence is the e.m.f. induced in the armature.

The same result is obtained by adding the IR drop directly to V, Fig. 153 (*b*), and then adding the IX drop, at right angles to I and leading, at the end of IR. The vector addition in this case is made by the use of the triangle of vectors described in Chap. I, page 27. The impedance-drop IZ is shown dotted in Fig. 153 (*b*), as it is not used in obtaining E' by this particular method.

It is to be noted that with a load of unity power-factor the current is in phase with the terminal voltage, but lags the generator *induced* voltage by an angle α.

It is a simple matter to find E' if the other quantities are known. E' is the hypotenuse of a right triangle of which $(V + IR)$ is one side and IX the other.

$$E' = \sqrt{(V + IR)^2 + (IX)^2}. \tag{51}$$

Example.—A 60-kv-a., 220-volt, 60-cycle alternator has an effective armature resistance of 0.016 ohm and an armature reactance of 0.070 ohm. What is its induced e.m.f. when the machine is delivering its rated current at a load power-factor of unity?

$$\text{The current } I = \frac{60{,}000}{220} = 273 \text{ amp.}$$

$$IR = 273 \times 0.016 = 4.37 \text{ volts.}$$

$$IX = 273 \times 0.070 = 19.1 \text{ volts.}$$

$$E' = \sqrt{(220 + 4.4)^2 + (19.1)^2} = 225 \text{ volts.} \quad \textit{Ans.}$$

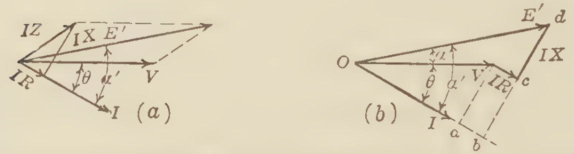


FIG. 154.—Alternator vector diagram for power-factor cos θ, current lagging.

Lagging Current.—When the current lags the terminal voltage by the angle θ, the same method is employed to calculate the induced e.m.f. Figure 154 (*a*) shows the current I lagging the terminal voltage V by the angle θ. The IR drop is along the current vector I, and the IX drop is in quadrature with I and leading, as before. The resulting impedance-drop IZ is then found, being the resultant of IR and IX. This impedance-drop is then added vectorially to V by completing the parallelogram

having V and IZ as two of its sides. The resultant gives the armature induced e.m.f., E'. It will be noted (Figs. 153 and 154) that the position of the armature-impedance triangle is determined by the current and not by the generator voltage. Therefore, when the current lags, this impedance triangle swings clockwise with the current.

As before, the impedance-drop may be added at the end of V, if the proper phase relations are observed. The most direct method of finding the induced e.m.f. E' is to use the method described under the triangle of vectors (page 27). IR, which is in phase with the current, is first added vectorially at the end of the terminal voltage V. Then the reactance-drop IX, at right angles to the current and leading, is added at the end of IR. The resultant voltage found by completing the polygon is the induced e.m.f. E'. This method is illustrated in Fig. 154 (b), where IR is parallel to I and IX is at right angles to I and leading.

The geometrical solution of this diagram is quite simple. If IR is projected on the current vector I, a right triangle of voltages, Obd, is formed, of which E' is the hypotenuse. The values of the two legs of this right triangle may be found as follows:

$$Oa = V \cos \theta$$
$$ab = IR$$
$$aV = bc = V \sin \theta$$
$$cd = IX$$
$$E' = \sqrt{\overline{Ob}^2 + \overline{bd}^2} = \sqrt{(Oa + ab)^2 + (bc + cd)^2}$$
$$= \sqrt{(V \cos \theta + IR)^2 + (V \sin \theta + IX)^2}. \qquad (52)$$

The current now lags the induced voltage E' by the angle α'.

Example.—Determine E' for a load in which the power-factor is 0.7, current lagging, using the constants of the example on page 168.

The rating of an alternator, as has already been pointed out, depends on the current or kilovolt-amperes rather than the kilowatts. Therefore, the current rating of the generator will remain unchanged, although the kilowatts in this problem are reduced to 0.7 of their former value.

$\cos \theta = 0.70$ $\qquad$ $IR = 4.37$ volts as before.

$\theta = 45.6°$

$\sin \theta = 0.714$ $\qquad$ $IX = 19.1$ volts as before.

$$E' = \sqrt{(220 \times 0.70 + 4.4)^2 + (220 \times 0.714 + 19.1)^2} = 237 \text{ volts.} \quad \textit{Ans.}$$

It is to be noted that the induced e.m.f. is now higher than before, although the value of the impedance-drop itself is the same. Therefore, for a fixed value of induced e.m.f., the terminal volts

become less with increasing lag of the current, even though the value of the current remains unchanged. This is due to the angle at which the impedance-drop subtracts from the induced e.m.f. It would be expected, therefore, that the drop in terminal voltage with increase of load would be much greater with lagging current than when the power-factor is unity. Such is the case.

At unity power-factor, the armature resistance-drop is the important factor in determining the value of E'. With lagging current, the resistance-drop plays but a small part and the armature *reactance*-drop is the important factor.

Leading Current.—Figure 155 (a) shows the alternator vector diagram when the current *leads* the terminal voltage by an angle θ. As before, IR and IX are laid off in phase and at right angles to the current respectively, and IZ their vector sum is found.

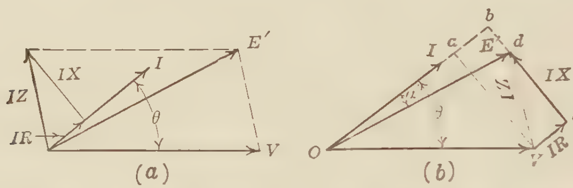


Fig. 155.—Alternator vector diagram for power-factor cos θ, leading current.

The parallelogram having IZ and V as its two adjacent sides is completed and the diagonal E' is their vector sum. This same vector addition is performed in (b) by adding IR, parallel to I, to the end of V and IX, at right angles to I and leading, to the end of IR. The resultant E' is the same as in (a).

The value of E' may be found trigonometrically as in Fig. 154.

The voltage-drop IR, parallel to the current, is projected on the current vector, at ab

$$Oa = V \cos \theta$$
$$ab = IR$$
$$aV = bc = V \sin \theta$$
$$cd = IX$$

$$E = \sqrt{\overline{Ob}^2 + \overline{bd}^2} = \sqrt{(Oa + ab)^2 + (bc - cd)^2} = \sqrt{(V \cos \theta + IR)^2 + (V \sin \theta - IX)^2}. \quad (53)$$

Equation (53) differs from equation (52) only in the sign of IX, which is now negative.

Example.—Repeat the foregoing problem when the power-factor is 0.7, current leading.

$$\cos\theta = 0.70 \qquad IR = 4.37 \text{ volts.}$$
$$\sin\theta = 0.714 \qquad IX = 19.1 \text{ volts.}$$

$$E' = \sqrt{(220 \times 0.70 + 4.4)^2 + (220 \times 0.714 - 19.1)^2} = 210 \text{ volts.} \quad \textit{Ans.}$$

The induced e.m.f. in the armature is now *less* numerically than the terminal voltage. This is a condition which cannot exist in a direct-current generator. It results from the phase position of the IZ drop with respect to V. A study of Fig. 155 shows that as the angle of lead of the current increases, IZ swings counter-clockwise and tends to make the magnitude of E' less.

100. Alternator Regulation with Lagging Current.—The voltage E', as determined in the preceding paragraph, is the voltage *induced* in the alternator armature under load conditions. In practice it is a quantity which is not easily measured and can be calculated only approximately, since it is difficult to determine the armature reactance X. E' is *not* the no-load terminal voltage of the alternator.

If there were no armature *reaction*, E' would be the no-load voltage of the machine.

For example, consider a separately excited, direct-current generator having an armature resistance of 0.1 ohm and delivering 50 amp. at 120 volts at the terminals. The induced voltage is obviously $120 + 50 \times 0.1$ or 125 volts. When the load is removed, the speed and field current being maintained constant, the no-load voltage is measured and found to be 128 volts. The difference between the 128 volts induced at no load and the 125 volts induced under load is due to the fact that under load, the armature reaction has weakened the field and reduced the induced e.m.f. (also see Part I, page 249, Fig. 224).

Figure 156 shows the characteristic of an alternator, taken with lagging current. At no load the induced e.m.f. is Oa' or da. At rated load the induced e.m.f. is db. The decrease ab from the no-load induced e.m.f. to the rated-load induced e.m.f. is due to the reduction in flux brought about by the weakening of the field due to lagging armature current. The voltage-drop bc is due to the armature impedance-drop, although bc is *not equal* to the armature impedance-drop since this drop is ordinarily out of

phase with the terminal voltage and the induced e.m.f. (see Fig. 154).

Thus with a lagging current, two factors tend to cause the terminal voltage to drop rapidly. First, the position of the armature coil when its current is a maximum is such that the field is weakened and the induced e.m.f. reduced. Second, the phase of the armature impedance-drop is such that it subtracts almost directly from the induced e.m.f. and this results in a lowered terminal voltage. Hence, a lagging current causes an excessive drop in terminal voltage with load. To maintain the voltage

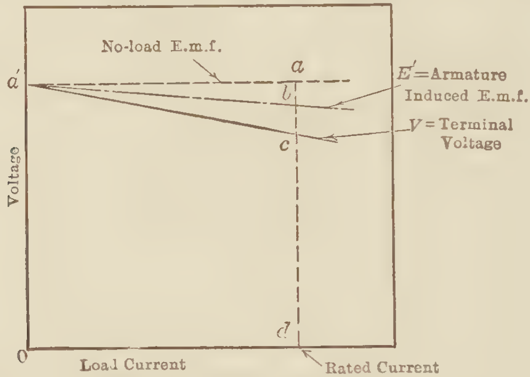


FIG. 156.—Alternator characteristic with a lagging current.

with a lagging current requires a considerable increase in field current and excitation.

Alternator regulation is defined as the difference between no-load and rated-load voltage, divided by rated-load voltage. For example, in Fig. 156, the regulation of the alternator is $\frac{da - dc}{dc}$ or ca/dc. The regulation is a measure of the ability of the alternator to maintain its terminal voltage and is not concerned directly with the armature induced e.m.f. E'.

Example.—The terminal voltage of a 400-kv-a., 60-cycle, 3-phase alternator is 2,300 volts when it is delivering its rated load of 100 amp. at 0.8 power-factor, lagging current. When this load is removed, the no-load voltage becomes 2,760 volts. What is the regulation of the alternator at this power-factor?

$$\text{Regulation} = \frac{2{,}760 - 2{,}300}{2{,}300} = \frac{460}{2{,}300} = 0.20 \text{ or 20 per cent.} \quad \textit{Ans.}$$

101. Alternator Regulation with Leading Current.—A study of Fig. 155 and the example, page 171, shows that with leading current the terminal voltage may *exceed* numerically the induced e.m.f. in the armature. With a fixed induced e.m.f., the terminal voltage may *rise* with increase of load. When the current leads, however, it also tends to *strengthen* the field through armature reaction. With a leading current, therefore, the induced e.m.f. E' may actually rise with increase of load, as shown in Fig. 157. Since the induced e.m.f. increases with load and the terminal voltage tends to exceed in magnitude the induced e.m.f. with

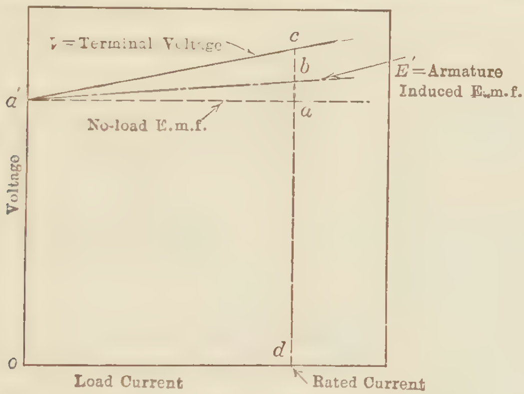


FIG. 157.—Alternator characteristic with a leading current.

increase of load, the terminal voltage at rated load may be considerably *greater* than its value at no load. In other words, the terminal voltage actually *drops* when the load is removed from the alternator.

Therefore, an alternator delivering a leading current may have a rising characteristic, similar to that shown in Fig. 157. This characteristic is not unlike that of the overcompounded, direct-current generator (see Part I, page 254, Fig. 228), except that in the overcompounded, direct-current generator the *induced* e.m.f. must be greater than the terminal voltage.

The regulation of the alternator under the conditions shown in Fig. 157 is, by definition,

$$\frac{da - dc}{dc} = -\frac{ac}{dc},$$

and is, therfore, *negative*.

Example.—The terminal voltage of the 400-kv-a. alternator (Par. 100) is 2,300 volts when it is delivering its rated load of 100 amp. at 0.8 power-factor, leading current. When this load is removed, the terminal voltage drops to 2,100 volts. What is the regulation of the alternator at this power-factor?

$$\text{Regulation} = \frac{2{,}100 - 2{,}300}{2{,}300} = \frac{-200}{2{,}300} = -0.087 \text{ or } -8.7 \text{ per cent.} \quad \textit{Ans.}$$

102. Alternator Regulation at Unity Power-factor.—When a load has a power-factor of unity, the current is in phase with the *terminal* voltage. It therefore lags the induced voltage. This is shown in Fig. 153 where the current I, in phase with the

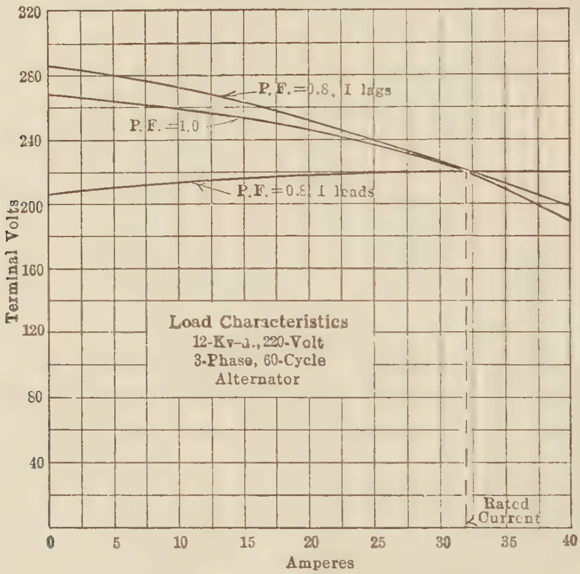


FIG. 158.—Typical load characteristics of a 12 kv-a., 3-phase alternator.

terminal voltage, lags the induced e.m.f. E' by the angle α. It was shown in Fig. 150 that there is no demagnetizing action of the armature on the field when the current is in phase with the *induced e.m.f.* Since with a unity power-factor load the current *lags* the induced e.m.f., there must be some demagnetizing action of the armature m.m.f. on the field and the induced e.m.f. must decrease with increase of load.

The combined reactance- and resistance-drops (Fig. 153) cause further decrease in terminal voltage. Therefore, with

unity power-factor load, the alternator terminal voltage drops with increase of load. For a given current, however, the terminal voltage is greater with unity power-factor load than with lagging current. In other words, the regulation at unity power-factor is better than with lagging current.

Figure 158 gives the characteristics of a 12-kv-a., 3-phase, 60-cycle, 220-volt generator at unity power-factor and at 0.8 power-factor for both lagging and leading current, all three characteristics being taken with the alternator field adjusted to give the same *rated* voltage of 220 volts in each case. The regulation at each power-factor is:

Unity power-factor: Regulation $= \frac{268 - 220}{220} = 21.8$ per cent.

Power-factor $= 0.8$, current lags: Regulation $= \frac{286 - 220}{220} = 30$ per cent.

Power-factor $= 0.8$, current leads: Regulation $= \frac{205 - 220}{220} = -6.8$ per cent.

103. The Saturation Curve.—The saturation curve of the alternator is obtained in precisely the same manner as the satura-

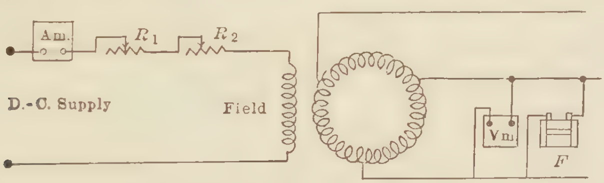


FIG. 159.—Connections for obtaining the saturation curve of an alternator.

tion curve of the direct-current generator. The field-circuit is connected across direct-current supply in series with rheostats and an ammeter (Fig. 159). A voltmeter is connected across one pair of line terminals to measure the induced e.m.f. Voltmeters across the other pairs of terminals are not necessary, since the e.m.fs. across all three phases should be equal. As a precaution it is advisable to measure the other e.m.fs. at some particular value of field current to determine whether or not all three are equal. The speed which should be maintained constant, is determined by a frequency meter connected across one of the phases (see page 97). The saturation curve is obtained by increasing the field current in steps and measuring the induced

e.m.f. at each value of field current. A saturation curve with decreasing values of field current may also be obtained. Figure 160 gives the saturation curve of a 1,500-kv-a., 2,300-volt, 60-cycle alternator taken with increasing values of field current.

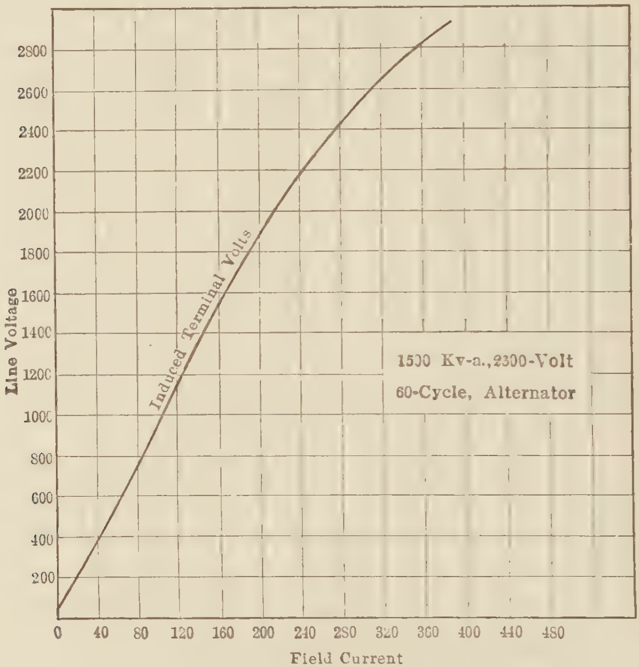


FIG. 160.—Saturation curve of a 1,500 kv-a. alternator.

104. Connections for Making Alternator Load Tests.—It is much more difficult to determine experimentally the characteristics of polyphase alternators than those of direct-current generators. If a 3-phase alternator is being tested, three loads connected either in Y or in delta are necessary. For each reading it is necessary to balance all three loads so that the three line currents are equal. If any one load with either the Y- or the delta-connection is changed, at least two currents are affected so that it requires time to make adjustments. If the alternator is Y-connected and the neutral is accessible, a Y-connection of the load, with the load neutral connected to the alternator

neutral, is most convenient since each line current may then be adjusted independently of the other two.

Figure 161 (*a*) shows the connections for obtaining the characteristic of an alternator at unity power-factor. The alternator is driven by a 230-volt shunt motor *M* and an ammeter and voltmeter are connected to measure the motor input, should it be desired to obtain the efficiency of the set. The alternator field is excited from 115-volt, direct-current supply, which may be obtained by connecting between neutral and outer of the Edison 3-wire system shown. The load is delta-connected and the power is measured with a polyphase wattmeter.

It is much more difficult to obtain loads whose power-factors are other than unity, particularly when it is desired that these power-factors remain constant under varying load. The most

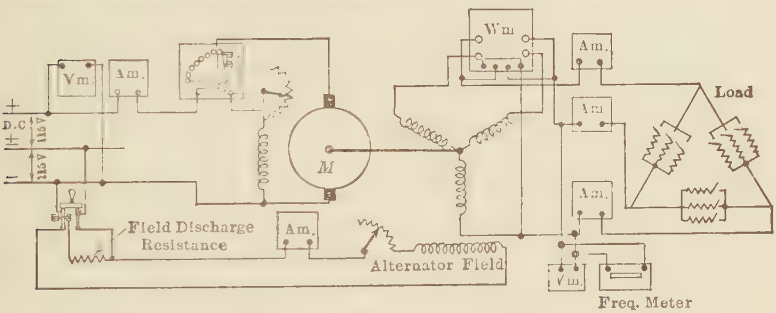


FIG. 161 (*a*).—Connections for making load test of a 3-phase alternator.

practicable method is to use a synchronous motor for a load. Its power load may be changed by applying more load to its shaft mechanically or by loading it with a direct-connected generator. The power-factor of the synchronous motor may be varied over a considerable range from lagging to leading current merely by varying the motor-field current (see page 281). Even with a synchronous motor, it requires considerable time to make the necessary frequency, load, and power-factor adjustments at each reading. It usually suffices to obtain the regulation by adjusting the load to rated voltage and current and the desired power-factor. The load is then removed and the voltage is again read, the frequency and field current remaining unchanged This gives sufficient data for obtaining the regulation.

Characteristic curves, similar to those shown in Figs. 156, 157 and 158, may be obtained by connecting as shown in Fig. 161 (*a*).

A synchronous motor load for obtaining power-factors other than unity may, however, be necessary.

When a highly inductive circuit is opened, a high e.m.f. of self-induction results. Since alternator fields have high self-inductance and considerable energy ($\frac{1}{2}LI^2$) is stored therein the field winding might puncture if the field-circuit were opened in the ordinary manner (see Part I, page 171). Puncture is prevented by the use of a *field-discharge switch.* This switch has an extra set of clips (Fig. 161 (*b*)) which make contact with one blade only when the switch is being opened. A resistance is connected between these clips and the other side of the line (Fig. 161 (*a*)). At the instant of opening the switch, the field (and the line temporarily) is paralleled by the field-discharge resistance. The energy of the field is dissipated partly in this resistance rather than at the switch contacts. Contact with switches opening inductive circuits should be carefully avoided, even in the case of very low voltages. There is danger not only of being burned by the arc, but also of being injured from the high induced voltage as well.

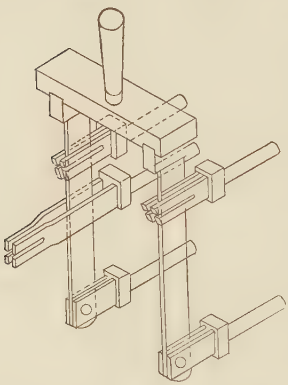


FIG. 161 (*b*).—Field discharge switch.

105. The Tirrill Regulator.—An automatic voltage regulator of the Tirrill type for direct-current machines is described in Part I, Chap. XI, page 255. An automatic voltage regulator is much more essential in the smaller alternating-current stations than in direct-current stations. The voltage changes in the generator and throughout the system are greater with alternating current than with direct current because of the added reactance-drop in the generator armature, transformers, feeders, etc. Alternating-current generators cannot readily be compounded to compensate for voltage-drop as direct-current generators are.

The Tirrill regulator is also designed to be used with alternating-current generators. As with large direct-current machines,

the regulator acts through the field of an exciter. The underlying principle of the regulator is the same whether used for alternating or for direct current, the voltage being controlled in each case by the rapid short-circuiting of the exciter-field rheostat.

When the alternator voltage drops, the contacts which short-circuit the exciter-field rheostat remain closed for longer periods of time, thus increasing the exciter voltage and hence the alternator-field current. If one exciter supplies the fields of all the alternators in the plant, it is obvious that one regulator can readily maintain the bus-bars at the desired voltage.

PARALLEL OPERATION OF ALTERNATORS

106. Division of Load between Alternators in Parallel.—The same reasons which make it necessary to operate direct-current generators in parallel (see Part I, page 296, Par. 222) apply to alternators.

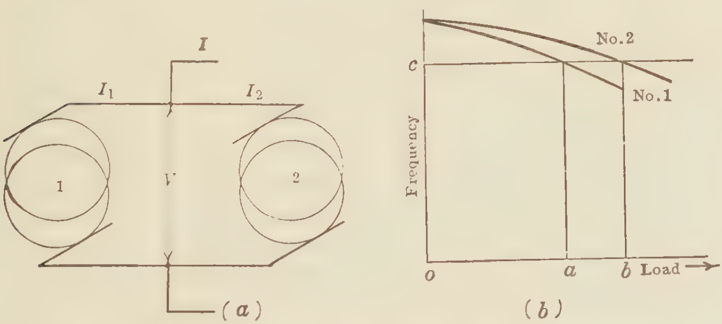


Fig. 162.—Alternators in parallel and speed-load characteristics.

In order to operate satisfactorily in parallel without equalizers, direct-current generators must have drooping-voltage characteristics.

The speed-load characteristic of the prime mover affects parallel operation only to the extent that it affects the load-voltage curve of the generator.

With alternators, load-voltage curves have practically no effect on the division of kilowatt load between machines. The division

of kilowatt load between alternators is determined almost entirely by the speed-load characteristics of their *prime movers*. Further, in order that alternators may operate satisfactorily in parallel, their *prime movers* must have *drooping* speed-load characteristics. Otherwise instability accompanies their operation in parallel.

The effect of prime mover, speed-load characteristics on the division of kilowatt load is shown in Fig. 162 (*b*). In Fig. 162 (*a*) are shown diagrammatically two alternators 1 and 2 connected in parallel.

If they are operating in parallel they must have the *same frequency* and the *same terminal voltage*. Figure 162 (*b*) shows the speed-load curve of each of the prime movers driving the alternators and it will be observed that each prime mover has a drooping speed-load characteristic. (Instead of plotting speed in r.p.m., the frequency or electrical speed is plotted. For example, a 6-pole alternator running at 1,200 r.p.m. would have the same electrical speed as an 8-pole alternator running at 900 r.p.m.) The speed-load curves of the prime movers are determined by their respective governors, if they are steam-, water-, or gas-driven units. If motor-driven, the speed-load characteristics depend upon the motor speed-load characteristics.

Let *oc* (Fig. 162 (*b*)), be the frequency at which the system is operating. By projecting horizontally to intersect the speed-load curves, the load taken by each prime mover at this frequency is obtained. *oa* is the load on prime mover 1 and *ob* is the load on prime mover 2, as both prime movers must be operating to give system frequency. Let the field of alternator 1 be strengthened by means of its field rheostat. At the same time weaken the field of 2 so that the line voltage does not change. If these were direct-current generators, alternator 1 would immediately take more load since its induced e.m.f. has been increased. But 1 *cannot take more load* because its prime mover can deliver only the load *oa* at this frequency. Alternator 2 cannot drop any load because its prime mover can deliver only the load *ob* at this frequency. Both machines must always operate at the same frequency which is not true of direct-current machines. *Therefore, the kilowatt load delivered by alternators in parallel cannot be shifted appreciably by means of the generator fields.*

To change the kilowatt load of either machine, the speed-load characteristic of its prime mover must be changed. In engine-driven units, this is done by changing the tension in the governor spring or altering the governing device in some manner. Assume, in Fig. 162 (*b*) that it is desired to make generator 1 take the same load as 2. The governor spring of 1 is so adjusted that the characteristic of 1 is raised, as shown in Fig. 163. Both machines now deliver the same load *oa′* at a frequency *oc′*. Under the conditions shown (Fig. 163) the frequency *oc′* is higher than the original frequency *oc* (Fig. 162). If the original frequency is to be maintained, the speed-load characteristic of 2 must be lowered at the same time that the characteristic of 1 is raised. Therefore, to adjust the load between alternators in parallel, the speed-load characteristics of the prime movers must be changed. If the alternators are driven by shunt motors, the speed-load characteristics of the motors may be changed by adjusting the motor-field rheostats. It will be noted, in Fig. 163, that the loads of the two machines are equal at one frequency only.

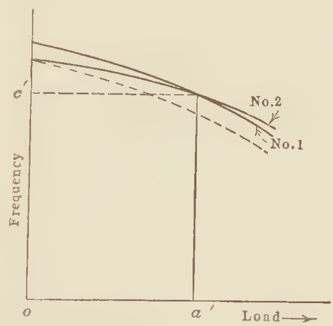


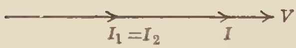
Fig. 163.—Speed-load curves of alternators in parallel—effect of changing governor control.

If the prime movers had flat speed-load characteristics, the operation of the alternators would be unstable. That is, very small disturbances or changes of frequency would cause very large fluctuations in the kilowatt load delivered by each machine. This condition would result in serious operating difficulties.

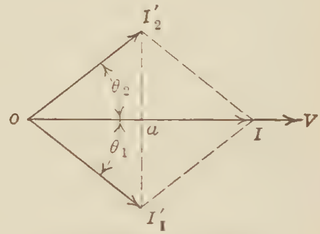
Alternators operating in parallel are in *stable equilibrium*. That is, any circumstance tending to throw the alternators out of parallel is opposed by reactions which tend to prevent the alternators pulling out. For example, assume that alternator 2 (Fig. 162) suddenly loses its driving torque, as might happen if the automatic trip on its turbine governor operated and shut off the steam supply. Alternator 2 automatically would take the entire load, the electrical reactions of the system would be such that motor action would develop in 1, and 2 would drive 1 as a

synchronous motor. (In practice large alternators are protected by reversed-energy relays which trip the circuit breakers when the alternator takes energy.)

107. Effect of Field Excitation.—It has been shown that changing the field excitation of alternators in parallel does not change the division of power load. It does, however, affect the kilovolt-ampere load. Assume in Fig. 162 (*a*) that alternators 1 and 2 deliver equal power and equal currents I_1 and I_2 to a load which has a power-factor of unity. Both 1 and 2 are so adjusted that their currents are in phase with the common terminal voltage V (Fig. 164 (*a*)) at which the system operates. The load requires a current I in phase with V. I is obviously the sum of I_1 and I_2.



(*a*) Equal loads and equal excitations



(*b*) Equal loads but different excitations

FIG. 164.—Effect of changing field excitation of alternators in parallel.

Now let the field of 1 be strengthened and that of 2 simultaneously weakened, so that the system voltage does not change. If instruments connected to these two alternators are read, the wattmeter readings will not change appreciably. The ammeters, however, will show a distinct increase of current in each alternator. The power-factor indicator in 1 will show that the power-factor of 1 has decreased from unity and that the current now lags. The power-factor indicator in 2 will show that the power-factor of 2 has also decreased from unity but that the current now leads.

These conditions are shown in Fig. 164 (*b*). The current I_1' lags V by $\theta_1°$ and I_2' leads V by $\theta_2°$. It has been assumed that the load current has not changed. The load current delivered by a central station is determined *entirely by the consumers' load*, and not by any conditions of operation in the station itself. Hence, the vector sum of I_1' and I_2' must still be equal to I. Therefore,

if I_1' lags I, I_2' must lead I by such an angle that the vector sum of I_1' and I_2' is still equal to the load current I. Under the foregoing conditions both I_1' and I_2' will terminate on a perpendicular to V at a (Fig. 164 (b)). This must be true since the *power* delivered by the two machines has not changed appreciably with the change in excitation. Hence, $VI_1' \cos \theta_1 = VI_1$, and $I_1' \cos \theta_1 = I_1 = oa$. Likewise, $VI_2' \cos \theta_2 = VI_2 = oa$, and $I_2' \cos \theta_2 = I_2 = oa$. In order that these conditions may hold, the line $I_1'aI_2'$ must be perpendicular to V at a. (The change of I^2R loss in the armature with change of excitation has been neglected since it is small.) Since the current of each alternator has now increased, but the power has not changed, the alternator losses have increased. Therefore, it is desirable to operate with the currents in phase so as to reduce heating losses. With loads other than unity power-factor the same general conditions exist. If the excitation of one generator is increased, its current lags further behind the resultant current and the current of the other generator leads the resultant current by a larger angle, the currents so adjusting themselves that their vector sum equals the load current.

The fact that conditions in the alternator adjust themselves to the changes of field excitation is obvious from the following: Through armature reaction a lagging current in an alternator *weakens* the field (page 165). The alternator with the lagging current operates with the higher induced e.m.f. (Fig. 154), resulting from the increased excitation.

Therefore, if the excitation be increased for an alternator in parallel with other machines, the machine delivers a *lagging* current which, through armature reaction, tends to prevent an increase in field strength; the vector giving the impedance-drop in the armature increases in magnitude and rotates in a clockwise direction so that it is more nearly in opposition to the terminal voltage. Hence, a given increase in the induced e.m.f. does not result in the same proportionate increase in the terminal voltage. Thus the reactions resulting from an attempt to increase the terminal voltage oppose this increase.

Likewise, the alternator with the weaker field delivers a *leading* current, which strengthens its field; at the same time the vector giving the impedance-drop in the armature increases in magni-

tude and rotates in a counter-clockwise direction, tending to increase the magnitude of the terminal voltage.

Thus a change of field excitation in alternators operating in parallel is accompanied by reactions in the system which oppose the change and cause all the alternators to have equal terminal voltages, thus giving stability to the system.

From the foregoing it is obvious that a change of excitation in alternators operating in parallel merely changes the power-factor for each alternator, and hence the kilovolt-ampere load of each machine. The kilowatt loads are not changed appreciably.

Figure 165 shows the connections for a single alternator ready to be connected across the bus-bars and thus put in parallel with the other alternators of the system.

108. Synchronizing.—Before direct-current generators can be safely put in parallel, two conditions must be fulfilled. The two terminal voltages must be equal, or substantially so, and the proper polarity must be observed.

These same two conditions must be fulfilled when alternators are connected in parallel. The equality of voltages can be readily determined by connecting a voltmeter first to one machine and then to the other. The voltmeter, when so connected, does not give any indication as to polarity, as the indications of an alternating-current voltmeter are independent of its polarity.

Lamps, however, can be used to determine the correct polarity. Figure 165 shows the connections for phasing a 3-phase alternator with the bus-bars. A lamp is connected across each pole of the 3-pole switch which connects the machine to the line. The voltage rating of the lamps should be 15 per cent. greater than that of the machine or line. For example, if the system is 220 volts, two 115-volt lamps in series may be used across each pole, although these lamps will be subjected to overvoltage during a part of the synchronizing period. If the machines are properly connected, the three lamps should all become bright and dim together. If they brighten and grow dim in sequence, it means that the phase rotation of the two machines is opposite, so that one phase must be reversed.

The lamps flicker at a frequency equal to the *difference* in the frequencies of the two machines. As the machines approach synchronism the flicker becomes slower and slower. When the

lamps are all dark the switch may be closed. The fact that the lamps are all dark indicates that the potential difference between each switch blade and its clip is nearly zero and that the alternator is in phase *opposition* to the bus-bars. Two points across which the potential difference is zero may be connected without any resulting disturbance, so that the switch may now be safely closed, thus putting the alternator in parallel with the bus-bars and the other alternators of the system.

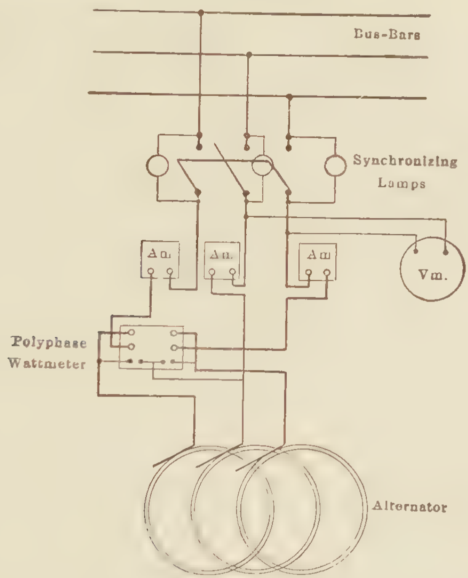


Fig. 165.—Connections for "3 dark" method of synchronizing with lamps.

The disadvantage of this method is that lamps are dark even though a very considerable voltage may exist across their terminals, and the machines may be connected in parallel therefore when considerable voltage difference exists between them. This may do no harm with slow-speed or small-capacity units, but with high-speed turbine units, which have little armature reactance and are quite "sensitive," there may be considerable disturbance if there exists a substantial phase difference at the time of connecting in parallel. Another objection to this "three-dark" method is that the lamps do not show whether the incoming machine is fast or slow.

The foregoing difficulties may be in part eliminated if the connections of two of the lamps, as 1 and 2 (Fig. 166) be crossed. When the machines are in synchronism, 1 and 2 are bright and 3 is dark. As one of the bright lamps is increasing and one is decreasing in brilliancy near the point of synchronism, it is possible to determine very accurately the instant at which the switch should be closed. This is called the Siemens-Halske or "two-bright-and-one-dark" method. By noting the sequence of brightness of the lamps, it can be determined whether the incoming machine is fast or slow. Before crossing the connections it is desirable to determine the correct phase rotation by the "three-dark" method (Fig. 165).

The best method is the use of the synchronism indicator or synchroscope described in Chap. III (page 97). Such an instrument shows accurately the position of synchronism. The synchroscope is connected across but one phase. It is possible that one phase of each machine may be in synchronism, but the other two out of phase due to wrong phase rotation. The correct phase rotation must be determined by lamps or by other means before depending entirely on the synchroscope. Synchronizing lamps are often used in conjunction with a synchroscope so that the operator has a check on the instrument.

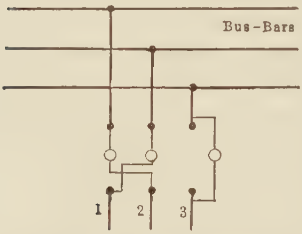


FIG. 166.—Connections for "2 bright and 1 dark" method of synchronizing with lamps.

109. Hunting of Alternators.—The driving torque of a reciprocating steam engine, or gas engine, is not uniform during a revolution of the flywheel, but varies from zero at the dead centers to a maximum at some intermediate position. Even with a heavy flywheel, this variation of torque may impart impulses to the induced e.m.f., causing it to be ahead of its proper position at some instants and behind it at other instants. This causes heavy currents to flow between machines in parallel and often causes their rotating members to "oscillate" as they are rotating. The angular effect of the crank position can be appreciated when it is realized that in a 60-pole alternator a displacement of one mechanical or space-degree in the rotating member makes a

difference of 30 electrical degrees in the phase angle of the e.m.f. The above impulses are often communicated to the system, causing synchronous motors and converters to oscillate. These oscillations are called "hunting." Hunting may become serious if the engine governors have a natural frequency of oscillation which is nearly the same as that of the machine rotors. The oscillations may then become cumulative and may even cause the machines to go out of synchronism.

Remedies for hunting are to use heavy flywheels, to put dashpots on the engine governors, and to use amortisseur or squirrel-cage windings around the field, such as is shown in Fig. 251 (page 287). Where several engine-driven units are used, they are often paralleled when their cranks occupy different angular positions. This minimizes the effect of the engine impulses on the system, although their effect is increased in the local interchange currents between generators. Because of the uniform driving torques of turbines, turbo-alternators seldom "hunt."

CHAPTER VII

THE TRANSFORMER

The static transformer is a device for transferring electrical energy from one alternating-current circuit to another without a change in frequency. This transference is usually, but not always, accompanied by a change of voltage. A transformer may receive energy at one voltage and deliver it at a *higher* voltage, in which case it is called a *step-up* transformer. A transformer may receive energy at one voltage and deliver it at a *lower* voltage, in which case it is called a *step-down* transformer. A transformer may receive energy at one voltage and deliver it at the *same* voltage, in which case it is called a *one-to-one* transformer.

This energy is transferred through the medium of an alternating magnetic flux. Hence, a static transformer has no rotating parts and, therefore, it requires little attention and its maintenance is low. The cost per kilowatt for transformers is low as compared with other apparatus and the efficiency is much higher. As there are no teeth, slots, or rotating parts, and the windings can be immersed in oil, it is not difficult to insulate transformers for very high voltages.

Because of these many desirable characteristics, the transformer is a very useful piece of apparatus, and as it can transform economically from low to high voltage and from high to low voltage, it is largely responsible for the extensive use of alternating current. For example, transformers may step up the voltage to very high values for transmission purposes, and thus effect a large saving in copper. Other transformers may step down the voltage from its transmission value to values which are safe to use for industrial purposes, such as for running motors, supplying lamps, etc. (see Fig. 267, page 309). Hence, the transformer permits a high degree of flexibility in a power system.

110. The Transformer Principle.—The transformer is based on the principle that energy may be efficiently transferred by

induction from one set of coils to another set by means of a varying magnetic flux, provided both sets of coils are on a common magnetic circuit.

Electromotive forces are induced by a change in flux linkages. In the generator, the flux is substantially constant in magnitude. The amount of flux linking the armature coils is changed by the relative *mechanical* motion of flux and coils (see Fig. 117, page 132). In the transformer, the coils and magnetic circuit are all stationary with respect to one another. The e.m.fs. are induced by the change in the *magnitude* of the flux with time. This is illustrated in Fig. 167 which shows diagrammatically the operation of a transformer.

A core is made up of rectangular stampings of sheet steel, clamped or bolted together.

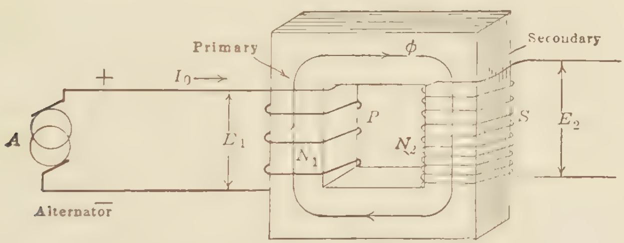


FIG. 167.—Simple transformer, secondary open-circuited.

A continuous winding P is placed on one side or leg of the iron core. Another continuous winding S, which may or may not have the same number of turns as P, is placed on the opposite side or leg. An alternator A supplies current, which varies nearly sinusoidally with time, to the primary winding P. Since the primary winding links the laminated iron core, its m.m.f. must produce in the core a flux which varies nearly sinusoidally with time (Fig. 168). This alternating flux links the turns of the winding S. As this flux is alternating, it induces in the winding S an e.m.f. of the same frequency as that of the flux. Because of this induced e.m.f., the secondary winding S is capable of *delivering* current and energy. Therefore, the energy is transferred from P, the primary, to S, the secondary, by means of the magnetic flux.

The winding P which *receives* the energy is called the *primary*. The winding S which *delivers* the energy is called the *secondary*. In a transformer, either winding may be the primary, the other being the secondary, depending upon which winding receives and which delivers energy.

111. Induced Electromotive Force.—The induced e.m.f. in a transformer is proportional to three factors: the *flux*, the *frequency*, and the *number of turns*. The complete equation for the induced e.m.f., assuming a sine wave, is

$$E = 4.44 f N \phi_{\text{max.}} 10^{-8} \text{ volts.} \tag{54}$$

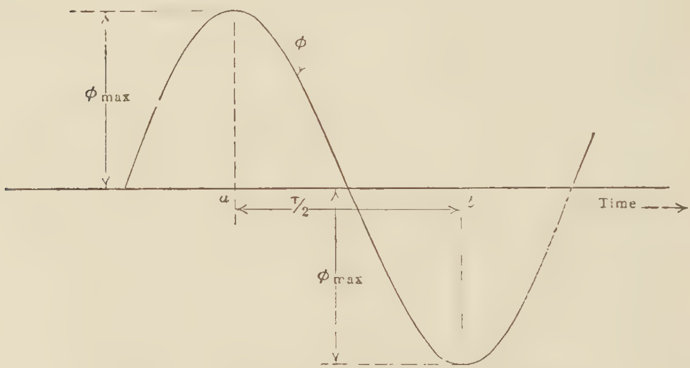


Fig. 168.—Sinusoidal variation of flux with time.

where f is the frequency in cycles per second, N is the number of turns and ϕ_{max} is the maximum value of the flux (see Fig. 168).

(Compare this equation with (49), page 156, which gives the induced e.m.f. in an alternator armature.)

Example.—The secondary of a 10-kv-a., 60-cycle transformer has 120 turns and the flux in the core has a maximum value of 720,000 maxwells. What is the voltage induced in this secondary?

Using equation (54)

$$\begin{aligned} E &= 4.44 \times 60 \times 120 \times 720{,}000 \times 10^{-8} \\ &= 4.44 \times 6 \times 1.2 \times 7.2 = 230 \text{ volts. } \quad \textit{Ans.} \end{aligned}$$

Since the flux ϕ (Fig. 167) is the same for each of the two windings it must induce the *same e.m.f. per turn* in each wind-

ing. The *total induced e.m.f.* in each winding must then be proportional to the number of turns in that winding. That is,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \tag{55}$$

where E_1 and E_2 are the primary and secondary *induced* e.m.fs. and N_1 and N_2 are the number of turns in primary and secondary, respectively.

In the primary, however, the induced e.m.f. is a counter e.m.f. tending to prevent current entering the primary. In the ordinary transformer, the terminal voltage differs from the induced e.m.f. only by a very small percentage, so that for most practical purposes it may be said that the primary and secondary terminal voltages are proportional to the number of turns in primary and secondary.

Example.—In the foregoing example, the primary has 1,200 turns. What is its counter e.m.f.?

$$N_1 = 1{,}200,\ N_2 = 120,\ E_2 = 230.$$

$$\frac{E_1}{230} = \frac{1{,}200}{120} = \frac{10}{1} \therefore E_1 = 2{,}300 \text{ volts.} \quad \textit{Ans.}$$

In the ordinary transformer the impressed terminal voltage would differ from the counter e.m.f. by only 1 per cent. or so.

112. Exciting Ampere-turns and Flux.—Figure 167 shows a current I_0 flowing into the primary winding from the upper wire. The directions of the flux, of the voltages and of the currents, as indicated on the figure, are those existing at the instant when the upper primary line is positive. There is no load on the secondary. Under these conditions the current I_0 flowing into the primary is usually from 3 to 8 per cent. of the rated current. This no-load current performs two functions. Flowing through the primary turns, it produces the flux in the core. It is, therefore, called the *exciting* or *magnetizing* current. Because of the alternating flux there are hysteresis and eddy-current losses in the core. Hence, the no-load current must also supply this core loss. Since this loss is small, the primary on open circuit is in reality an impedance in which the inductive reactance predominates. Hence, the no-load current lags the terminal voltage by a large angle and the no-load power-factor is of the order of 5 to 10 per cent.

The exciting current produces a flux ϕ whose direction at each instant is determined by the corkscrew rule. At the instant

shown in Fig. 169, when the upper primary line is positive the flux must have a clockwise direction through the core. With the frequency and turns fixed, the induced e.m.f. in a transformer is *proportional to the flux* (equation (54)). Since the counter e.m.f. differs in magnitude but slightly from the terminal voltage, this flux *must adjust itself to such a value as to make the counter e.m.f. substantially equal to the terminal voltage.* Even at full load and above, the induced or counter e.m.f. differs from the terminal voltage by only 1 or 2 per cent. Hence the flux can change by this amount only. For all practical purposes, it may be said that the flux in a constant-potential transformer is constant under all normal operating conditions.

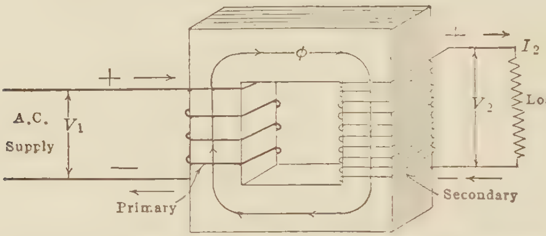


FIG. 169.—Simple transformer, load applied to secondary.

113. Primary and Secondary Ampere-turns with Load.—Figure 169 shows the transformer of Fig. 167 with a load I_2 applied to the secondary. At the instant shown the direction of the current I_2 must be such as to *oppose* the flux ϕ.

This is in accordance with Lenz's law that an induced current always has such a direction as to oppose the cause which produces it (see Part I, page 168). If the secondary current I_2 were producing the flux ϕ, then by the corkscrew rule the current would flow *in* at the upper terminal (Fig. 169). Since I_2 opposes the flux ϕ, it must actually flow *out* at the upper terminal. The secondary current I_2 then tends to *reduce* the value of the flux in the transformer core. If the flux is reduced, the back e.m.f. of the primary is also reduced. As in the direct-current motor (see Part I, page 271), a reduction in the counter e.m.f. results in less opposition to the flow of current into the primary. Hence, the primary current increases in value and supplies the energy which is being delivered to the load by the secondary. The fore-

going is the sequence of reactions which follow the application of load to the secondary and which enable the primary to take from the line the energy to supply the increased power demanded by the secondary.

It has already been stated that the mutual flux ϕ remains substantially constant over the working range of the transformer. If the flux does not change the *net* ampere-turns acting on the core cannot change. The effect of the secondary turns is *to reduce* the flux. The effect of the primary ampere-turns is to *increase* the flux. If the *net* ampere-turns are to remain constant, the increase in primary ampere-turns over the no-load ampere-turns must be equal to the secondary ampere-turns. Therefore, any primary ampere-turns in excess of the exciting ampere-turns must be balanced by equal and opposing secondary ampere-turns.

The exciting current is usually of small magnitude and differs considerably in phase from the total primary current. In most constant-potential transformers, this no-load current may be neglected. If the no-load current be neglected in comparison with the total primary current, the *primary* and *secondary ampere-turns* are *equal*, and

$$N_1I_1 = N_2I_2.$$

Therefore,

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}. \tag{56}$$

That is, *the primary and secondary currents are inversely as the primary and secondary turns.*

The above relation also follows from the law of the conservation of energy. If the transformer losses be neglected and unity power-factor be assumed,

$$V_1I_1 = V_2I_2$$

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1}.$$

Example.—The secondary of the transformer in the problem, page 190, delivers 89 amp. at 230 volts and at unity power-factor. (*a*) How much power does the secondary deliver? (*b*) What is the primary current?

(*a*) $P = 230 \times 89 = 20{,}470$ watts $= 20.47$ kw. *Ans.*

(*b*) The secondary ampere-turns (effective)

$$N_2I_2 = 120 \times 89 = 10{,}680$$

$I_1 = \dfrac{10{,}680}{1{,}200} = 8.90$ amp., since there are 1,200 primary turns. *Ans.*

Also

$$\frac{I_1}{I_2} = \frac{I_1}{89} = \frac{120}{1{,}200}$$

$$\therefore \quad I_1 = 89\,\frac{120}{1{,}200} = 8.9 \text{ amp.} \quad \textit{Ans.}$$

114. Leakage Reactance.—In the preceding discussion it has been assumed that *all* the flux which links the primary also links the secondary. In practice it is impossible to realize this condition. All the flux produced by the primary does not link the secondary, but a part completes its magnetic circuit by passing through the air rather than around through the core, as shown by ϕ_1 (Fig. 170). This flux ϕ_1 is called the *primary leakage flux.*

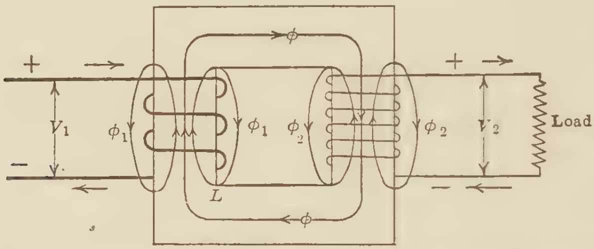


Fig. 170.—Mutual flux, primary leakage flux, and secondary leakage flux in a transformer.

The mutual flux ϕ is produced by the joint ampere-turns of primary and secondary. The primary leakage flux ϕ_1 is produced by the ampere-turns of the primary alone and is, therefore, proportional to the primary current. Since ϕ_1 does not link the secondary it does not induce any e.m.f. in the secondary, so it is not useful. ϕ_1 does induce an e.m.f. in the primary. This e.m.f. is a counter e.m.f. tending to prevent the flow of current. It is proportional to the current and frequency and lags the current by 90°. It is, therefore, a reactance e.m.f. and causes a reactance-drop I_1X_1 in the primary. X_1 is called the *primary leakage reactance.* It is obvious that a portion of the primary impressed voltage is utilized in supplying this reactance-drop. This in turn reduces the counter e.m.f., hence the flux and thus the secondary induced e.m.f.

It was stated in Par. 112 that the secondary ampere-turns acted upward (Fig. 170), opposing the mutual flux ϕ. Further application of the corkscrew rule shows that these ampere-turns act to send flux downward outside the secondary. Some flux actually does complete its circuit in the local path about the secondary, as shown by ϕ_2 (Fig. 170). This flux ϕ_2 links the secondary but not the primary and is, therefore, proportional to the secondary ampere-turns. Hence, it is called the *secondary leakage flux.*

The mutual flux ϕ induces an e.m.f. which tends to send current out of the secondary at its upper terminal. The flux ϕ_2 acts in opposition to ϕ and, hence, tends to prevent current flowing out of the secondary at the upper terminal. Hence, the flux ϕ_2 tends to reduce the voltage across the secondary. Since the effect of ϕ_2 is proportional to the secondary current and the frequency, it is also considered a reactance voltage I_2X_2. X_2 is called the *secondary leakage reactance.*

The effect of both primary and secondary leakage reactance is to reduce the secondary terminal voltage. If it is desired that the transformer regulate closely, ϕ_1 and ϕ_2 must be reduced to as low a value as is possible. Transformers constructed with primary and secondary on separate legs as shown in Figs. 167 and 169, which are diagrammatic only, would have too great a leakage reactance for practicable purposes. To reduce the magnitude of the leakage fluxes and hence of the leakage reactances, the primary and secondary are split up into sections and closely interwoven (see Figs. 176 and 177). Moreover, the paths of the leakage flux are not so simple as is indicated in Fig. 170. Some of the flux links a portion of the primary turns, some a portion of the secondary, etc.

In transformers of very large rating, a *high* value of leakage reactance is desirable since it limits the short-circuit current. Large transformers, having low leakage reactance, have been wrecked on short-circuit by the enormous stresses between turns caused by the large value of the short-circuit current.

115. Transformer Testing.—Theoretically, it is possible to determine the efficiency of a transformer by measuring simultaneously the input and the output with electrical instruments. The efficiency of the smallest power transformer is from 95 to 97

per cent. and the efficiency of large units is from 98 to 99 per cent. In the larger units there is but 2 per cent loss. Unless carefully calibrated, ordinary wattmeters may be in error by 1 per cent. which would cause a 50 per cent. error in the losses. Errors greater than 1 per cent. may be introduced by the use of instrument transformers (see page 222). In fact it is possible to have the input and the output wattmeters reading so nearly alike that an almost imperceptible difference exists between their readings. Likewise the voltage regulation of a transformer is from 1 to 3.5 per cent., depending on the power-factor (see Fig. 174). Again, poor precision is obtained if this regulation is determined by loading the transformer and measuring the small difference between the no-load and rated-load voltages.

It is far more accurate, in determining the efficiency, to measure the losses directly and then from the losses to determine the efficiency. The voltage regulation may also be obtained by measuring the primary and secondary resistances and leakage reactances and then calculating the regulation.[1]

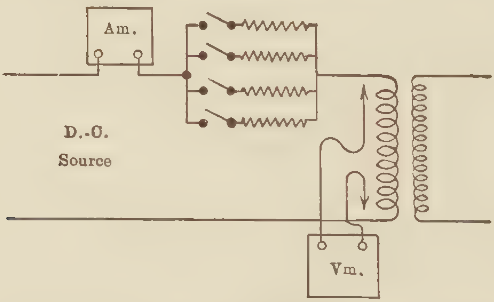


Fig. 171.—Connections for measuring direct-current resistance of transformer windings.

116. Transformer Copper Losses.—Three separate losses occur in transformers: the primary copper loss, the secondary copper loss, and the core loss in the iron.

The copper losses may be calculated if the primary and secondary resistances are known. Since the windings are not embedded in slots, their resistances with alternating current are but slightly greater than with direct current. Hence, it is sufficiently

[1] For further description see Dawes, "A Course in Electrical Engineering," Vol. II.

accurate in most cases to measure the primary and secondary resistances with direct current and to use these values of resistance in calculating the copper losses. The resistance may be measured by the usual drop in potential method (Fig. 171). It is desirable to use small values of current, since large values cause the core to be magnetized so strongly in one direction as to leave it permanently magnetized. Since the windings have very high values of self-inductance, the *voltmeter should be disconnected before opening the circuit.* Otherwise the high e.m.f. of self-induction is almost certain to injure the voltmeter.

If R_1 is the primary resistance and R_2 the secondary resistance, the total copper loss is

$$P_C = I_1^2R_1 + I_2^2R_2.$$

117. Transformer Core Losses.—Since the magnetic flux in the core is reversed several times a second, there must be hystere-

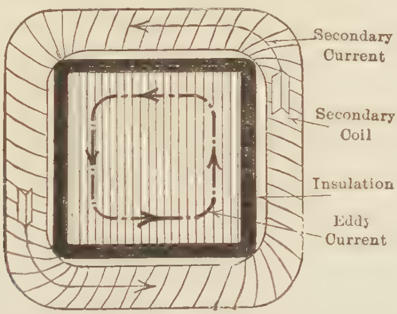


FIG. 172.—Induced current in secondary and in core.

sis loss (see Part I, page 163). Because of its low hysteresis loss silicon steel is used almost universally for transformer cores. Laminating does *not* reduce hysteresis loss.

The alternating flux induces e.m.fs. in the iron core as well as in the primary and secondary windings. If the iron core were of solid metal, the accompanying eddy currents would cause prohibitive losses. The dotted line (Fig. 172) shows the paths that the eddy currents tend to take. By laminating the core, the paths of these eddy currents are broken up by the oxide and varnish on the laminations.

To measure these core losses (hysteresis and eddy current), connections are made as shown in Fig. 173. Rated voltage and

frequency are impressed across one of the windings, preferably the winding whose voltage rating most nearly corresponds to a convenient instrument voltage, such as 110 or 220 volts. For example, if the transformer (Fig. 173) were rated at 2,200/220 volts, the measurement would be made on the 220-volt or low side. Since the no-load losses are small, instrument losses should be investigated. For example, in Fig. 173 the wattmeter measures the power taken by the voltmeter, hence the voltmeter should either be open-circuited when reading the wattmeter or correction made for the power that it consumes.

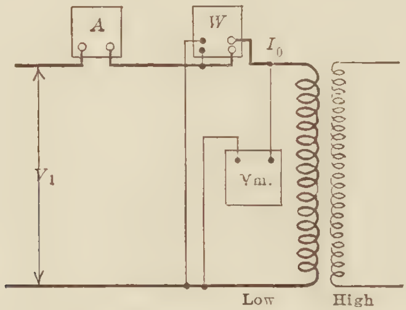


FIG. 173.—Connections for core loss measurement.

Since rated voltage is impressed across the winding, rated flux exists in the core, and therefore the core loss is at its normal value. Since the flux remains substantially constant at all loads, the core loss also remains substantially constant at all loads. Let the measured core loss be P_0.

118. Transformer Efficiency.—Since the losses in the transformer may be determined experimentally, the efficiency can be readily calculated. Thus, the efficiency

$$\eta = \frac{\text{output}}{\text{output} + \text{losses}}$$

$$= \frac{V_2 I_2 \cos \theta_2}{V_2 I_2 \cos \theta_2 + I_1^2 R_1 + I_2^2 R_2 + P_0} \qquad (57)$$

V_2 = secondary terminal voltage; I_2 = secondary current; $\cos \theta_2$ = secondary power-factor.

Example.—A 100 kv-a., 60-cycle, 2,200/122-volt transformer is tested for its core loss by impressing 122 volts, 60 cycles, across its low side with its

high side open (see Fig. 173). The value of core loss is found to be 465 watts. (See Fig. 174.)

The primary or high-side resistance is measured and found to be 0.338 ohm and the secondary or low-side resistance is found to be 0.0011 ohm. (*a*) Find the efficiency of the transformer at rated load and unity power-factor. (*b*) Find the efficiency at half-load and unity power-factor.

(*a*) The rated primary current

$$I_1 = \frac{100,000}{2,200} = 45.5 \text{ amp.}$$

The rated secondary current

$$I_2 = \frac{100,000}{122} = 820 \text{ amp.}$$

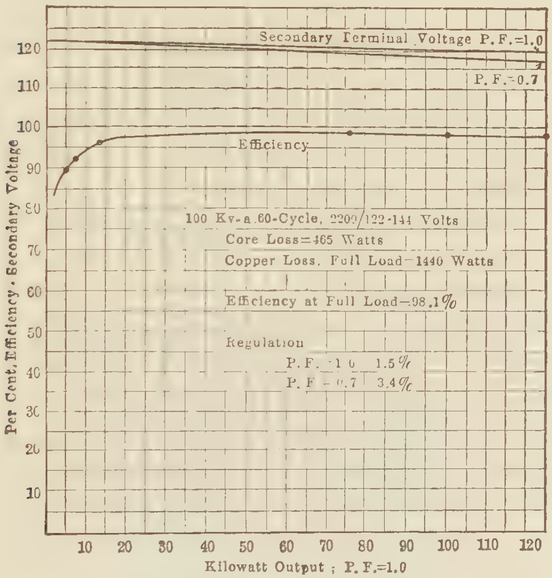


FIG. 174.—Characteristics of a 100 kv-a., 60-cycle transformer

Using equation (57), $\left(\eta = \dfrac{\text{output}}{\text{output} + \text{losses}}\right)$

$$\eta = \frac{122 \times 820}{122 \times 820 + (45.5)^2 0.338 + (820)^2 0.0011 + 465}$$

$$= \frac{100,000}{100,000 + 700 + 740 + 465}$$

$$= \frac{100,000}{101,905} = 0.981. \quad \textit{Ans.}$$

(*b*) The core loss remains unchanged. Since the load current is one-half the rated value, the copper losses must be *one-fourth* the values given in (*a*).

Hence,

$$\eta = \frac{122 \times 410}{122 \times 410 + 175 + 185 + 465}$$
$$= \frac{50{,}000}{50{,}825} = 0.984. \quad \textit{Ans.}$$

Example.—Determine the efficiency of the foregoing transformer when the power-factor is 0.7, lagging current: (*a*) at rated load; (*b*) at 25 per cent. rated load.

(*a*) The output of the transformer, like that of the alternator, is determined by heating, hence by the current and not by the kilowatt output. The rated current, therefore, remains unchanged by power-factor. The efficiency

$$\eta = \frac{122 \times 820 \times 0.7}{122 \times 820 \times 0.7 + (45.5)^2 0.338 + (820)^2 0.0011 + 465}$$
$$= \frac{70{,}000}{70{,}000 + 700 + 740 + 465} = \frac{70{,}000}{71{,}905} = 0.974. \quad \textit{Ans.}$$

It will be noted that the losses are the same as in (*a*) in the previous example.

(*b*) The primary current now becomes $45.5 \times 0.25 = 11.4$ amp. and the secondary current becomes $820 \times 0.25 = 205$ amp.

$$\eta = \frac{122 \times 205 \times 0.7}{122 \times 205 \times 0.7 + (11.4)^2 0.338 + (205)^2 0.0011 + 465}$$
$$= \frac{17{,}500}{17{,}500 + 44 + 46 + 465} = \frac{17{,}500}{18{,}055} = 0.969. \quad \textit{Ans.}$$

It will be observed that the output is one-fourth that in (*a*), the copper losses are one-sixteenth, and the core losses remain unchanged.

The efficiency of the transformer is high even at light load and low power-factor, as is shown by the foregoing example. Figure 174 gives the efficiency curve of this 100-kv-a. transformer at unity power-factor. It will be observed that the efficiency is high from 10 per cent. load to 25 per cent. overload and that the curve is flat from 15 per cent. load to 25 per cent overload.

TRANSFORMER CONSTRUCTION

119. Core- and Shell-type Transformers.—Transformers are divided into two general types, the core and the shell type. These two types differ in the arrangement of the iron and copper with respect to each other.

In the core type of transformer the winding or copper surrounds the iron core. Although Figs. 167, 169, and 170 are diagrammatic merely, they represent core-type transformers. Figure

175 (*a*) shows the general arrangement of the core-type transformer. The core is in the form of a hollow square made up of sheet-steel laminations about 14 mils thick. These laminations

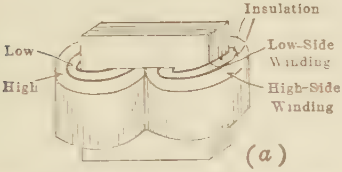


FIG. 175 (*a*).—Arrangement of coils and core in a core-type transformer.

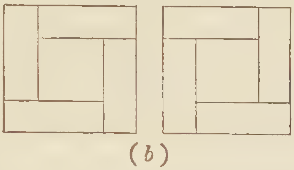


FIG. 175 (*b*).—Arrangement of joints in adjacent lamination layers.

are usually built up with rectangular strips, the joints of which butt, in the individual layers. The joints lap in alternate layers, however, as indicated by Fig. 175 (*b*), which shows the arrangement of joints in two adjacent layers. When a large

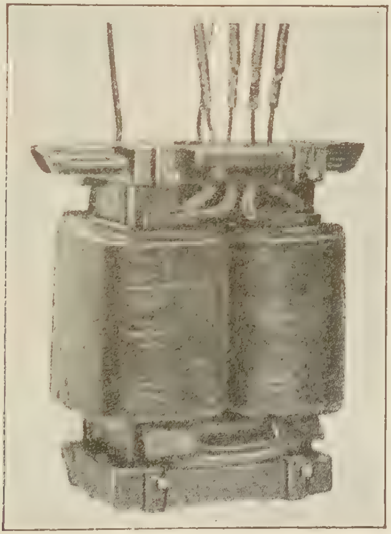
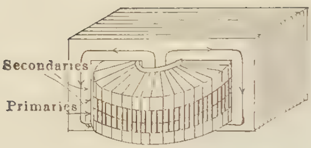


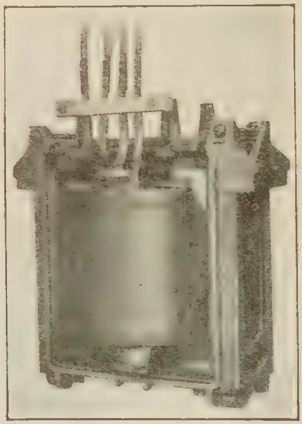
FIG. 176.—Coil and core assembly of Wagner core-type distribution transformer.

number of transformers of a single type are being manufactured, the laminations are often made of L-shaped stampings stacked so that the joints alternate. Figure 176 shows a core-type transformer assembled, with leads, etc., but without the case.

If a transformer were made with the primary and secondary coils on separate legs, as indicated in Figs. 167, 169, and 170, an unsatisfactory transformer would result, as the large leakage flux for both primary and secondary would result in very poor regulation. By having both a primary and a secondary on each leg, as shown in Figs. 175 (*a*) and 176, the leakage flux is reduced to a very small value. If the high-voltage winding were placed next the core, it would be necessary to insulate it, both from the core and from the low-voltage winding. Thus, two layers of high-voltage insulation would be necessary. By placing the high-voltage winding outside, and around the low-voltage wind-



(*a*).—Arrangement of coils and core in shell-type transformer.



(*b*).—Coil and core assembly of Wagner shell-type distribution transformer.

FIG. 177.—Shell-type transformer.

ing, only the one layer of high-voltage insulation, that between the high- and low-voltage windings, is necessary.

The core type of transformer is well adapted to high voltages, especially in the smaller ratings, because the insulation problem is not difficult.

In the shell type of transformer, the iron surrounds the copper, as shown in Fig. 177. The core has the form of a figure 8. The entire flux passes through the central part of the core, but outside this central core it divides, half going in each direction as shown in Fig. 177 (*a*). The coils are made in pancake form, usually

wound with strip copper. These coils are taped, and the primary and secondary are usually stacked so that each primary is adjacent to a secondary. In this manner the leakage flux of both primary and secondary is reduced to a very small value. In Fig. 177 (*a*) the primaries are the high side and the secondaries are the low side. The secondaries, or low-side coils, are placed adjacent to the iron in order to minimize the amount of high-voltage insulation required. Figure 177 (*b*) shows a shell-type transformer removed from its case.

120. Type H Transformer.—In designing a transformer it is desirable that the mean length of turn be as short as possible. This reduces both the weight of copper and the resistance and reactance of the winding. This is accomplished in the Type H transformer of the General Electric Company by making a shell-type transformer in which the core is cruciform in shape, as shown in Fig. 178. The central core around which the coils are wound is operated at much higher flux density than the four wings. Although the reluctance and losses in this core are high, they are not excessive when the entire magnetic circuit is considered. These transformers are used mostly as distribution transformers for stepping down from 2,200 and 1,100 volts to 220 and 110 volts, so that the primary is the high side. It will be observed that the low side, the secondary, is next the iron. That is, one of the two low-side coils is next the central core and the other is next the iron of the four wings. The two high-side coils lie between the two low-side coils, and are not adjacent to the iron. The advantage of this design is that only moderate insulation is required between the low-side coils and the core. As high-voltage insulation, such as the mica shields, need be used only between the high- and the low-voltage coils, a minimum amount of high-voltage insulation is required.

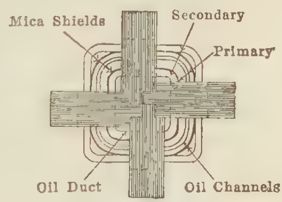


Fig. 178.—Core and windings of type H transformer.

In designing a transformer, provision should be made for keeping it cool. Spaces or ducts should be left between coils and between coils and core. Such ducts, or channels, are shown in Fig. 178. The oil in these ducts becomes heated, its specific

gravity decreases, and the oil rises. When it comes in contact with the transformer case it cools, which increases its specific gravity, and it therefore flows downward outside the transformer coils, and is subjected to further cooling. There is a continuous circulation of oil up through the coils and through the ducts in the core and this carries away the heat.

Figure 179 shows a Type H transformer assembled, but removed from its case.

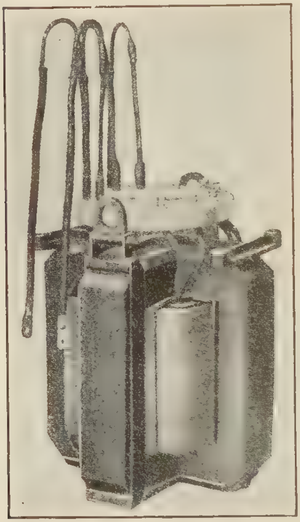


Fig. 179.—Type H transformer removed from tank.

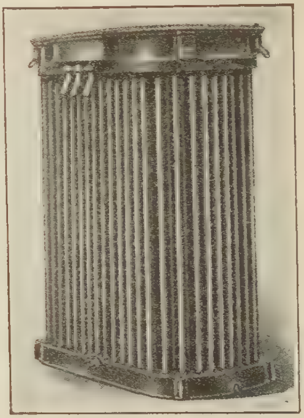


Fig. 180.—Wagner oil-filled, self-cooled, three-phase, distribution-type transformer.

121. Cooling of Transformers.—All the energy lost in a transformer must be dissipated as heat. Although this energy is but a small proportion of the total energy undergoing transformation, it becomes quite large in amount in the transformers of larger rating. The larger the transformer, the more difficult it becomes to dissipate the heat, for the kilowatt capacity of the transformer increases much faster than the radiating surface.

Transformers are divided into two classes, self-cooled types and artificially cooled types. The self-cooled types are usually immersed in oil. The oil within the windings and core becomes

heated and, because of the lesser density of the heated oil, it rises to the top of the case, where it becomes cooled. The cooled oil has a greater density than the warm oil and so flows downward, in close contact with the case, where it is still further cooled.

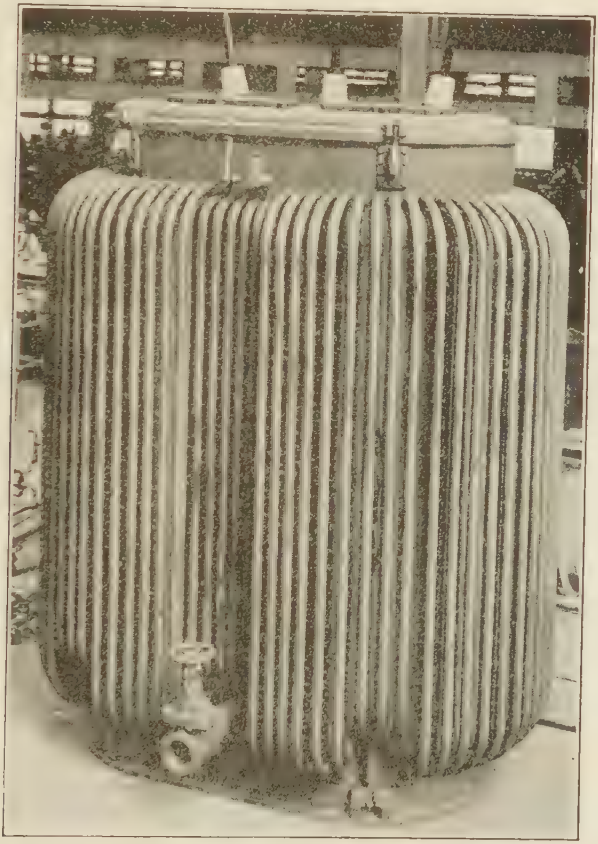


FIG. 181.—Westinghouse 1,000-kv-a., 3-phase, 60-cycle, 46,000–23,000-volt tubular-tank transformer.

After it reaches the bottom of the transformer it again rises, passing up through the windings and core. In addition to carrying heat away from the windings and core, the oil is an excellent insulator and dielectric.

In the transformer of moderate rating, the radiating area is increased by corrugating the case (Fig. 180). As transformers increase in rating, it becomes difficult to dissipate the heat by means of the surface of the case alone. One method of increasing the radiating area is to use exterior tubes running from the top of the case to the bottom (Fig. 181). The hot oil passes out

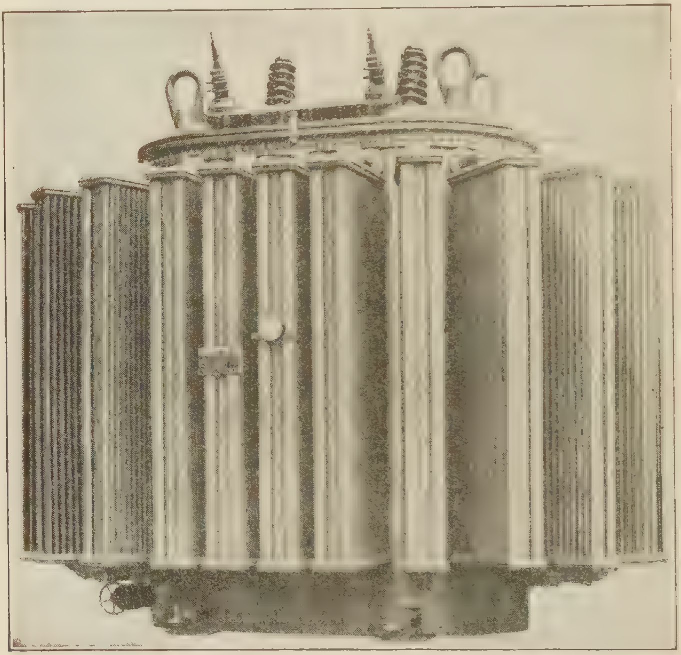


Fig. 182.—General Electric 8,000 kv-a., 25-cycle, 4,400–6,600/6,390/6,270-volt, outdoor transformer. (Radiator tank.)

from the tank into the top of the tubes, is gradually cooled and descends through the tubes to the bottom of the tank where it again passes up through the windings and core.

Transformers having this tubular construction are limited in size by the side and overhead clearances of the railroads. The tubular principle can be utilized, however, by bolting radiators to the casing to take the place of the tubes (Fig. 182). As these radiators are held by bolts, they may be removed during shipment and bolted in place when the transformer is installed.

Figure 183 shows the core and windings of the radiator type of transformer (Fig. 182). It will be observed that this is a shell type of transformer. When the leads are carried out through bushings in the transformer cover it is necessary to support the core and windings entirely from the cover. The cover with the core and windings can then be lifted as a unit.

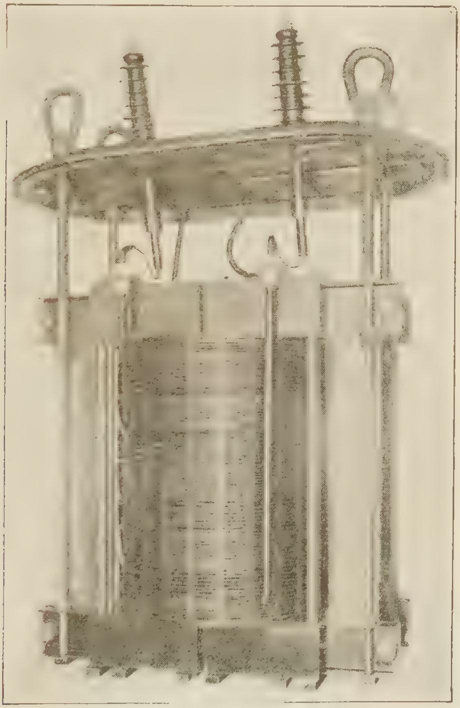


FIG. 183.—Core and windings of the radiator type of transformer shown in Fig. 182.

There are two different types of artificially cooled transformers, air- and oil-cooled transformers. The air-cooled or air-blast type is ordinarily mounted on a platform under which air pressure is maintained by means of blowers (Fig. 184). The air is forced up through the transformer windings and core, keeping them at the proper temperature. An advantage of air-cooled transformers is that fire risk is reduced because the danger of flooding the

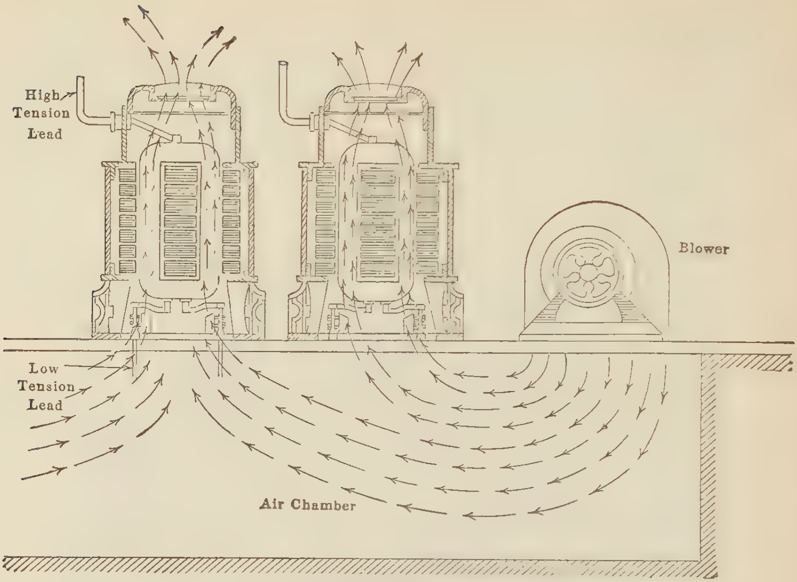


FIG. 184.—Cooling of air-blast transformers.

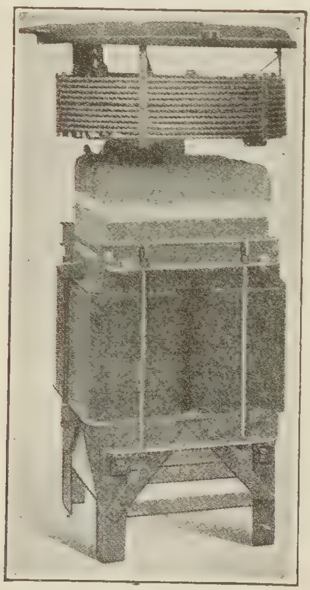


FIG. 185.—Water-cooled, shell-type transformer removed from its case.

station with burning oil is eliminated. They are therefore used extensively in sub-stations which are located in congested districts. On the other hand, this type of transformer does not have the added dielectric strength which the oil gives to the insulation. For this reason air-cooled transformers are seldom manufactured for potentials in excess of 30,000 volts.

The most common method of artificially cooling the oil-cooled type of transformer is to place a copper coil in the top of the transformer tank and circulate cooling water through the coil, the surface of which is in contact with the hot oil. Careful tests for leaks in the cooling coils should be made at regular intervals, as even a very small amount of water in the oil greatly impairs its insulating and dielectric properties.

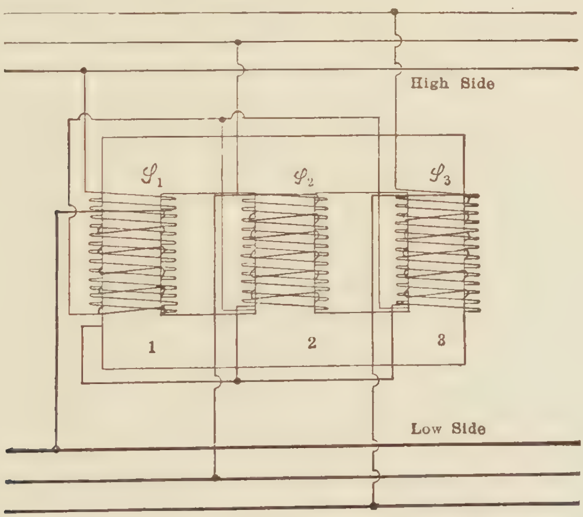


FIG. 186.—Practical arrangement of windings on a 3-phase, core-type transformer connected Y-Y.

122. Three-phase Transformers.—Three-phase transformers have considerably less weight and occupy much less floor space than three single-phase transformers of equal rating. Their use is therefore advantageous in many instances. Figure 186 shows the construction of the core type of 3-phase transformer. The primary and secondary of each phase are wound over one of the three legs of the transformer. Each of the three legs acts in

turn as return path for the fluxes from the other two. For example, the three fluxes, φ_1, φ_2, and φ_3 will obviously differ in phase by 120°, as shown in Fig. 187. That is φ_2 reaches its maximum instantaneous value acting upward, 120° after φ_1, etc. Figure 187 shows that at any instant the sum of the three fluxes is zero. When $t = 0$, $\varphi_1 = 0$, $\varphi_2 = -0.866\,\Phi_{max}$ and therefore acts downward, and $\varphi_3 = +0.866\Phi_{max.}$ and therefore acts upward. At this instant, the total flux acts upward through 3 and downward through 2, there being no flux through 1. When $\omega t = 30°$, φ_1 and φ_3 are each equal to $\frac{\Phi_{max.}}{2}$ and act upward. φ_2 is negative maximum and acts downward. At this instant, therefore, the two fluxes in 1 and 3 act upward and combine to flow downward through 2. Thus one or more of the legs of the three phases will always act as return path for the fluxes of the others.

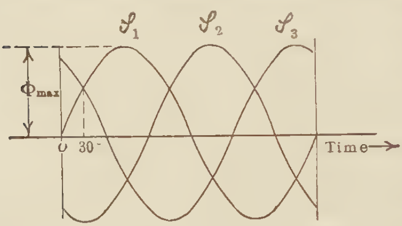


Fig. 187.—Time variation of fluxes in a 3-phase transformer.

As it has a single tank, the 3-phase transformer costs less and occupies less floor space than three single-phase transformers of equal rating. These advantages are often balanced by the fact that if any one phase becomes disabled, the whole transformer must ordinarily be removed from service. If one transformer of a 3-phase bank of single-phase transformers becomes disabled, the transformer may be replaced by a single spare which can be readily substituted. The tubular-tank transformer (Fig. 181) is a 3-phase transformer.

123. Auto-transformers or Compensators.—It is possible to transform alternating-current energy with a transformer which does not have a separate primary and secondary. Such a transformer, having a 2:1 voltage ratio, is shown diagrammatically in Fig. 188 (*a*). The winding *ac* may consist of a single winding on one leg of a transformer and having a tap at its midpoint *b*, or windings *ab* and *bc* may each consist of several coils on the same leg or on separate legs, the coils being interwoven to reduce magnetic leakage. These coils are connected in series in such a manner that they all aid or tend to send flux in the same direc-

tion around the core. Assume that a voltage of 100 volts is impressed across *ac*. Since *b* is at the midpoint of the winding, the voltages from *a* to *b* and from *b* to *c* must each be 50 volts. If a non-inductive load of 2.5 ohms is connected across *bc* at *dd'* the current in *bd* and *d'c* will be 20 amp. and the power 1,000 watts. The losses in this type of transformer are small and in this discussion may be neglected. Also the magnetizing or no-load current and the leakage reactances are small and usually may be neglected. If the losses are neglected, the input to the transformer is 1,000 watts and is delivered at 100 volts. The current entering the winding *ab*, therefore, must be 1,000/100 or 10 amp. This current is shown flowing downward at the instant in question. Since the current flowing from *b* to *d* through *dd'* is 20 amp., then by Kirchhoff's first law the current flowing from *c* to *b* through the winding *cb* must be 10 amp. flowing *upward*. This current combines with the 10 amp. flowing downward in *ab* and gives the 20 amp. required by the load.

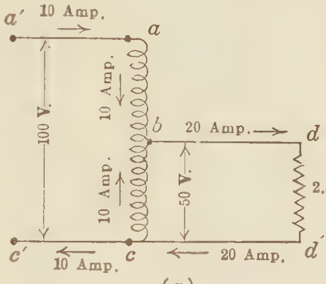


FIG. 188.—Auto-transformer giving 2 to 1 voltage ratio.

This 10 amp. flowing upward from *c* to *b* is a transformed or secondary current. The 10 amp. flowing from *a* to *b* drops 50 volts in potential, representing 500 watts. By the law of the conservation of energy this power must either be dissipated or appear elsewhere. Actually it is transferred to the flux in the core. This flux by transformer action, raises the potential of the current flowing from *c* to *b* by 50 volts, since losses are neglected. Therefore, 500 watts only of the 1,000 watts delivered to the load *dd'* are *transformed*. The remaining 500 watts flow *conductively* to *dd'*, in virtue of the current of 10 amp. flowing through *ab*, *bd*, and thence to *dd'*. So far as power is concerned *ab* may be considered as the primary and *bc* the secondary of the transformer. It is obvious that the transformer may be reversed; that is, power may be supplied to *bc* at 50 volts and delivered to the line *a'c'* at 100 volts.

Figure 189 shows an auto-transformer which transforms from 100 to 75 volts. The load *dd′* takes 20 amp. at 75 volts, or 1,500 watts. Neglecting losses, the input to the system must therefore be 1,500 watts. At 100 volts the current input must be 15 amp. as shown. This current of 15 amp. drops 25 volts in potential in flowing from *a* to *b*, hence 25 × 15 or 375 watts are transferred to the magnetic field. This power is utilized in raising the potential of 5 amp., by 75 volts (375 watts) in winding *bc*. Hence of the 1,500 watts, 375 watts are transformed and 1,125 watts flow conductively to the load through *ab*.

In the foregoing examples two facts should be noted. All the power is not *transformed*, but a certain proportion flows *conductively* to the load. Hence a smaller transformer can be used than would be necessary were all the power transformed as in the regular transformer. The smaller the ratio of transformation, the smaller the proportion of power *transformed*. Thus, with a 4:3 ratio only one-fourth the total power is transformed whereas with a 2:1 ratio, one-half the total power is transformed. Hence, the auto-transformer becomes economical when the ratio of transformation is small. It is not economical with large ratios of transformation. Moreover, since the primary and secondary are connected conductively, there is always danger of high potentials to ground on the low-side unless special precautions are taken.

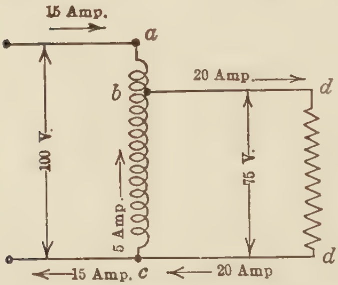


FIG. 189.—Auto-transformer giving a 4 to 3 voltage transformation.

124. Industrial Uses of Auto-transformers.—It is sometimes necessary to transmit power at voltages slightly greater than it is feasible to generate. For example, it might be desirable to transmit power at 26,400 volts, particularly if it is necessary to use underground cables through cities. It is difficult to insulate generators for voltages exceeding 15,000 volts. By using auto-transformers, however, it is possible to generate at 13,200 volts and transform economically to 26,400 volts by means of auto-transformers. Figure 190 shows the connections such as would

be employed if a 3-phase, Y-connected auto-transformer were used for this purpose. The transformer and generator under these conditions operate as a unit.

By means of an auto-transformer it is possible to obtain very readily and very economically a 3-wire system from a 2-wire

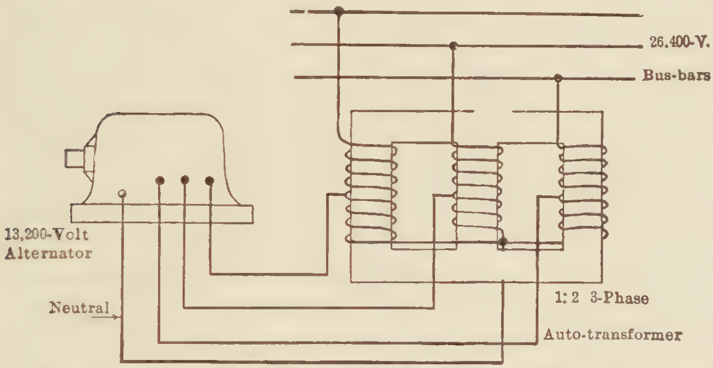


FIG. 190.—Alternator and 3-phase auto-transformer operating as a unit.

supply. Figure 191 shows a 220-volt alternating-current supply from which it is desired to obtain 110-volt service for lighting. An auto-transformer is connected directly across the 220-volt wires and a wire is carried from its midpoint (Fig. 191). This wire is called the *neutral*. Obviously the voltage from neutral to either wire is 110 volts and 110-volt lamps can be connected between either outer wire and the neutral. This gives a 220-110 volt, 3-wire system (not 3-phase). (For further discussion of the 3-wire system, see page 325.) An auto-transformer used in this manner is called a *balance coil*.

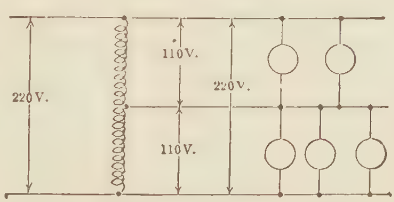


FIG. 191.—Compensator used to obtain a 3-wire lighting system.

Auto-transformers are also used extensively as alternating current motor starters (see page 244).

125. Phasing Transformer Windings.—The primaries and secondaries of transformers usually consist of two or more coils each, rather than a single coil. By connecting these coils either

in series or in parallel various voltage and current ratings may be obtained with the same transformer. It is obvious that the connections must always be so made that the proper phase relations between coil voltages are obtained. In many standard types of transformer the coil terminals which are brought to a terminal board or lead support are arranged in such a manner that these correct phase relations are obtained, if the well-known conventions are followed in making the connections (Fig. 193). Wiring diagrams usually accompany transformers when they leave the factory. There is always the possibility, however, of the wiring diagrams being lost, the internal connections are often changed during repairs, and it may be necessary to carry leads from the individual coil terminals through conduit, etc. so that

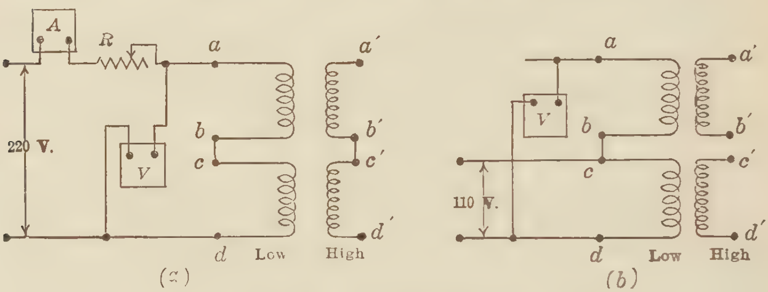


FIG. 192.—Methods of phasing transformer coils.

the terminal designations cannot be readily traced. In such cases it is necessary to "phase" the windings.

Figure 192 shows a 2,200–1,100/220–110-volt distributing transformer which is the most common type of power transformer. The low-side winding consists of two 110-volt coils *ab* and *cd*. The high-side winding consists of two 1,100-volt coils *a′b′* and *c′d′*. Let it first be desired to connect the transformer so that the low side has a 220-volt rating and the high side a 2,200-volt rating. Since it is desirable from the point of view of safety and convenience to utilize low voltage when testing, a 220-volt source, if available, is utilized. The low-side windings are first phased.

If coils *ab* and *cd* were connected in series across 220 volts and happened to act in opposition on their magnetic circuit, a virtual

short-circuit would result, the current being limited only by the small resistance and leakage reactance of these coils. To prevent short-circuit, a resistance or an impedance R is inserted in series with the coils to limit the current (Fig. 192 (a)). An ammeter A may be inserted in series or a voltmeter V may be connected across the two coils. If coils ab and bc are in opposition, the ammeter current will be large since the transformer coils offer but little impedance, the current being limited almost entirely by R. The voltmeter will simultaneously read low, since the impedance-drop across ad is small. On the other hand if the ammeter reading is small, or if the voltmeter reads practically 220 volts, the coils are aiding. (Test lamps may be used in place of the voltmeter.) If the coils are in opposition, the connections of one should be reversed. On the other hand if it is desired to operate the coils ab and cd in parallel at 110 volts, points a and d may be connected together if the test shows that the coils are in opposition.

The foregoing procedure may be safely pursued with the high-side coils, utilizing the 220-volt supply.

Another method of phasing primaries is shown in Fig. 192 (b). If a 110-volt supply is available, it may be connected across one low-side coil as cd without the possibility of damage. Terminal b of coil ab is connected to c, one terminal of coil cd, and a voltmeter is connected across the two outer ends a and d. If the voltmeter reads approximately 220 volts, the connection is correct for series operation. If it reads zero, points a and d may be connected together and the coils operated in parallel at 110 volts.

The high side may also be phased by tests made on the low side if the connections in Fig. 192 (a) are used. With the low side disconnected from the line, connect high-side coils $a'b'$ and $c'd'$ in series at $b'c'$. With the resistance or impedance R in series with the low side, connect a' to d'. Then apply voltage to the low-side circuit. If the coils $a'b'$ and $c'd'$ are aiding, they are short-circuited on themselves. This will be shown by the ammeter in the primary circuit reading high and the voltmeter reading low. If series operation is desired, then the low side is disconnected and the lead short-circuiting $a'd'$ is removed. On the other hand if it is desired to operate the high side at 1,100 volts, that is if the coils $a'b'$ and $c'd'$ are to be in parallel, the connections giving

the lowest reading of the ammeter and the highest reading of the voltmeter should be retained.

Figure 193 shows the conventional method of connecting the coils of a 2,200–1,100/220–110-volt distributing transformer. The high side consists of two 1,100-volt coils connected to terminals *ad* and *be* respectively. The line terminals *a′e′*, which enter the transformer through porcelain bushings at the wide side of the case, are permanently connected to terminals *a* and *e*. If it is desired to employ the 2,200-volt connection, the coils *ad* and *be* are connected in series by the two links *bc* and *cd* which connect terminals *b* and *d* together. If it is desired to employ the 1,100-volt connections, the coils are connected in parallel by connecting these links across terminals *ab* and *de*. A study of the arrows shows that in both cases the coils act in conjunction so far as the external circuit is concerned. The leads from the low-side coils 1-3 and 2-4 are carried outside the case through a porcelain bushing. By connecting 2 and 3 together, a 220–110 volt, 3-wire system is obtained. By connecting 1-2 and 3-4 together, the coils are connected in parallel and a 2-wire, 110-volt system is obtained.

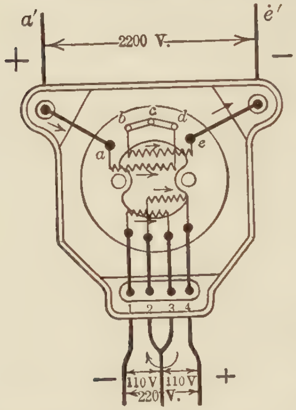


Fig. 193.—Connections of a distributing transformer.

126. Polyphase Transformer Connections.—There are several methods of connecting 3-phase transformer banks, as for example, Y-Y, Δ-Δ, Δ-Y, Y-Δ, V-V, T-T, etc.

Figure 194 shows a Y-Y connected transformer bank, which may be either a step-up or a step-down bank. For simplicity a 1:1 ratio and 100 volts between lines on the primary are assumed. With this connection, unbalanced loads to neutral cannot be applied on the secondary without seriously unbalancing the three voltages. This voltage unbalancing can be eliminated by connecting the primary neutral to the generator neutral. If it is desired to obtain a 3-phase, 4-wire system, the delta-Y system (Fig. 196) may be used, since it is not open to the objection

of extreme voltage unbalancing when unbalanced loads are connected to neutral.

The delta-delta bank shown in Fig. 195 is often used, especially for moderate voltages. Its chief advantage is that if one transformer becomes disabled, the system may operate in "V" or open delta. In both the Y-Y and the delta-delta connections, the ratio between the primary and secondary line voltage is the same as the individual transformer ratio.

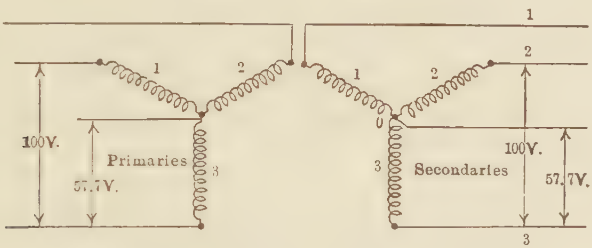


FIG. 194.—Y-Y connection of transformers.

The delta-Y connection shown in Fig. 196 is a very useful connection for stepping up the voltage. A distinct advantage of delta-Y connection over the delt-delta connection is that for high voltages the transformers need not be so well insulated. For a 100,000-volt system, the Y-connected transformers need

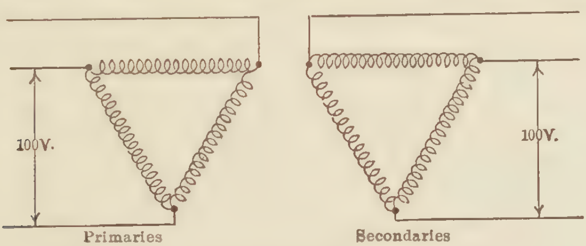


FIG. 195.—Delta-delta connection of transformers.

be insulated only for 58,000 ($100{,}000/\sqrt{3}$) volts, whereas delta-connected transformers must be insulated for 100,000 volts. The Y-delta system is often used for stepping down the voltage (see page 309, Fig. 267).

The ratio between line voltages in the delta-Y and Y-delta systems is not the individual transformer ratio, for the line volt-

age on the Y-side is $\sqrt{3}$ times that given by the transformer ratio. A delta-Y bank cannot be paralleled with a Y-Y or a delta-delta bank, even although the voltage ratios are correctly adjusted, as there will be a 30° phase difference between corresponding voltages on the secondary side.

The primaries of three single-phase transformers may be connected in either Y or delta without any attention being paid

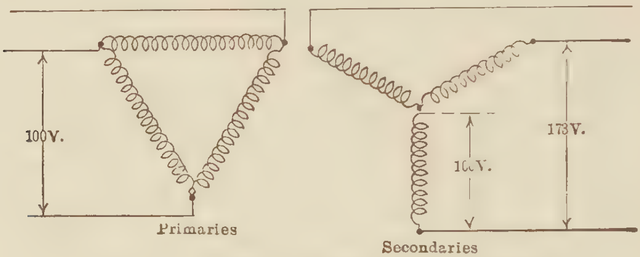


FIG. 196.—Delta-Y connection of transformers.

to phase relations. The secondaries must be phased like the alternator coils in Par. 94 (page 157). The primaries of *3-phase transformers*, however, must be correctly connected as regards phase relations. That is, the relations between the three fluxes must be those shown in Fig. 186 (page 209). The actual phasing

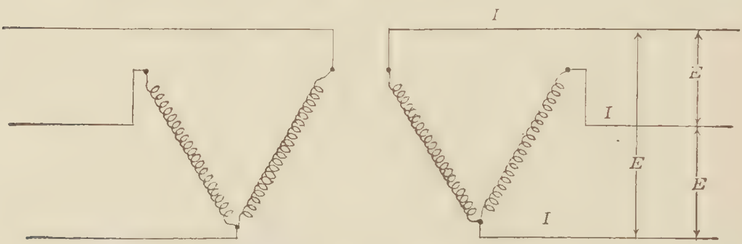


FIG. 197.—V or open-delta connection of transformers.

is often avoided as the primary and secondary connections are brought out of the case symmetrically.

127. The V-connection.—It was pointed out in Par. 94 (page 158) that line voltage must exist between the open ends of the two coils of the delta before the third coil is connected. At no load, with only two transformers, three equal 3-phase voltages exist around the secondaries and a 3-phase transformation is

therefore possible with two transformers only. This is called the "V" or open-delta connection (Fig. 197).

At first thought it might appear that the V-connection would have two-thirds the rating of the delta-connection. Both transformers work at a reduced power-factor when connected in V, even though the power-factor of the load remains fixed. Therefore, the kilovolt-ampere rating of the V-connection is less than two-thirds of the kilovolt-ampere rating of the delta-connection having individual transformers of equal rating. The ratio of the V-rating to the delta-rating is $1/\sqrt{3} = 58$ per cent. rather than $66\frac{2}{3}$ per cent. This can be proved as follows:

Let I be the rated current of each transformer and E the line voltage. The power, at unity power-factor Fig. 197 is

$$P_1 = \sqrt{3}EI.$$

As the transformer rating is determined by the *current*, the output of three of these transformers in delta will be

$$P_2 = 3EI.$$

Therefore,

$$\frac{P_1}{P_2} = \frac{\sqrt{3}EI}{3EI} = \frac{1}{\sqrt{3}} \text{ or 58 per cent.}$$

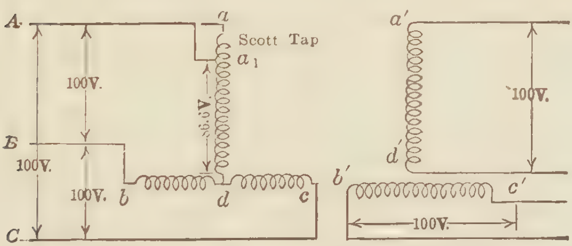


Fig. 198.—Scott or T-connection, 3-phase to 2-phase.

Often in practice a V-bank of transformers is first installed. The third transformer is added when the increase in load on the system warrants it. The rating of the bank is then increased 73 per cent. with an investment increase of but 50 per cent.

128. The Scott or T-connection.—By means of the Scott or T-connection it is possible to transform not only from three phase to three phase by means of two transformers, but also from

three phase to two phase or from two phase to three phase. This connection is seldom used, however, for 3-phase to 3-phase transformation. The method of connecting for 3-phase to 2-phase transformation is shown in Fig. 198. Two transformers, having primaries *ad* and *bc* and secondaries $a'd'$ and $b'c'$, are used. The middle point *d* of the winding *bc* must be accessible. One end *d* of the primary winding *ad* is connected to the middle point *d* of the primary *bc*. The ends of the three coils are connected to the three-phase supply *ABC* as shown. The transformer *bc* is called the *main* transformer and the transformer *ad* the *teaser* transformer.

The 3-phase supply is assumed to have a line voltage of 100 volts and the transformers have a 1:1 ratio. The voltages E_{dc} and E_{db} are each equal to 50 volts and differ in phase by 180°, since coil *dc* and coil *db* are both on the same magnetic circuit. Since the total voltage impressed across any system must be equal to the vector sum of the component voltages, the three component voltages E_{da}, E_{db}, and E_{dc} must combine in such a way as to give the three 3-phase line voltages E_{ab}, E_{bc}, and E_{ca}. In order that these component voltages may so combine to give the 3-phase line voltages, the voltage E_{da} must be equal to 100 $\sqrt{3}/2$ or 86.6 volts, and it must differ in phase from E_{db} and E_{dc} by 90°. Obviously the secondary voltages $E_{a'd'}$ and $E_{b'c'}$ must also differ in phase by 90°. Hence, a 2-phase system is obtainable from the secondaries.

Since the voltage across transformer *ad* is 86.6 volts, its secondary voltage will be only 86.6 volts, whereas the voltage across $b'c'$ the secondary of transformer *bc* is 100 volts. This gives two different voltages in the two phases of the 2-phase system, which is undesirable. By connecting line *A* to a_1, a tap which makes $E_{a_1d} = 0.866E_{ad}$, the volts per turn in transformer *ad* are increased in the ratio of 1:0.866. That is, a_1d is now a step-up transformer having a ratio of 0.866:1. The secondary voltage $E_{a'd'}$ will now be 100 volts, which gives a symmetrical 2-phase system. The tap a_1 is called a *Scott tap*.

It is obvious that various 2-, 3-, and 4-phase systems may be obtained from these secondaries (see pages 123 to 128). Furthermore, it is evident that this system may also operate to transform from 2 to 3 phase.

129. Constant-current Transformers.—The transformers heretofore considered are constant-potential transformers; that is, both the primary and the secondary voltages remain substantially constant and a change of load is accompanied by a corresponding change of current. Street lamps, however, are ordinarily connected in series and require constant *current*. Constant alternating current is ordinarily obtained from a constant-current or "tub" transformer.

The construction of the transformer is such that the primary and the secondary can move with respect to each other. The

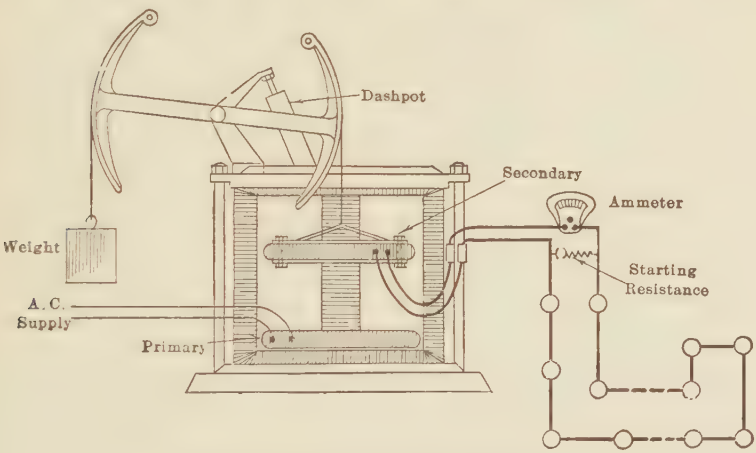


FIG. 199.—Constant-current transformer.

primary coil may be fixed and the secondary may move or the secondary coil may be fixed and the primary may move. Both types are found in practice. Figure 199 shows a transformer in which the primary is stationary and the secondary is movable. The load consists of a number of lamps connected in series. The secondary is suspended from a lever which is counter-weighted. A dashpot is provided to prevent rapid fluctuations in the position of the moving coil.

The operation of the transformer is as follows: Assume that the secondary coil is "floating;" that is, it is free to move either up or down and is delivering a certain current to a series load. The currents in the primary and secondary flow in opposite direc-

tions. Therefore, there is *repulsion* between the two coils (see Part I, page 20). Assume that the load changes, for example, that it decreases. This change of load would be produced by *short-circuiting* one or more lamps, causing a *decrease* in the load resistance. Because of the decreased load resistance, first the secondary and then the primary current tends to increase. This increases the repelling force between the two coils, resulting in the secondary moving further away from the primary. As a result a smaller proportion of the primary flux links the secondary, and the secondary e.m.f. is reduced. The secondary coil will move away from the primary until the secondary current is again at its normal value. When the secondary moves away from the primary, the leakage flux of both secondary and primary must increase (see page 194). Because of its large proportionate leakage flux this type of transformer has a very low power-factor except at or near its maximum load. This is one objection to its use.

Since this type of transformer attempts to deliver constant current under all conditions of load, it should never be open-circuited, since on opening the circuit, a high resistance in the form of an arc occurs and a very high voltage results. When not in service the secondary is held up against the core by a special latch. On starting, a low resistance is shunted across the secondary (Fig. 199). The resulting current causes the coil to rise and releases the latch.

This type of transformer is used alone for supplying "Mazda C" incandescent series street lamps. When used to supply magnetite lamps (see page 336) a mercury-arc rectifier (see page 303) is necessary in addition, since this type of lamp requires unidirectional current.

INSTRUMENT TRANSFORMERS

130. Electrical Measurements at High Voltages.—It is not usually practicable to connect instruments or meters directly to high-voltage circuits. Unless the high-voltage circuit is grounded at the instruments, they may be subjected to high-voltage stresses to ground. This makes it dangerous for anyone to come in contact with the switchboard apparatus. Further,

instruments become inaccurate when connected directly to high voltage, because of the electrostatic forces which act on the indicating element.

By means of instrument transformers, instruments may be entirely insulated from the high-voltage circuit and yet indicate accurately the current, voltage, power, etc., in the high-voltage circuit. Moreover, low-voltage instruments having standard current and voltage ranges may be used for all high-voltage circuits, irrespective of the voltage and current ratings of the circuits.

131. Potential Transformers.—Except that their power rating is small, potential transformers do not differ materially from the constant-potential transformers already discussed. As instruments only and sometimes pilot lights are connected to their secondaries, such transformers ordinarily have ratings of from

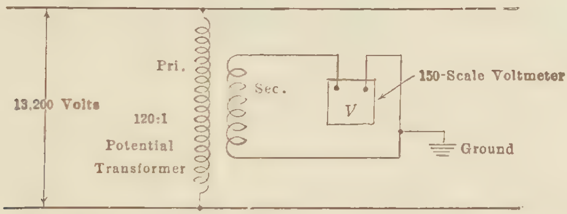


FIG. 200—Use of potential transformer on a 13,200-volt single-phase circuit.

40 to 200 watts. The low-tension side is almost always wound for 110 volts and the ratio is then determined by the voltage of the high-voltage winding. For example, a 13,200-volt potential transformer would have a ratio of 13,200/110 = 120:1. Figure 200 shows a simple connection for measuring voltage in a 13,200-volt circuit by means of a potential transformer. The secondary should always be grounded at one point to eliminate "static" from the instrument and further to insure safety to the operator. Figure 203 shows a potential transformer used in conjunction with a current transformer for measuring power by means of a wattmeter.

132. Current Transformers.—To avoid connecting alternating-current ammeters and the current-coils of other instruments directly in high-voltage lines, current transformers are used. In addition to insulating the instruments from high-voltage, they

step down the current in a known ratio. This enables a lower-range ammeter to be used than would ordinarily be required if the instrument were connected directly in the primary line.

The current or series transformer has a primary, usually of few turns, wound on a core and connected in series with the line Fig. 201. When the primary has a large current rating, it may

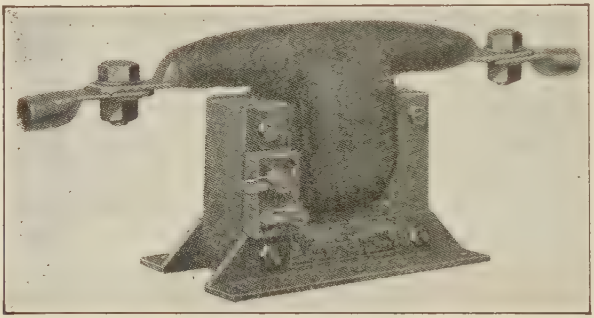


Fig. 201.—Indoor current transformer. (General Electric.)

consist of a straight conductor passing through the center of a hollow core, as shown in Fig. 202. The secondary, consisting of several turns, is wound around the laminated core. The ratio of current transformation is approximately the inverse ratio of turns. For example, if the primary has two turns and the secondary 60 turns, the ratio will be 30:1.

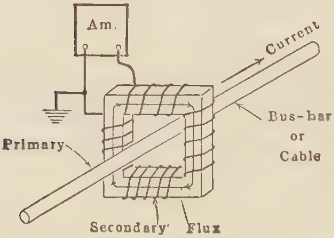


Fig. 202.—Construction of one type of current transformer.

The secondaries of practically all current transformers are rated at 5 amp., regardless of the primary current rating. For example, a 2,000-amp. current transformer has a ratio of 400:1 and a 60-amp. transformer has a ratio of 12:1.

The current transformer differs from the ordinary constant-potential transformer in that its primary current is determined entirely by the load on the system and not by its own secondary load. If the secondary becomes open-circuited when the transformer is carrying even a moderate current, a high voltage will

exist across the secondary because the large ratio of secondary to primary turns causes the transformer to act as a step-up transformer. *This high voltage is dangerous.*

Therefore, a current transformer should always have its secondary short-circuited.

Figure 203 shows the method of connecting a typical instrument load, through instrument transformers, to a high-voltage line. The load on the instrument transformers includes an

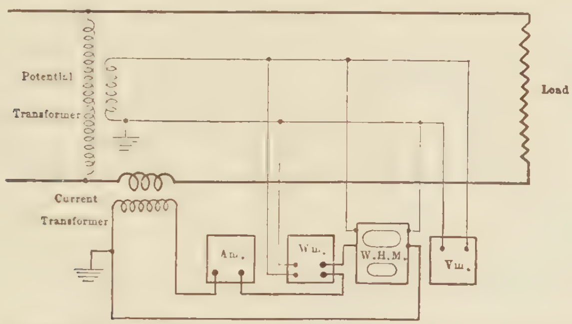


FIG. 203.—Typical connections of instrument transformers and instruments for single-phase measurements.

ammeter, a voltmeter, a wattmeter, and a watthour meter. Each secondary is grounded at one point. Correction for ratio of transformation must be applied to all the instrument readings, the wattmeter and watthour meter involving the ratio of both the current and the potential transformers. Usually in permanent installations, as on switchboards, the instrument scales themselves are so marked as to take into consideration these ratios. Therefore, the primary power may be read directly.

CHAPTER VIII

THE POLYPHASE INDUCTION MOTOR

The induction motor is the most widely used type of alternating-current motor. This is due to its ruggedness and simplicity, to the absence of a commutator, and to the fact that its operating characteristics are well adapted to constant-speed work.

133. Fundamental Principle of the Induction Motor.—The principle of the induction motor may be illustrated as follows:

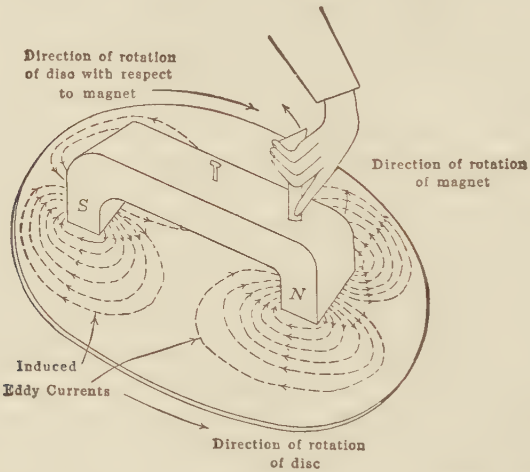


Fig. 204.—Rotation of metal disc produced by rotating magnet.

A metal disc (Fig. 204), is free to turn upon a vertical axis. The disc may be of any conducting material, such as iron, copper, or aluminum. A magnet, free to rotate on the same axis as the disc, is placed above the disc and the ends of the magnet are bent down so that its magnetic flux cuts the disc When this magnet is rotated, the magnetic lines cut the disc and induce currents in it, as shown in the figure. As these currents find themselves in a

magnetic field, they tend to move across this field, just as the currents in the conductors of a direct-current motor tend to move across the magnetic field of the motor. By Lenz's law, the direction of the force developed between these currents in the disc and the magnetic field producing them will be such that the disc tends to follow the rotating magnet, as shown in the figure.

Power is required to overcome the friction of the disc and to supply the I^2R loss due to the induced currents in the disc. This power must come from the mechanical power required to rotate

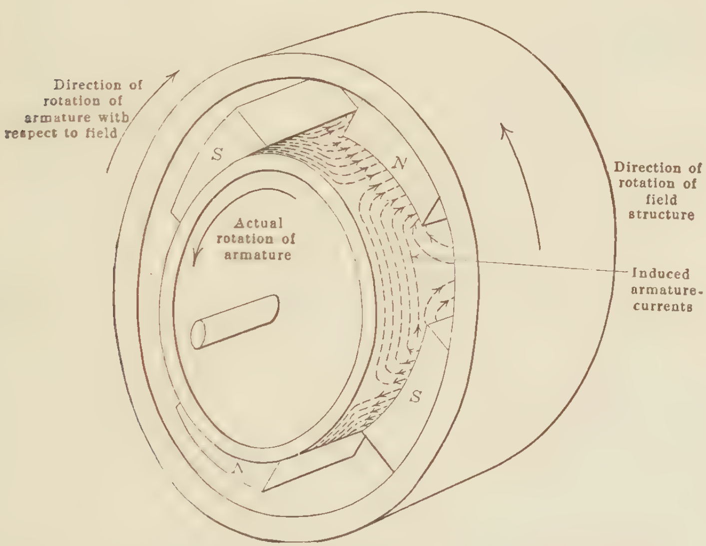


FIG. 205.—Rotation of conducting cylinder due to induced currents.

the magnet. The power transferred to the disc from the magnet can be transferred only by means of a force acting between the two. If force acts between the two, the magnet must pull the disc and the directions of the induced currents in the disc must be such as to cause the disc to *follow* the magnet.

It is obvious that the disc tends to attain the same speed as the magnet. This principle is further illustrated by the retarding action in watthour meters. The retarding magnets tend to cause the disc to attain their own speed which in this case is zero.

The disc can never attain the speed of the magnet, for were it to reach this speed, there would be no relative motion of the disc and the magnet and, therefore, no cutting of the disc by the magnetic flux. The currents in the disc would then become zero and no torque would be developed, which would result in the speed of the disc becoming less than that of the magnet. Because the disc cannot attain the speed of the magnet, there must always exist a *difference* of speed between the two. This difference of speed is called the revolutions *slip*.

A cylinder may be used instead of the disc, as shown in Fig. 205. In the figure are shown four poles, the magnetic lines of which cut the cylinder. If the frame carrying these poles be revolved by mechanical means, the currents induced in the cylinder will cause the cylinder to rotate in the same direction as that of the rotating frame. This cylinder and the magnetic poles which are associated with it are more representative of the commercial induction motor than the disc is, although both operate on the same principle.

It is to be noted that the currents in the disc or armature of this type of motor are *induced* therein, rather than being *conducted* into the armature as in the ordinary direct-current motor.

The induction motor, therefore, may operate without any conducting connection between the armature and the external circuit.

134. Rotating Fields Produced by Two-phase Windings.—The rotating magnetic fields described in the previous paragraph were produced by rotating the magnetic poles mechanically. This is practically the same as the rotating poles of an alternator field. Rotating magnetic fields may, however, be produced by sending polyphase currents through polyphase windings, such as alternator windings. Such rotating fields are produced entirely by electrical means, there being no mechanical rotation of the pole-pieces themselves.

The windings which produce the rotating field are ordinarily wound on the fixed member, called the *stator*. The stator is made of circular laminations of iron, and has slots and a winding identical to that of alternators. In fact, an alternator armature can be used for an induction-motor stator, if the frequency and number of poles are the same. The rotating member in which currents are induced by the rotating field is called the *rotor*.

Figure 206 (*a*) shows a cross-sectional diagram of a 4-pole, 2-phase induction motor. The single-layer stator winding, in which there are three slots per pole per phase is shown diagrammatically in Fig. 207. For simplicity, the connections of the *A*-phase only are shown. The connections of the *B*-phase are

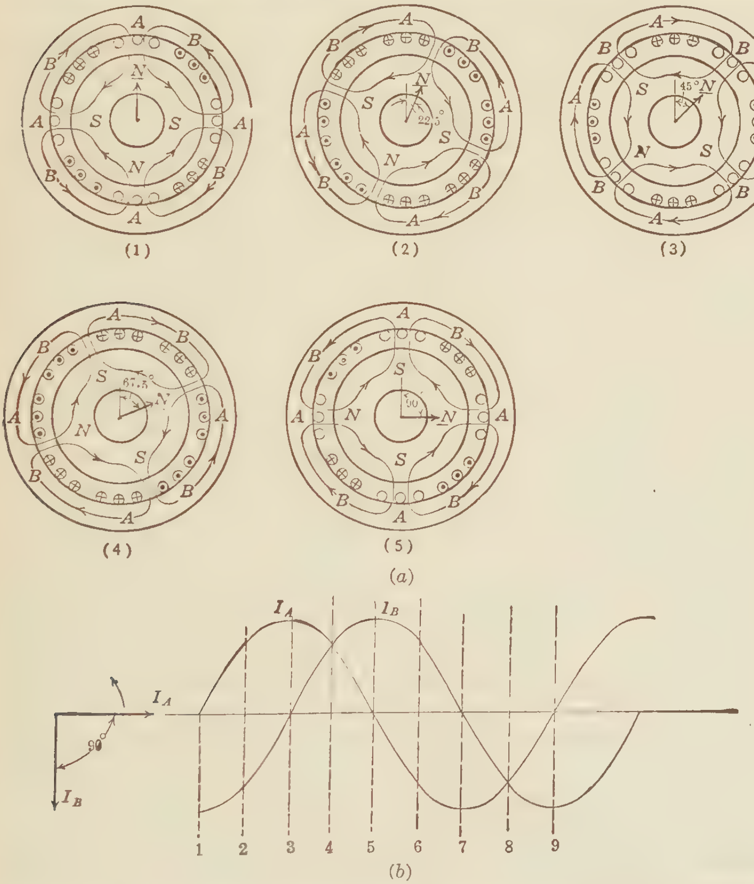


FIG. 206.—Production of rotating field by 2-phase currents in a 4-pole winding.

identical with those of the *A*-phase. The time-variation of the currents in the two phases is shown in Fig. 206 (*b*). The current I_A in the *A*-phase leads current I_B in the *B*-phase by 90°. The radius-vectors I_A and I_B associated with these time-curves are also shown in Fig. 206 (*b*).

At the instant (1) (Fig. 206 (*b*)), the current I_A is zero and I_B is negative maximum. There is, therefore, no current in the conductors of the *A*-belt in diagram (1) (Fig. 206 (*a*)). The armature is so wound that when I_B is negative (Fig. 206 (*b*)) the current in the upper left-hand conductor-belt of the *B* phase is inward, that in the adjacent *B*-belts is outward, and that in the opposite *B*-belt is inward. By applying the corkscrew rule, the m.m.fs. of the two upper *B*-belts send flux into the armature in the region between them forming a north pole *N*. Likewise, the m.m.fs. of these same belts when combined with the m.m.fs. of the two remaining belts form alternate south and north poles as shown.

At the instant (2) the current in the *B*-phase is still negative, but is 0.707 times its maximum value. The current, therefore, still flows inward in the same *B*-belts as at instant (1), but is less in magnitude. The direction of the winding of the *A*-phase is such that when the current in the *A*-phase is positive the current is inward in the top conductor-belt as shown. The current in the *A*-belt at instant (2) is 0.707 times its maximum value, hence equal currents flow in all the conductors of both phases. Therefore, all adjacent phase-*A* and phase-*B* conductor-belts carrying current in the same direction at the given instant act as a single belt. Application of the corkscrew rule shows that the flux produced by the various conductor-belts has the direction shown in diagram (2) (Fig. 206 (*a*)), north and south poles being formed midway between those belts in which the current flows in opposite directions. It will be observed that in diagram (2) all the *N*- and *S*-poles have advanced 22.5 space-degrees in a clockwise direction from their position in diagram (1).

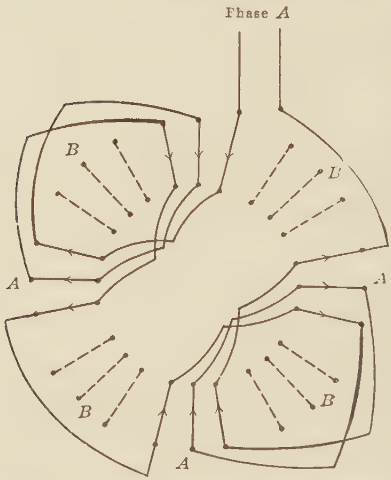


Fig. 207.—Single-layer, 4-pole, 2-phase induction-motor winding, lap-connected.

At instant (3) the current I_B in the B-phase is zero. The current I_A in the A-phase is positive maximum and has the same direction that it had at instant (2). Therefore, the direction of the fluxes produced by A-phase conductor-belts will be the same as in diagram (2). N- and S-poles will be found midway between adjacent conductor-belts, as shown in diagram (3). Note that all north and south poles have again advanced 22.5 space-degrees in a clockwise direction from their position in (2).

At instant (4) both I_A and I_B are positive and each equal to 0.707 times its maximum value. The direction of the currents in the A-phase conductors will not have changed from the direction in (2) and (3), but the direction of the currents in the B-phase conductors will be opposite to that shown in (1) and (2). Adjacent A-phase and B-phase conductor-belts carrying currents in the same direction act as a single belt and produce fluxes as shown in (4). N- and S-poles are formed midway between belts in which the currents have opposite directions. Again in (4) all north and south poles have advanced 22.5 space-degrees in a clockwise direction from their position, as shown in (3).

A study of the currents and the direction of the fluxes which they produce at instant (5) shows that all north and south poles have again advanced 22.5 space-degrees in a clockwise direction from their position in (4). Between (1) and (5) the currents have gone through 180 time-degrees and in the same interval all north and south poles have advanced 90 space-degrees in a clockwise direction.

Further analysis of the direction of the currents and the fluxes which they produce shows that during each time-interval, 6-7, 7-8, 8-9, the north and south poles have advanced 22.5 space-degrees. At (9) the end of the current cycle, all north and south poles have advanced 180 space-degrees. After two complete cycles of current all north and south poles will again be in their original positions, as shown in (1). The foregoing illustrates the production of a rotating magnetic field by means of polyphase currents in polyphase windings.

The machine (Fig. 206) is a 4-pole, 2-phase motor. Had it been wound for two poles only, the resulting north and south pole would have completed one revolution in *one* cycle of the current. Figure 208 shows such a 2-pole stator and also shows

the poles existing at instants (1) and (2) of Fig. 206 (*b*). The poles have advanced 45° in this interval 1-2 instead of advancing 22.5°, as in Fig. 206. Hence, in one cycle of the current the poles (Fig. 208) advance 360 space-degrees.

In a 6-pole machine, the poles complete one revolution in three cycles of the current; in an 8-pole machine they complete one revolution in four cycles of the current, etc.

Furthermore, when 3-phase currents are supplied to any type of machine with 3-phase windings, similar rotating fields result. Such fields complete one revolution per cycle of current in 2-pole machines, one revolution in two cycles of current in 4-pole machines, etc.

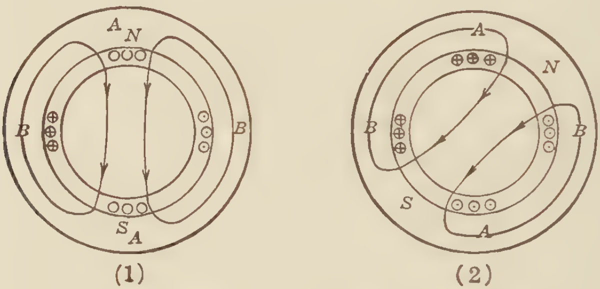


FIG. 208.—Rotating field in a 2-pole, 2-phase machine.

The direction of the rotating field in a 2-phase machine may be reversed by reversing the connections to *either* phase; the direction of the rotating field in a 3-phase machine may be reversed by interchanging any *two* leads.

135. Synchronous Speed; Slip.—It has just been shown that the angular speed of an alternating-current rotating field depends on two factors, the frequency of the current and the number of poles for which the machine is wound. The relation between speed, frequency, and poles is given by the following equation:

$$N = \frac{f \times 120}{P} \qquad (58)$$

where N is the speed of the field in revolutions per minute, f the frequency in cycles per second, and P the number of poles (see equation (2), page 15). This speed N of the rotating field, is called the *synchronous speed* of the motor. The common

synchronous speeds for commercial motors at 25 and at 60 cycles per second are as follows:

Poles	r.p.m. = N	
	$f = 25$	$f = 60$
2	1,500	3,600
4	750	1,800
6	500	1,200
8	375	900
12	250	600

Slip.—If an armature whose conductors form closed circuits be placed in a rotating field, it will develop torque because the induced currents act in conjunction with the rotating magnetic field, as in the disc (Fig. 204).

As has already been pointed out, the armature can never attain the speed of the rotating field, for if it did, the cutting of conductors by flux would cease, there would be no rotor current and, therefore, no torque.

The difference between the speed of the rotating field and that of the rotor is called the *revolutions slip* of the motor.

Example.—The rotor of a 4-pole, 60-cycle motor has a speed of 1,720 r.p.m. Find its revolutions slip.

The synchronous speed from (58)

$$N = \frac{60 \times 120}{4} = 1{,}800 \text{ r.p.m.}$$

Revolutions slip = 1,800 − 1,720 = 80 r.p.m. *Ans.*

It is more convenient to express the slip as a fraction of the synchronous speed. Denote the speed of the rotor by N_2 and the synchronous speed by N. Then the slip

$$s = \frac{N - N_2}{N}. \tag{59}$$

Example.—Determine the slip in the foregoing, 4-pole motor.

$$s = \frac{1{,}800 - 1{,}720}{1{,}800} = \frac{80}{1{,}800} = 0.0444 \text{ or } 4.44 \text{ per cent.} \quad \textit{Ans.}$$

The rotor speed is obviously

$$N_2 = N(1 - s) \text{ (from equation (59)).} \tag{60}$$

Since from (58),

$$N = \frac{f \times 120}{P}$$

$$N_2 = \frac{f \times 120}{P}(1 - s). \quad (61)$$

The full-load slip in commercial motors varies from 1 to 10 per cent, depending on the size and the type of motor.

136. Rotor Frequency and Induced Electromotive Force.—If the rotor of a 2-pole, 60-cycle motor is at standstill and voltage is applied to the stator, each rotor conductor will be cut by a north pole 60 times per second and by a south pole 60 times per second, as this is the speed of the rotating field. If the stator be wound for four poles, the speed of the rotating field is halved, but each conductor is then cut by two north and two south poles per revolution of the field and, therefore, by 60 north and 60 south poles per second, the same as in the 2-pole motor. Consequently, the frequency of the rotor currents at standstill ($s = 1.0$) will be the same as the stator frequency. This holds true for any number of poles. At standstill, the motor is a simple static transformer, the stator being the primary and the rotor being the secondary.

If the rotor of a 60-cycle motor revolves at half speed in the direction of the rotating field ($s = 0.5$), the rotor conductors are cut by just one-half as many north and south poles per second as when standing still and the frequency of the rotor currents is, therefore, 30 cycles per second.

By taking other rotor speeds, it can be seen that the rotor frequency

$$f_2 = sf, \quad (61a)$$

where f_2 is the rotor frequency, s the slip, and f the stator frequency. *The rotor frequency is equal to the stator frequency multiplied by the slip.*

Example.—What is the frequency of the currents in the rotor of a 60-cycle, 6-pole induction motor, if the rotor speed is 1,164 r.p.m.

The synchronous speed

$$N = \frac{60 \times 120}{6} = 1{,}200 \text{ r.p.m. (equation (58), page 232).}$$

The slip $$s = \frac{1{,}200 - 1{,}164}{1{,}200} = 0.03.$$

$$f_2 = 0.03 \times 60 = 1.8 \text{ cycles per second.} \quad \textit{Ans.}$$

From the foregoing, it is clear that under ordinary operating conditions the rotor frequency is low. At standstill or when starting, the rotor frequency is that of the stator. The rotor frequency has a very important bearing on the operating characteristics of the induction motor.

The induction motor can be used as a frequency changer if the rotor is driven mechanically at the proper speed. Current is taken from the rotor, or secondary, through slip-rings. Under these conditions, some of the power is supplied electrically and some mechanically.

137. Torque of the Induction Motor.—With either series or shunt direct-current motors operating at constant voltage, the torque will always increase with increase of load until the armature is brought to a standstill. If the motors are of commercial

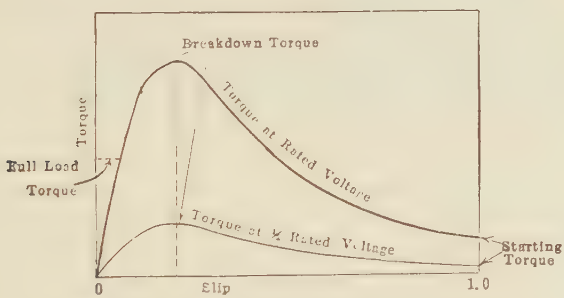


FIG. 209.—Slip-torque curves for squirrel-cage motor.

sizes, the torque at standstill with rated voltage across the armature would be very large as compared with rated-load torque. It would be impracticable to allow the motor to develop this very large standstill torque since the current would be prohibitively large, the maximum torque that such motors develop being limited by the current which the armature can safely carry. Even so, such motors are able to develop several times full-load torque on starting.

With induction motors the torque increases with increase of load only up to a certain maximum torque, further increase of load resulting in a *decrease* of torque, the rotor coming to a standstill. This maximum torque is from two to four times the rated-load torque. This is illustrated in Fig. 209, which shows the slip-

torque curves for the ordinary squirrel-cage motor. When the slip is nearly zero, the rotor is running at its maximum speed or at nearly synchronous speed. As the load is increased, the torque at first increases almost in proportion to the slip. At larger values of slip, the torque increases less rapidly than the slip and ultimately a maximum value of torque, the *break-down torque*, is reached (Fig. 209). Further increase of load results in *less* torque being developed by the motor and the rotor must, therefore, come to a standstill. With the squirrel-cage motor, the starting or standstill torque ($s = 1.0$) is very small (Fig. 209).

For a given value of slip, the torque is proportional to the impressed voltage *squared*. For example, if at any given value of slip, the impressed voltage is halved, the air-gap flux is halved and as a result the induced rotor current is halved. The rotor currents of half their former magnitude, therefore, find themselves in a magnetic field of half its former strength, and the torque is, therefore, one-fourth its former value. This is illustrated by the torque curve at one-half rated voltage (Fig. 209).

It is important to keep in mind that the torque varies as the *square* of the impressed voltage. A 10 per cent. reduction in voltage results in a 19 per cent. reduction in break-down torque.

138. The Squirrel-cage Motor.—The squirrel-cage motor is the simplest type of induction motor and is the most generally used. The core of the rotor or armature (Fig. 210) like that of the direct-current armature, is usually built up of slotted steel punchings. The winding consists of copper bars placed in slots. These bars have their ends connected together by conducting rings called *end-rings*. The bars are usually bolted to the end-rings and then welded or brazed. Formerly solder was used, but considerable trouble was encountered by its melting and then being thrown out of the joint by centrifugal action. Another method is to place the rotor in a mold and cast the ends of the bars in a ring of cast copper. The General Electric Company manufactures a rotor in which an aluminum grid is cast integral with the end-rings. The methods in which the end-rings are cast integral with the bars of the winding are the best from the operating point of view, as the rotor conductors have no opportunity to work loose.

In nearly all induction motors, except those of largest capacity, the stator slots are of the semiclosed type (Fig. 211). If open slots are used, as is sometimes done, magnetic wedges are employed so as to give the effect of semiclosed slots.

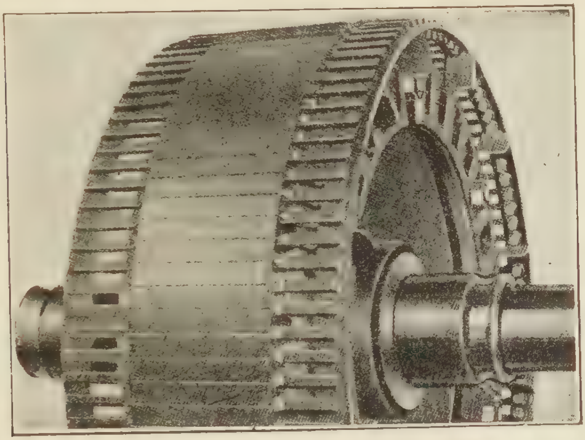


FIG. 210.—Squirrel-cage rotor.

The advantage of the semiclosed slot is that the effective sectional area of the air-gap is increased and the magnetizing current is, therefore, reduced. The magnetizing current, whose

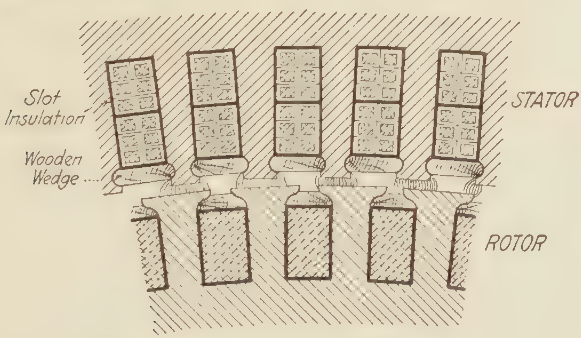


FIG. 211.—Stator and rotor slots of squirrel-cage induction motor.

m.m.f. produces the flux, lags the terminal voltage by 90°. A reduction in magnetizing current increases the power-factor, which is a distinct advantage since the power-factor of induction motors

is low under the best conditions. On the other hand, the semi-closed slot gives a much higher slot inductance than the open slot and this inductance in the stator and in the rotor lowers the power-factor and decreases the starting torque and the break-down torque of the motor.

Figure 212 shows a stator partially wound. The method of placing the coils in the semi-closed slots should be noted.

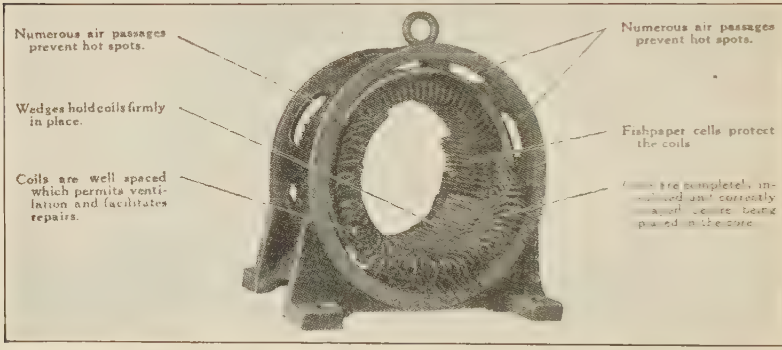


FIG. 212.—Windings being placed in a Westinghouse 600–800 frame stator.

139. Operating Characteristics of the Squirrel-cage Motor.—The squirrel-cage motor, like the direct-current shunt motor, operates at substantially constant speed. As the rotor cannot reach the speed of the rotating magnetic field, it must at all times operate with a certain amount of slip. At no load, the slip is very small. As load is applied to the rotor, more rotor current is required to develop the necessary torque in order to carry the increased load. Consequently, the rotating magnetic field must cut the rotor conductors at an *increased* rate, in order to produce the necessary increase of current. The slip of the rotor must, therefore, increase, so that the rotor speed drops. As the resistance of the squirrel cage is very low, a small increase in slip produces a large increase in current. The slip, therefore, for ordinary loads is small. In large motors, 50 hp. or greater, the slip is of the order of 1 to 2 per cent at full load. In the smaller sizes of motor, the slip may be as high as 8 to 10 per cent at full load.

Figure 213 shows the usual characteristic curves of a 10-hp., squirrel-cage motor. It will be noted that the torque, speed, and efficiency curves are very similar to those of a shunt motor. The power-factor increases with the load.

At no load the losses are small, just as with the transformer. The no-load current produces the m.m.f. which sends the flux across the air-gap and around the magnetic circuit. Hence, the no-load current lags the voltage by a large angle and the no-load

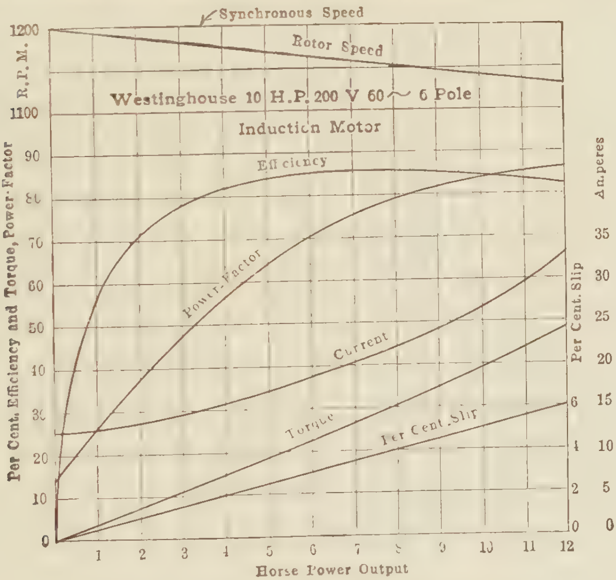


FIG. 213.—Operating characteristics of a squirrel-cage induction motor.

power-factor is low. Because the no-load or magnetizing current must send the flux across the air-gap, its value is comparatively large being from 20 per cent. of rated current in large motors to 40 to 60 per cent. of rated current in small motors. As the load on the motor increases, more power is required and the power-factor must increase.

The efficiency curve is similar to that for other types of electrical apparatus (see Part I, page 272, Fig. 241). At all loads there are certain fixed losses, such as core loss, friction, and windage. In addition there are the load losses (I^2R) which increase nearly as the square of the load. At light loads, there-

fore, the efficiency is low because the fixed losses are large as compared with the input. As the load increases, the efficiency increases to a maximum. Beyond this point the I^2R losses become relatively large, causing the efficiency to decrease.

One disadvantage of the squirrel-cage motor lies in the fact that it takes a very large current at low power-factor on starting, and in spite of this large current it develops but little torque. When the motor is at standstill, the squirrel cage acts as the short-circuited secondary of a transformer, causing the motor to take an excessive current on starting, if full voltage is applied.

Figure 209 shows the variation of torque with slip for two different values of line voltage. It will be noted that for small values of slip up to and beyond full load, which is the ordinary range of operation, the torque is substantially proportional to the slip. At higher values of slip, however, the torque reaches the maximum or *break-down torque* and beyond this maximum point the torque decreases as the slip increases until at standstill ($s = 1.0$) the torque is relatively *small*. This low starting torque is due to the fact that the torque is proportional to the power transferred across the gap. The greater the power transferred across the gap, the greater the force acting between stator and rotor and the greater the torque. At standstill, practically all the power transferred across the gap must be accounted for in the rotor I^2R loss. The resistance of the squirrel-cage rotor is so low that a *large* current is required to produce any considerable I^2R loss. Hence, the starting torque must be small even with abnormally large rotor currents. Therefore, even when full line voltage is applied to the stator, the *starting torque* of the squirrel-cage motor is *small*.

This low starting torque of the squirrel-cage motor is further reduced by the necessity of applying reduced voltage to the motor on starting (see the next paragraph). If 50 per cent. line voltage is applied on starting, only 25 per cent. normal starting torque is obtained (Fig. 209). The squirrel-cage motor, therefore, cannot be used where it must be started under load.

Because of its low rotor resistance, the squirrel-cage motor has excellent operating characteristics for constant-speed work. The slip is small and the speed regulation is good. In addition, the motor is simple, rugged, and requires but little attention.

Some of its fields of application are in machine shops, in woodworking shops, in cement mills, and in textile mills; in fact, it is used in most cases where the load requires constant speed with but little starting torque.

In addition to its low starting torque, another disadvantage of the squirrel-cage motor is that its speed is not adjustable.

Figure 214 shows a 6,000-hp. induction motor driving a rail mill, which rolls hot steel billets into steel rails.

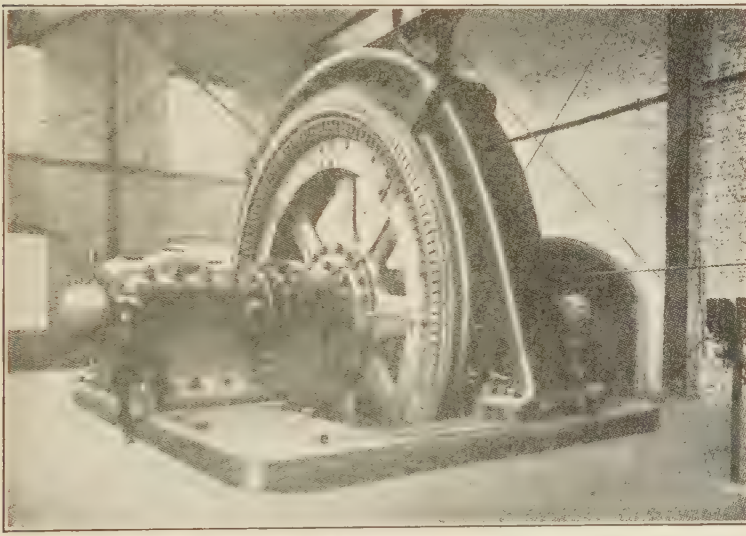


FIG. 214.—6,000-hp., 75-r.p.m., 6,600-volt, General Electric induction motor driving, rail mill. (Illinois Steel Co., Gary, Ind.)

140. Starting Squirrel-cage Motors.—A squirrel-cage winding has very low resistance and at standstill corresponds to the short-circuited secondary of a transformer. Therefore, if the motor is connected directly across the line, the no-load current is large and with large motors the resulting disturbance to the line voltage may be so great as to be prohibitive. Small induction motors up to 5 hp. can usually be connected directly across the line without undue disturbance of the line voltage, since they take but 2.5 to 3 times full-load current on starting.

Figure 215 shows the connections often used for the smaller-sized motors where special starting devices are not required.

A double-throw switch, when in the starting position, puts the motor in series with three high-capacity fuses, one in each line. Because of the action of a spring, the switch can make contact on this side only while it is held in position. When the switch is thrown to the running position, the current is supplied through three fuses designed to carry only the safe operating current of the motor. This gives the motor overload protection that would not otherwise be obtained if fuses sufficiently large to carry the starting current were used during normal operation. The high-capacity fuses at the motor (Fig. 215) are sometimes omitted, the line fuses giving the required protection at starting.

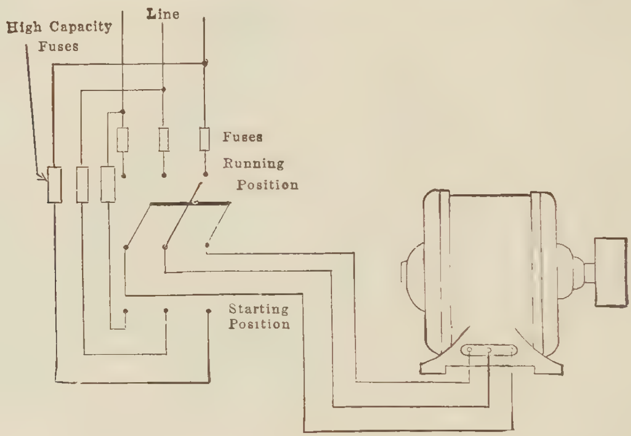


FIG. 215.—Switching connections when motor is connected directly across line at starting.

With motors of larger rating it is necessary to limit the starting current. To do this an automatic starter employing graphite compression resistors is now on the market. The three carbon piles are located in steel tubes lined with refractory material. As the motor accelerates, the resistance is reduced automatically by compression of the carbon piles. When the motor reaches its rated speed, the carbon piles are short-circuited.

Another method of limiting the starting current is to use a delta-connected motor (Fig. 216). By means of a triple-pole, double-throw (T.-P., D.-T.) switch, the windings are first connected in Y across the line, thus applying only $1/\sqrt{3}$ or 58 per

cent. of the normal voltage to each coil. This makes the *line* current one-third the value it would have if the motor were directly across the line. When the motor has attained sufficient speed, the switch is thrown over, connecting the motor in delta across the line.

The most common method of starting the squirrel-cage motor, however, is to use an auto-starter or starting-compensator, similar to those shown in Figs. 217, and 218. In the General Electric compensator shown in Fig. 217, the three coils of a 3-phase auto-transformer are connected in Y. When the switch is in the starting position, the compensator is connected across the line with only the line fuses for protection. Under these conditions, the three motor lines T_1, T_2, and T_3 are connected to three taps, one in each phase of the auto-transformer. Hence, the motor voltage is

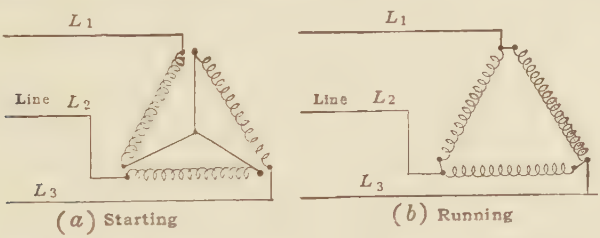
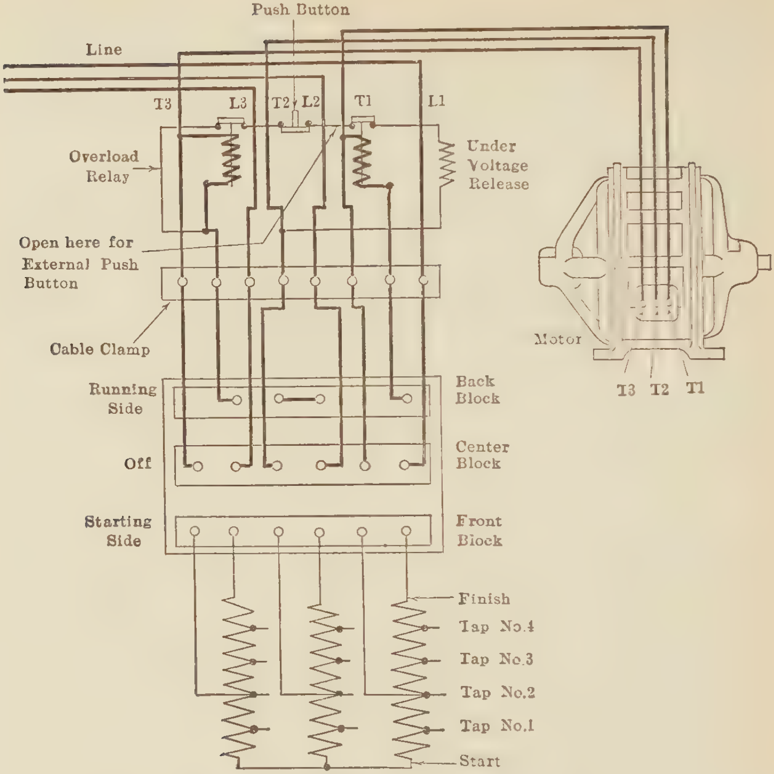


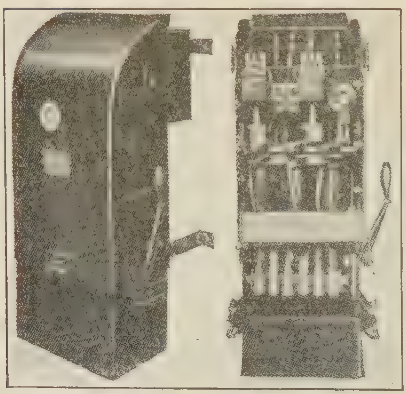
FIG. 216.—The Y-delta method of starting an induction motor.

reduced, usually to one-fourth or to one-half its rated value. When the switch is in the running position, the compensator is entirely disconnected from the line and the motor is connected directly across the line through the overload relays. In Fig. 217 the heavy lines show the path of the current when the compensator is in the running position. When the line voltage drops to a value which is too low for the motor to operate satisfactorily, the under voltage release allows the starting arm to spring to the "off" position. The under voltage release is also operated by its circuit being opened either by the overload relays or the push button.

It should be remembered that a compensator supplying a motor with half voltage reduces the line current to one-fourth its normal value. The motor being at half voltage takes one-half the current that it would take if directly across the line. As this current



(a)



(b)

FIG. 217.—Auto-starter for squirrel-cage induction motor. (General Electric Co.)

is supplied by the secondary of a two-to-one transformer, the line current is but half the motor current and is, therefore, one-fourth the current that would have been taken had the motor been directly across the line.

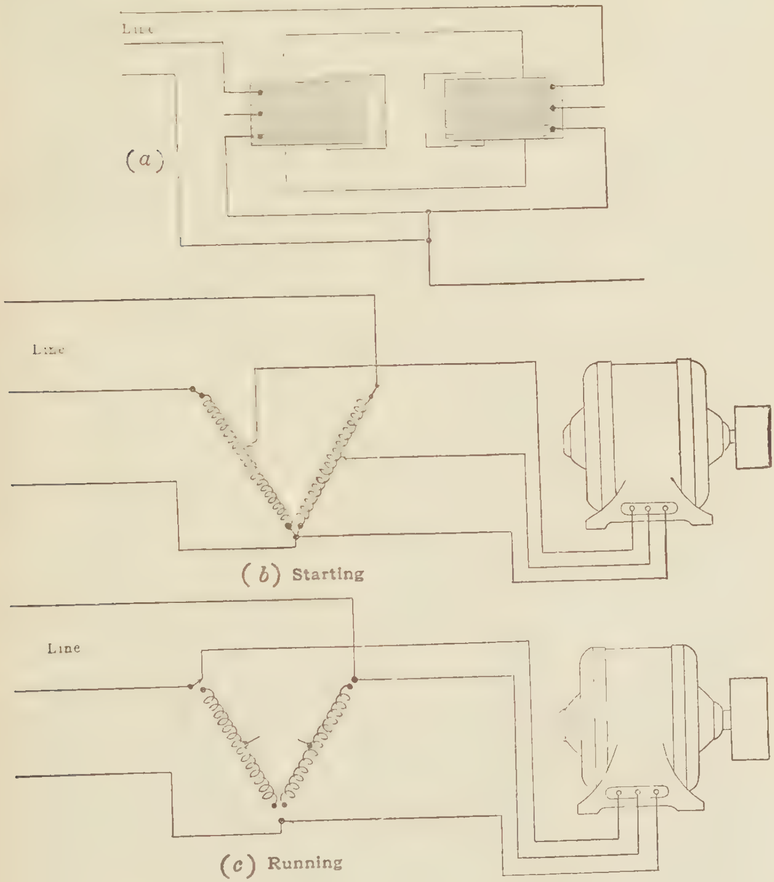


FIG. 218.—V-connected starting compensator.

It is not necessary to use a 3-coil auto-transformer. In the Westinghouse starting compensator, two coils are mounted on the two outer legs of a transformer core (Fig. 218 (*a*)) similar to the core used for 3-phase, core-type transformers (see page 209, Fig. 186). On starting, these two coils are connected in V across

the line and two motor taps are taken off as shown in Fig. 218. The motor is thus supplied at a reduced 3-phase voltage. When the starting handle is placed in the running position, the two motor taps are connected directly to their corresponding lines (Fig. 218 (*c*)), and at the same time the compensator is entirely disconnected from the line. One advantage of this type of starter is that it can be readily used on 2-phase as well as on 3-phase circuits.

Practically all starting compensators have an under-voltage release, as indicated in Fig. 217. When the line voltage decreases to a low value, a solenoid plunger drops, releasing the starting handle which springs back to the "off" position.

141. The Wound-rotor Induction Motor.—If resistance be introduced in the rotor circuit of an induction motor, the slip for any given value of torque will increase.

A given value of torque requires a definite value of flux and a definite value of current. The flux of the induction motor is practically constant, since the terminal voltage and, hence the back e.m.f. are practically constant. If resistance be introduced in the rotor circuit, the rotor impedance is increased. (At slips which give the ordinary values of torque, the armature reactance is small as compared with its resistance, hence the armature impedance is practically all resistance.) If the slip remains constant, the induced e.m.f. of the rotor does not change. The armature current, which is equal to this e.m.f. divided by the rotor impedance, decreases. The torque, therefore, decreases.

To bring the torque back to its original value, the armature current must be increased. To increase the armature current, the armature induced e.m.f. must increase. Since the flux is constant, the increase in the induced e.m.f. may be obtained only by this flux cutting the rotor conductors at a greater rate. For a given value of torque, therefore, the slip must *increase* when resistance is introduced in the rotor circuit.

The slip-torque curve will be changed from curve (1) to curve (2) (Fig. 219). It will be noted that full-load torque is obtained at increased values of slip as the rotor resistance is increased. The *value* of the maximum or break-down torque will not be affected, but the point of maximum torque moves toward the point of zero speed. ($s = 1.0$.) That is, the maximum torque

occurs at a greater value of slip. The rotor now runs at reduced speed, but the reduced speed is obtained at the expense of efficiency, for the I^2R losses in the rotor-circuit are increased.

It is evident that speed control may be obtained by the introduction of resistance in the rotor-circuit. This method of speed control is very similar to the armature-resistance method of speed control in the direct-current motor (see Part I, page 281, Par. 213). The speed-load curve is almost identical with that of the shunt motor with resistance in the armature (Part I, page 281, Fig. 250 (*b*)). The lowering of the speed is accompanied by a material lowering of the efficiency and by poor speed regulation. The electrical efficiency of the rotor is equal to the ratio of actual speed to synchronous speed. For example, at 25 per cent. slip,

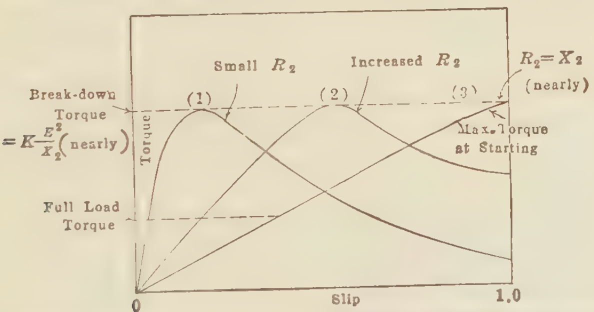


FIG. 219.—Effect on the slip-torque curve of inserting resistance in rotor circuit.

the rotor efficiency is 75 per cent. That is, of the power transmitted across the air-gap, 25 per cent. is lost as heat in the rotor resistance. The other 75 per cent. is converted into mechanical power, although this is not all available at the pulley because of rotor friction and core losses.

If sufficient resistance be introduced in the rotor circuit, maximum torque may be made to occur at standstill, as shown by curve (3) (Fig. 219). That is, break-down torque is obtained at starting.

This increase of torque is due to the increased losses in the rotor-circuit, due to the current flowing through added resistance. The power required to supply these losses must be transferred across the air-gap, causing an increase in the pull between stator and rotor, hence an increase in torque.

An adjustable resistance cannot be placed readily in the squirrel-cage rotor, so that rotors requiring external resistance are usually wound either two or three phase (Fig. 220). The 2-

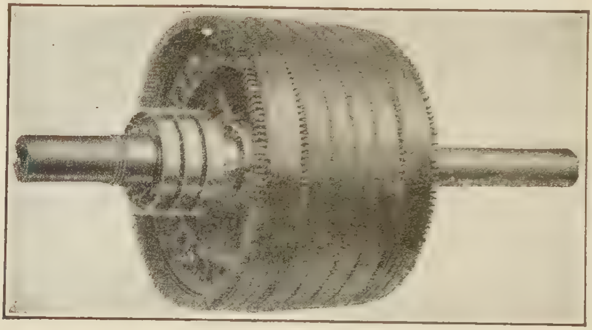


FIG. 220.—Wound rotor of 100-hp., 440-volt induction motor.

phase windings may be connected either star or mesh and the 3-phase windings may be connected either Y or delta. Such rotor windings are in every way similar to stator windings. The

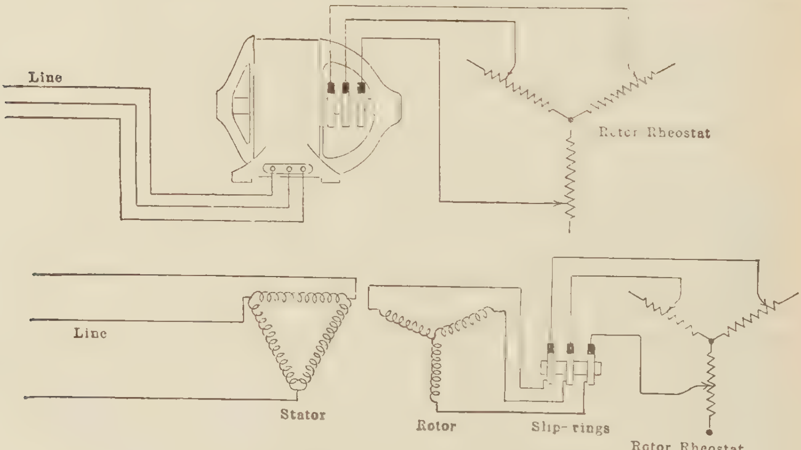


FIG. 221.—Connections for a wound-rotor induction motor.

three ends of the 3-phase winding are brought out to three slip-rings, as shown in Figs. 220 and 221. Brushes, bearing on each of these three rings (Fig. 221) connect to Y-connected external resistances, usually through a controller. The entire resistance

of each phase is in circuit on starting. In addition to producing a very good starting torque, the starting current of the motor does not greatly exceed the rated current. As the motor comes up to speed, the external resistance is cut out. The motor then operates on curve (1) (Fig. 219).

Figure 222 shows a controller and grid resistance such as are used with wound-rotor induction motors. Various sections of the starting resistances are short-circuited by the upper row of finger contacts as the drum is turned toward the running position. The lower contacts are in series with the stator leads and serve to open the circuit when the handle is in the "off" position. When

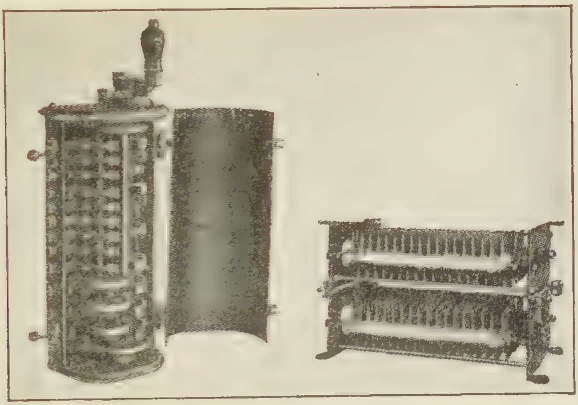


FIG. 222.—Controller and grid resistors used with wound-rotor induction motor. (General Electric Co.)

the handle is thrown to "reverse," two stator leads are interchanged, reversing the direction of rotation.

Even without the controller, the wound-rotor type of motor is more expensive than the squirrel-cage motor, due to the greater cost of winding and connecting the rotor coils. The controller and resistors add further to the cost. In the running position, this type of motor has a greater slip than the ordinary squirrel-cage motor, because it is not possible to secure the very low resistance obtainable with the squirrel-cage winding. As has been pointed out, such external resistance may be used to obtain speed control at reduced efficiency and with poor speed regula-

tion. Hence, this type of motor has better starting characteristics, but poorer running characteristics than the squirrel-cage motor.

Where a rheostat is used for starting duty only, the rotor conductors may be connected to resistance grids within the rotor itself. Such grids can be short-circuited by copper brushes operated by pushing a rod which protrudes from the center of the rotor shaft. Such a rotor is shown in Fig. 223. This type of rotor cannot be operated with the grids in circuit continuously because of the difficulty of dissipating the heat which is developed within the rotor. As the resistors in the rotor burn out frequently, the manufacture of this type of motor has been practically discontinued.

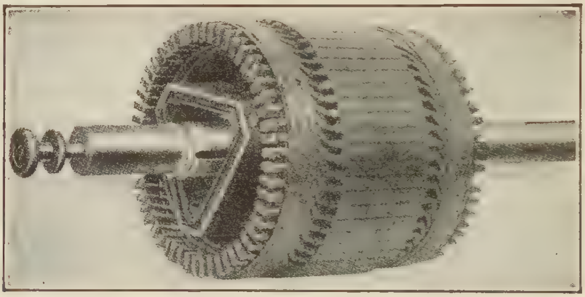


FIG. 223.—Rotor of induction motor having starting resistance within rotor.

Industrial Applications.—Wound-rotor induction motors are used where considerable starting torque is required, and frequently where speed adjustment is desired. Common applications of this type of motor are in cranes, elevators, pumps, hoists, railways, calenders, etc.

Where induction motors are used for railway locomotives, as in the Great Northern Ry. and the Norfolk and Western Ry. the wound-rotor type is necessary.

Another recent use of these wound-rotor induction motors is in the electric propulsion of battleships. The motors are connected directly to the propeller shafts. Two synchronous speeds are obtained by changing the number of poles. Intermediate speeds are obtained by changing the frequency of the generator.

142. Speed Control of Induction Motors.—The speed of the rotor of an induction motor is given by

$$N_2 = \frac{f \times 120}{P}(1 - s) \text{ (page 234, equation (61))}$$

where N_2 is the rotor speed in revolutions per minute, f is the frequency of supply in cycles per second, P is the number of poles and s is the slip.

Obviously, the three factors, frequency, slip, and number of poles determine the speed of the induction motor. In order to change the speed, it is necessary to change at least one of these factors.

Changing the Slip.—The slip may be changed by introducing resistance in the rotor circuit. This has already been discussed in connection with the wound-rotor type of motor. At a given slip, any value of torque up to the break-down torque may be obtained by this method. Its disadvantages are lowered efficiency and poor speed regulation.

These disadvantages may be avoided by introducing counter e.m.fs. instead of resistance in the rotor circuit, either at line frequency, which requires that the rotor have a commutator, or by means of an auxiliary commutating machine which introduces counter e.m.fs. at rotor frequency through slip-rings. This last method necessitates the use of a commutating type of machine to produce e.m.fs. at rotor, or slip frequency, and ordinarily a third machine to drive or be driven by this commutating machine. The investment and complications make this method applicable only to the largest units, such as are used for driving rolls in steel mills.

Change of Frequency.—Commercial power systems operate at constant frequency and it is impossible to control the speed of induction motors by change of frequency when the motors take their power from such systems. In a few special instances, such as in the electric propulsion of battleships, the motors are the only loads connected to the turbo-alternators and it is possible to obtain speed control by changing the speed of the turbines themselves. Even here the range of speed variation is limited, because the efficiency of turbines decreases very rapidly when their speed departs from the speed for which they are designed.

Change of Poles.—By means of a suitable switch, the stator connections may be changed in such a manner that the number of poles is changed. This changes the synchronous speed of the motor and, therefore, the speed of the rotor. If the poles be changed in the ratio of three to two, the winding will probably be designed for two-thirds pitch at the higher speed making it a full-pitch winding for the lower speed. In such a motor the best possible design is not usually obtainable at both speeds. That is, desirable characteristics such as high power-factor, etc., are sacrificed at one speed in order that a reasonably good motor may be obtained at the other speed. Sometimes the stator

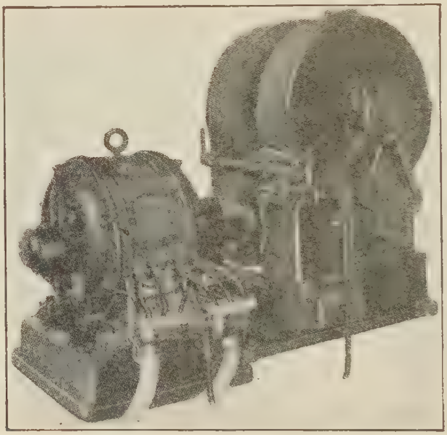


FIG. 224.—Westinghouse two-speed induction motor driving, elevator winding machine.

connections are changed from delta to Y at the same time that the pole connections are changed. This changes the voltage per phase and makes possible a better motor at each speed. Because of the complications involved in changing the connections, it is not desirable to attempt to obtain more than two speeds by changing the number of poles. To avoid these complicated switching connections, induction motors sometimes have two distinct windings, the two windings being connected for a different number of poles. Figure 224 shows such a motor driving an elevator winding machine, a very common application. This

motor has a squirrel-cage rotor with high resistance end-rings, giving higher running slip but much greater starting torque than the low-resistance, squirrel-cage rotor. The 7,500-hp., wound-rotor induction motors used to drive the electrically propelled battleship *Tennessee* also have this type of winding. One winding is connected for 36 poles and the other for 24 poles.

In the electrically propelled battleship *New Mexico*, the stators can be connected for 24 or for 36 poles, giving a speed change of three to two. In wound-rotor types of motors it is necessary to change the rotor as well as the stator connections. Otherwise negative torque will be developed by certain of the rotor conductor-belts.

It will be observed that speed control of induction motors is not easily obtained. The inducton motor is inherently a constant-speed motor.

143. The Induction Generator.—If an induction motor be driven above synchronous speed, the slip becomes negative. The rotor conductors then cut the flux of the rotating field in a direction opposite to that which occurs when the machine operates as a motor. The rotor currents are then reversed with respect to the direction which they had when the machine operated as a motor. By transformer action these rotor currents induce currents in the stator which are substantially 180° out of phase with the inducing currents. They cause a transfer of energy, therefore, from the rotor to the stator, and the machine operates as a generator. The machine is then called an *induction generator*.

In many respects the operation of the induction generator is materially different from the operation of the synchronous generator or alternator.

The rotor must be driven above synchronous speed in order to obtain generator action, and the power output is practically proportional to the slip as it is with the induction motor. Hence, the machine does not have a definite speed for a given frequency as the synchronous alternator has, but the speed with constant frequency varies with the load. Because its speed is not in synchronism with line frequency, the machine is often called an *asynchronous* generator. The frequency and voltage of the induction generator are *that of the line to which it is connected*, irrespective of its speed.

With the induction machine the change from motor to generator action is not unlike the change from motor to generator action of a direct-current shunt machine across constant-voltage bus-bars when the motor speed is raised. The excitation, hence the north and south poles of the direct-current machine, does not change in the transition from motor to generator action, but the direction of current in its armature does change. Likewise the excitation and the north and south poles of the induction machine do not change in its transition from motor to generator action. The excitation is supplied by the polyphase currents in the stator windings which cause the north and south poles to rotate in the gap at synchronous speed. This excitation is obtained from the alternating-current *lines* to which the machine is connected. The rotating north and south poles of the synchronous alternator obtain their excitation from a direct-current source.

This exciting current which comes from the line is a lagging current with respect to the motor current as has already been stated. This exciting current does not change in the transition from motor to generator, but the motor current reverses or changes its phase by 180°. The exciting current, therefore, must *lead* the generator current. Hence, the induction generator delivers only *leading* current. As most loads demand *lagging* current, the induction generator cannot supply these loads, whereas the synchronous alternator can deliver current at any power-factor, leading or lagging current. The induction generator, therefore, must always operate in parallel with a synchronous machine, the synchronous machine supplying not only the lagging current to the load, which the induction machine cannot supply, but the exciting current required by the induction machine as well. These are serious objections to the use of induction generators.

The distinct advantage of the induction generator is the fact that it does not hunt or drop out of synchronism; it is simple and rugged, and when short-circuited it delivers little or no power because its excitation at once becomes zero. Its principal use seems to be in the development of small water powers, where the cost of attendance would prohibit the use of synchronous apparatus. The induction generator connected to the water wheel does not need to be synchronized, requires no direct-current

excitation, and does not fall out of synchronism. It delivers power if there is sufficient water; if not, it merely runs idle as an induction motor. Such machines would feed into a main generating station located in the vicinity and so could be under the occasional inspection of an operator.

The induction generator is also very useful for braking purposes in railway work. If the induction motors be left connected across the line on a down grade, any tendency of the train to drive them above synchronism will be accompanied by generator action. In addition to braking the train, the generators pump power back into the line and so relieve the main generating station of some of its load. The machine requires no complicated control apparatus when used for regenerative braking, such as is required by direct-current motors operating under similar conditions.

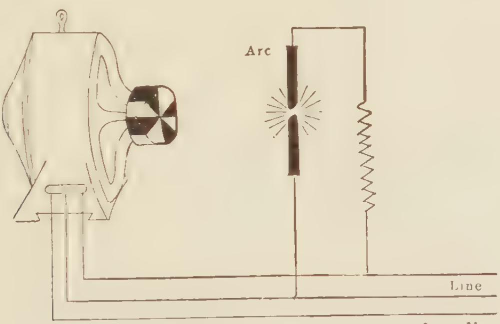


FIG. 225.—Stroboscopic method for measuring slip.

144. Measurement of Slip.—There are various methods for measuring slip. The slip may be determined by measuring the rotor speed and subtracting this speed from that of the rotating field as determined from the frequency. As the slip is but a very small percentage of either the synchronous speed or the rotor speed and is the difference of two nearly equal quantities, it is not possible to determine it accurately by the measurement of each of these quantities and so finding their difference.

A simple method of measuring slip is shown in Fig. 225. A "target" or disc is fastened to the end of the shaft or to the pulley of the motor. This disc has the same number of black

and the same number of white sectors as the motor has poles. This disc is illuminated by an arc lamp which is fed from the motor mains. When the current in the arc is passing through its zero values, the arc emits but little light and during these periods the sectors on the disc are but dimly illuminated. In one-half cycle the armature of the motor would advance one pole if there were no slip. During this time each black sector would advance to the position just occupied by the adjacent black sector which preceded it. The same is true of the white sectors. During the period of advancement the sectors are but faintly visible because the current in the arc is passing through zero. Therefore, each black sector and each white sector is not clearly visible until it has reached the position just occupied by the

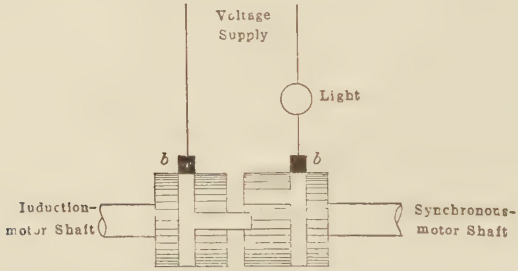


FIG. 226.—Measurement of slip by means of synchronous motor.

sector of the same color just preceding it. As the disc is intensely illuminated twice every cycle, while the arc current is passing through its maximum values, all the sectors are clearly visible twice every cycle. Therefore, if the disc rotated at synchronous speed it would *appear* stationary. Due to the fact that each conductor on the rotor does *not* advance one pole each half-cycle, the sectors will not reach the position of the next adjacent sector of the same color, but will fall short of this distance, due to the slip. The sectors on the disc will then appear not stationary, but will seem to be rotating slowly backward. The number of revolutions per minute that they *appear* to rotate is the revolutions slip of the rotor. Figure 225 shows a stroboscope for a 4-pole machine. Occasionally, black and white stripes are painted on the face of the pulley (Fig. 225), to serve the same purpose.

A mechanical-electrical method of measuring slip is shown in Fig. 226. Two cylinders of insulating material are driven, one by the induction motor shaft and the other by a small synchronous motor having the same number of poles as the induction motor. Each of these cylinders is fitted with a slip-ring, to which a small contact piece is connected. The synchronous motor always runs at the speed of the rotating field. Every time, therefore, the induction motor slips one revolution, the contact pieces touch each other, closing the circuit between the two slip-rings. This is indicated by a flash of the light connected in series with the rings through the brushes *b* (Fig. 226).

In the Electrical Engineering Laboratories at Harvard University, the induction motor and the synchronous motor jointly drive a differential through gears, a method developed in these laboratories. The speed of the differential is the revolutions slip of the induction motor. If desired, the speed of the differential, and hence the slip, may be measured with considerable accuracy with a speed counter. By changing gears, the apparatus is adapted to machines having any number of poles.

145. The Induction Regulator.—Without auxiliary apparatus, it is practically impossible to maintain the proper voltage at all the distribution points of a system, because with a fixed voltage at the station bus-bars, the voltage at the ends of short feeders will ordinarily be greater than the voltage at the ends of long feeders. Owing to the ohmic and reactive drops in the lines, the voltage at the load end of the feeder may vary considerably with the load on the feeder. In order to maintain a more constant voltage at the distribution point, without using an excessive amount of copper, an induction regulator is often connected to each feeder. This maintains the voltage at the distribution point practically constant.

The induction regulator is a transformer having a movable secondary. In this way it closely resembles the induction motor. The general principle of the single-phase type is shown in Fig. 227. An ordinary winding is placed in the slots on the stator and a drum-winding is placed in the rotor slots. When the secondary is in the plane of the primary, the maximum e.m.f. is induced in the secondary, because the mutual inductance of the windings is a maximum when the secondary is in this position. When the

secondary is at right angles to the primary, the primary flux does not link the secondary so that the induced e.m.f. in the secondary is zero. As the mutual inductance of the windings is zero under these conditions, the secondary acts like a choke-coil of very high

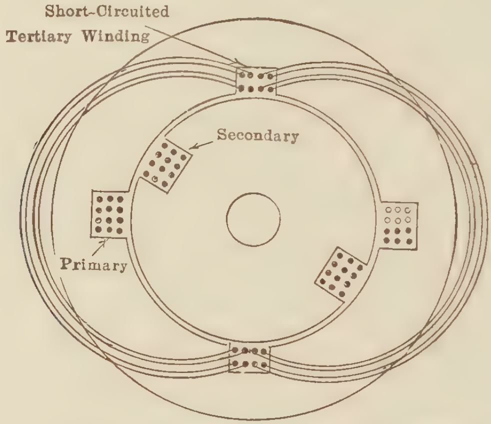


FIG. 227 (*a*).—Single-phase induction regulator.

impedance. To prevent this, a short-circuited tertiary winding is placed on the stator. This acts like a short-circuited transformer secondary, and, therefore, reduces the inductance of the regulator secondary to a very small amount. The primary

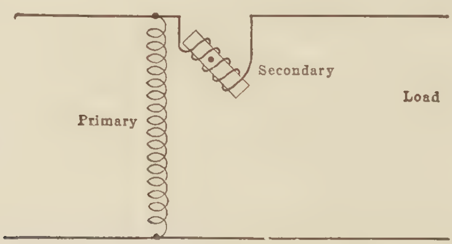


FIG. 227 (*b*).—Connections of a single-phase induction regulator.

winding is shunted across the line (Fig. 227 (*b*)) and the secondary is connected in series with the line. That is, the regulator operates as an auto-transformer. The kilovolt-amperes *transformed* is the product of the secondary current and the secondary voltage which is a small portion of the total load voltage

When the secondary is in the plane of the primary in one position, its induced e.m.f. is a maximum and it is connected to act

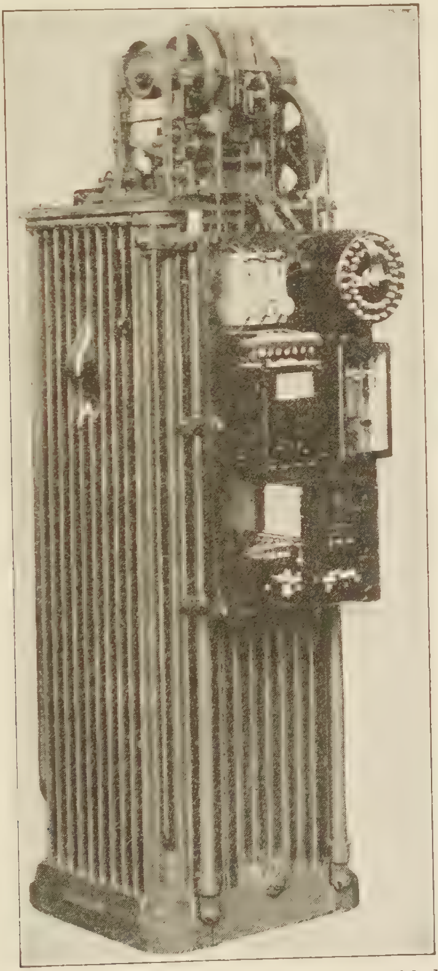


FIG. 228.—General Electric feeder voltage regulator assembled, with panel board.

as a booster. When the secondary is turned 180° from this position, its e.m.f. is also a maximum, but it now bucks the line voltage. Any value of voltage between that corresponding to

these two positions may be obtained by varying the position of the secondary.

The secondary is turned by a small motor, controlled by relays (Fig. 228). The relays are actuated by a contact-making voltmeter. If the voltage is too high, one set of contacts causes the motor to turn in such a direction as to make the secondary reduce the line voltage. If the voltage is too low, another set of contacts causes the motor to reverse its direction and the secondary boosts the line voltage.

The 3-phase induction regulator closely resembles the 3-phase, wound-rotor induction motor. The three stator windings or primaries are connected across the line in either Y or delta. The three secondaries, which correspond to the three phases of a rotor winding, are insulated from one another and each is connected in series with one of the 3-phase lines. As the stator produces a uniform rotating field, the induced e.m.fs. in the secondaries are constant and are independent of the position of the rotor. Their boosting and bucking effect, however, depends upon the phase relations existing between each induced secondary e.m.f. and its respective line voltage.

The 3-phase regulator requires no short-circuited tertiary winding.

CHAPTER IX

SINGLE-PHASE MOTORS

146. Alternating-current Series Motor.—The direction of rotation of either the direct-current shunt motor or the direct-current series motor is the same irrespective of the polarity of the line voltage. If the line terminals be reversed, both the field current and the armature current are reversed, and the direction of rotation remains unchanged. If such motors be supplied with alternating current, therefore, the *net* torque developed acts in one direction only.

With alternating current, the shunt motor develops but little torque. The high inductance of the shunt field causes the field current and, therefore, the main flux to lag nearly 90° in time-phase with respect to the line voltage. The armature current cannot lag the line voltage by a large angle if the motor is to develop any substantial power at a reasonably high power-factor. There will be considerable time-phase difference, therefore, between the flux and the armature current. When the flux is a maximum, the armature current is near its zero value and when the armature current is a maximum, the flux is near its zero value. Hence but little torque wi l be developed under these conditions, and such a motor is therefore impracticable.

In the series motor, however, the same current flows in both the armature and the field. Hence, the flux and the armature current are practically in time-phase with each other. The flux is therefore a maximum when the armature current is a maximum, etc. Consequently, the series motor develops approximately the same torque per ampere with alternating current as it does with direct current. With a few changes, the series motor will operate satisfactorily with alternating current.

Since the flux is alternating, the field structure must be laminated, as well as the armature.

The series-field winding has a comparatively high self-inductance. In order to prevent the voltage-drop across the field

becoming so high as to make the motor impracticable, as few turns as possible are used. The series-field reactance is further reduced by operating at low frequency, 25 cycles being the maximum frequency for which a good motor can be designed.

Owing to the fewer number of turns per pole, it is necessary to make the air-gap of the motor as short as mechanical clearance will permit. Hence, armature reaction is increased (see Part I, page 237, Par. 189). Moreover, the cross-magnetizing flux resulting from armature reaction has the further disadvantage in that it increases the *reactance* of the armature, increasing the voltage-drop in the armature and thereby lowering the power-factor of the motor. The armature reaction is almost entirely

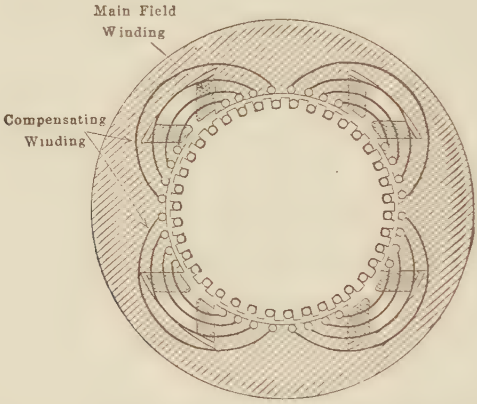


FIG. 229.—Windings of an alternating-current series motor.

suppressed by a compensating winding in the pole-faces (Fig. 229). This winding is ordinarily in series with the motor and is so connected that the current in each slot is opposite in direction and equal in magnitude to the current in the armature slot directly opposite. Hence, the amature m.m.f. is opposed at each point on the armature by a m.m.f. which is opposite in direction and nearly equal in magnitude. The armature reaction is therefore reduced to a minimum. If the compensating winding be short-circuited on itself, it becomes a short-circuited secondary of a transformer, the primary of which is the armature. Hence, its ampere-turns are nearly opposite in phase and equal in magnitude to those of the primary. The motor is then said to

be *inductively* compensated. If the winding is in series with the motor, the motor is said to be *conductively* compensated.

Owing to transformer action between the fields and armature, commutation becomes more difficult than with direct-current motors, and greater refinements in design are necessary.

This type of motor owes its development to the necessity of securing a single-phase motor suitable for railway electrification. Its operating characteristics (Fig. 230) are similar to those of the direct-current series motor. The power-factor decreases with increase of load, whereas the power-factor of most other apparatus

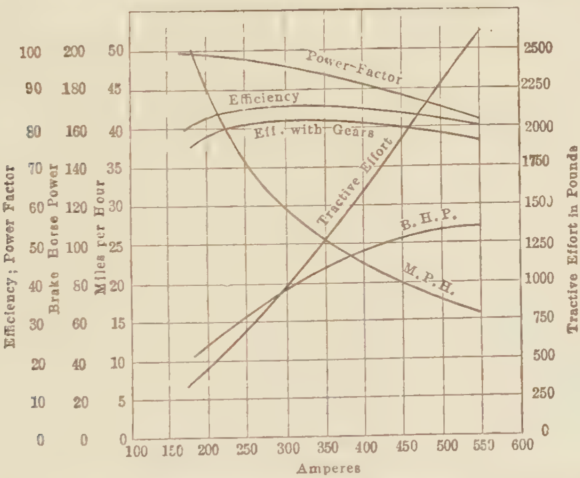


FIG. 230.—Characteristic curves of 430-amp., 235-volt, 25-cycle, single-phase Westinghouse railway motor. Continuous rating, 200 amp., 235 volts.

increases with increase of load (see page 239, Fig. 213). Initially, this motor was designed and developed by the Westinghouse Electrical and Manufacturing Company for the New York, New Haven, and Hartford R.R. From New Haven to Harlem the locomotives take power at 11,000 volts, 25 cycles, from an overhead trolley wire, by means of a pantograph trolley. An auto-transformer on the locomotive reduces this voltage to 250 volts, the rated voltage of the series motors. The electric locomotives run from Harlem into the Grand Central Station, New York City, over the New York Central, 600-volt, direct-current system. The same motors are used for both direct- and alter-

nating-current service, the control devices switching over automatically when transition is made from one to the other. The motors which operate at 250 volts each on alternating current are connected two in series for direct-current operation.

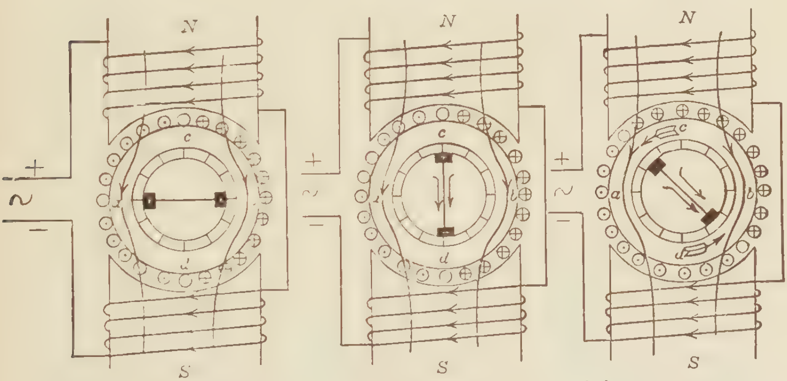
Owing to the many refinements in design and to its greater size per horsepower, this type of motor has been practically limited to the large units which are used in electric locomotives.

147. Universal Motors.—Simple series motors of fractional-horsepower sizes will operate satisfactorily on alternating-current circuits at frequencies as high as 60 cycles. Such motors obviously must have laminated-field structures. The advantages of such motors are that they operate satisfactorily with both alternating and direct current, they have high starting torque, and their speed is readily controlled, usually with series resistance. In order to have high power-factor and output they must run at high speeds, from 4,000 to 10,000 r.p.m. in the smaller sizes. Simple motors are available up to ¼ hp., and compensated motors are available up to ½ hp. Typical industrial applications of this type of motor are sewing-machines, vacuum cleaners, and portable electric drills.

148. The Repulsion Motor.—Figure 231 shows diagrammatically a machine having a 2-pole, drum-wound armature and commutator, such as would be used for a direct-current motor. This armature is placed in a 2-pole magnetic field, the field being connected to a single-phase, alternating-current source. A single-phase flux, varying sinusoidally with time, links the armature winding and induces e.m.fs. in it. That is, the armature is the secondary of a transformer of which the field winding is the primary.

At the instant shown in Fig. 231 (*a*), the current in the upper line is positive and is increasing; the current in the lower line is negative. The field is wound in such a direction that the upper pole is north and the lower pole is south at this instant. Hence, the direction of the flux is downward through the armature. By Lenz's law (see Part I, page 168) the direction of the induced e.m.f. must be such that if current flows, it *opposes* the increase of flux. Therefore, the direction of the induced e.m.f. on the right-hand side of the armature is inward and that on the left hand side is outward. Points *a* and *b* on opposite sides of the armature lie

in the geometrical neutral. Since some coil side at a connects to some coil side at b to complete a coil, the difference in potential between points a and b is equal only to the induced e.m.f. in a single coil. In fact, with a large number of armature slots the potential difference between a and b is practically zero. For simplicity, assume that each commutator connection is carried radially inward to the commutator rather than spirally as is usually done. If the brushes short-circuit commutator segments in the neutral plane (Fig. 231 (a)), no current flows since they short-circuit points between which no potential difference exists. The brushes are in such a position that torque would be developed if current did flow in the armature and brushes, since the currents



(a) No current, no torque (b) Current, but no torque (c) Current, and torque

Fig. 231.—The repulsion motor.

in all conductors under each pole flow in the same direction, and the direction of current under the two poles would be opposite.

Due to the transformer action between field and armature, the maximum potential difference between points in the armature occurs along the pole axis or between points c and d. Therefore, if the commutator be short-circuited along the plane cd, current will flow through the armature. It will behave like a short-circuited secondary opposing the flux. But no torque will be developed since an equal number of ampere-conductors under each pole carry current in opposite directions. The net torque developed is zero.

In diagram (*a*), therefore, the brushes are in a position to develop torque but there is no armature current; in diagram (*b*) there is a large armature current but the brushes are in such a position that the net torque is zero.

If the brushes be moved to some position intermediate between (*a*) and (*b*), as in (Fig. 231 (*c*)), the brushes short-circuit points of the armature between which a difference of potential exists and the direction of the current in the conductors directly under the poles is such that a net torque results, tending to cause rotation, the direction of rotation being counter-clockwise. By changing the position of the brushes so that they are in opposite sides of the plane *cd*, the direction of rotation may be reversed. The brush position for a 4-pole repulsion motor is shown in Fig. 232.

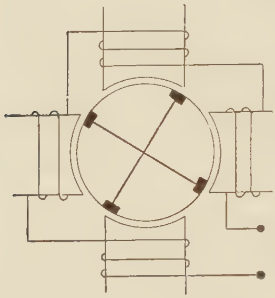


FIG. 232.—Brush position in a 4-pole repulsion motor.

As a rule, repulsion motors do not have salient poles such as are shown in Figs. 231 and 232, which are diagrammatic only. As the reluctance of the air-gap must be kept as low as possible, the motor is made like the induction motor having a stator completely surrounding the rotor, a short air-gap, and a distributed winding. Its characteristics are those of the series motor, namely high starting torque and high speeds at light loads. The simple motor cannot ordinarily be used for continuous duty because of severe sparking, but it can be used for starting certain types of motors which otherwise would have little or no starting torque.

149. Single-phase Induction Motor.—An induction motor may be made to operate with single-phase current applied to its stator, although this single-phase current of itself cannot produce a rotating field. Such a single-phase motor is shown diagrammatically in Fig. 233. At the instant shown the current enters the lower line and is increasing. By the corkscrew rule the upper pole is north and the lower pole is south. The flux ϕ may be assumed to be increasing. The effect of the squirrel-cage rotor is as if each conductor on one side of the armature were connected

with a conductor on the opposite side of the armature to form a closed turn (Fig. 233). Since the rotor is a transformer secondary, the directions of its currents must be such as to *oppose* the increase of flux. As a result, the motor when stationary is a short-circuited static transformer. The magnetic field alternates but always acts along the same axis. As there is no lateral motion of the field, there is no tendency to rotate.

If, however, the rotor, by some means, be caused to rotate in either direction, the rotor conductors will *cut* the flux ϕ and develop a rotational e.m.f. as in a generator. As this e.m.f. occurs in conductors which are short-circuited on themselves, comparatively large currents flow. These currents, acting in conjunction with the stator current, do produce a rotating field, whose direction of rotation is the same as that of the armature. Hence, the motor continues to accelerate in this direction until it reaches nearly synchronous speed. Near synchronous speed it develops considerable torque and will carry load. Like the polyphase motor its slip increases with load and it has a definite breakdown torque. For the same speed and weight the single-phase motor has approximately but 50 per cent. of the output of the polyphase motor. It is slightly less efficient and has lower power-factor.

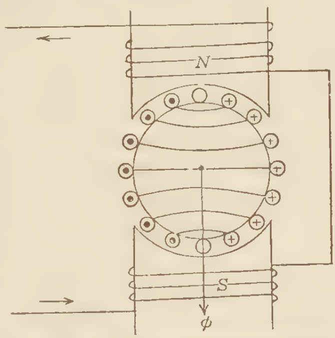


FIG. 233.—Transformer currents in the rotor of a single-phase induction motor.

150. Operation of the Polyphase Motor as a Single-phase Motor.—If one phase of a polyphase induction motor be opened, the motor will operate as a single-phase induction motor, although it will not start under these conditions. The rating and the breakdown torque of a polyphase motor, operating single-phase, are considerably reduced and if rated polyphase load is applied continuously, the motor may overheat.

Ordinarily, in starting a polyphase motor, all three lines are closed when the compensator is in the starting position and the motor starts as usual. When the compensator is thrown to the

running position, however, a phase may become open through the compensator. The motor then operates single-phase and the only indication that it may give of this condition is overheating if the load is near the rated value. It is difficult to locate the open-circuit by a voltmeter or by test lamps, since the motor generates a counter e.m.f. almost equal to the line voltage and the voltage difference across the open-circuit is small. The best test for an open phase is to insert an ammeter in each line.

151. Starting Single-phase Induction Motors.—As the single-phase induction motor is not self-starting, auxiliary means must

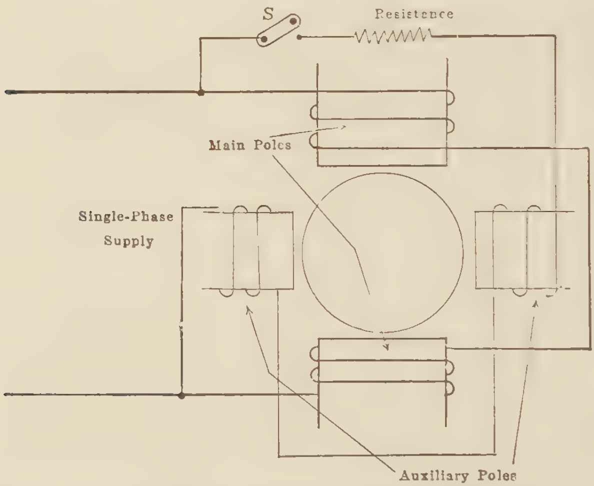


Fig. 234.—Split-phase method of starting a single-phase induction motor.

be used to supply initial torque. One method is to split the phase by combinations of inductance, resistance, and capacitance.

Split-phase Methods.—Figure 234 shows one method of splitting the phase, a 2-pole motor being shown. The main winding, which is highly inductive, is connected across the line in the usual manner. Between the main poles are auxiliary poles which have a high-resistance winding and this winding is also connected across the line. As the auxiliary winding has a high resistance, its current will be more nearly in phase with the voltage than the current in the main winding. For the best conditions, the two currents should differ in phase by 90°, but this

is not readily obtainable, and in fact is not necessary. These two sets of poles produce a sort of rotating field which starts the motor. When the motor comes up to speed, a centrifugal device in the rotor opens the switch S and disconnects the auxiliary winding. The motor then operates with the single-phase pulsating field. Actually, such motors have non-salient poles. The air-gap is uniform and the windings are embedded in slots.

This same starting principle is obviously applicable to motors having more than two poles.

To reverse the direction of rotation the starting winding is reversed with respect to the main winding.

Another method of splitting the phase is to use a 3-phase winding, as shown in Fig. 235, and to connect resistance and

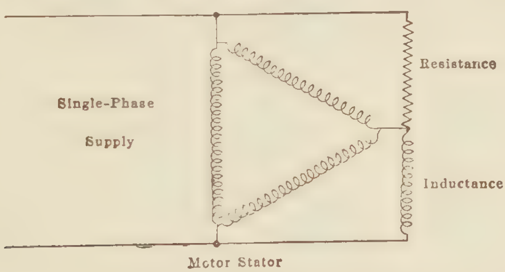


Fig. 235.—Splitting the phase with resistance and inductance.

inductance as shown. Resistance and capacitance may also be used. Either a delta- or a Y-connected stator may be used. The resistance and inductance, when connected as shown, displace the phase relations of the currents in the different phases of the stator with respect to one another and so produce a sort of rotating field. All these phase-splitting devices produce an elliptical rotating field. Because of the characteristics of the field combined with the squirrel-cage characteristics of the rotor, the resulting torque is barely sufficient to start the motor, even without load.

Shaded-pole Method.—The shaded-pole method is shown in Fig. 236. A short-circuited coil of low resistance is connected around one pole tip. When the flux is increasing in the pole a portion of the flux attempts to pass down through this shaded tip. This flux induces a current in the coil which by Lenz's law

is in such a direction as to oppose the flux entering the coil. Hence at first the greater portion of the flux passes down the right-hand side of the pole, as shown in Fig. 236. Ultimately, however, the main flux reaches its maximum value, where its rate of change is zero. The opposing e.m.f. in the shading coil then becomes zero, and later the opposing m.m.f. of the short-circuited coil ceases, the current in this coil lagging its e.m.f. Considerable flux then penetrates the short-circuited coil. After the main flux begins to decrease, the induced current in the shading coil tends to prevent the flux then existing in the shaded portion of the pole tip from decreasing. The flux first reaches its maximum value, therefore, at the right-hand or non-shaded side of the pole, and later reaches its maximum at the left-hand or shaded side. The effect of the shading coil is to retard in time-phase a portion of the flux, so that there is a sweeping of the flux across the pole-face from the right- to the left-hand side in the direction of the shading coil. This flux cutting the rotor conductors induces currents, which in turn produce a torque sufficient to start the motor. The shaded pole is not a common method of starting single-phase induction motors and is used only in motors of very small size.

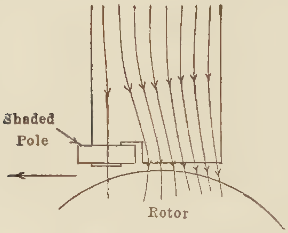


FIG. 236.—Shaded-pole method of starting a single-phase induction motor.

It will be remembered that this same shaded-pole principle is used as the light-load adjustment in the induction watthour meter (see page 94).

Repulsion-motor Starting.—The preceding methods of starting the single-phase induction motor produce very weak starting torques which are insufficient to start the motor except under the lightest loads. The Wagner single-phase induction motor starts as a repulsion motor and has a large starting torque. A cross-section of the motor and a view of the armature are shown in Fig. 237. The armature is similar to the type used in the ordinary direct-current motor, except that the brushes J press on the end of the commutator L rather than radially on its surface. These brushes are short-circuited on themselves and are set in a

position corresponding to those in Fig. 231 (*c*) or Fig. 232 (pages 265 and 266) so that the motor starts as a repulsion motor. It has a large starting torque and comes up to speed rapidly. As

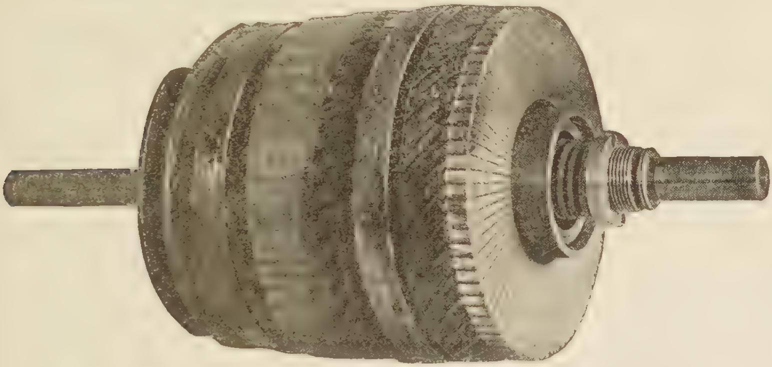


FIG. 237 (*a*).—Rotor of a Wagner single-phase, type BA motor.

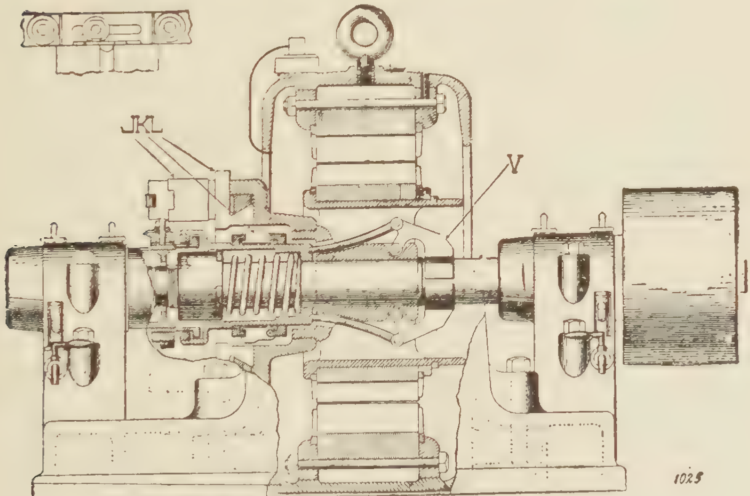


FIG. 237 (*b*).—Wagner single-phase induction motor.

it approaches synchronism, a centrifugal device *V* (Fig. 237 (*b*)) is thrown outward and pushes the brushes away from the commutator, while at the same time a metal ring *K* presses against the commutator bars on the inside and short-circuits them.

The motor now operates as a single-phase induction motor. Since the brushes press on the commutator only during the starting period, there is little deterioration even though there is considerable sparking while they are in service.

Other manufacturers are now employing this method for obtaining large starting torque.

CHAPTER X

SYNCHRONOUS MOTORS AND CONVERTERS: RECTIFIERS

152. The Synchronous Motor.—A direct-current generator, when supplied with electrical energy at its rated voltage, operates satisfactorily as a motor and at the same rating as when operating as a generator. Likewise an alternator, when supplied with electrical energy at its rated voltage and frequency, operates satisfactorily as a motor, and at the same rating as when operating as a generator. When a synchronous machine operates as a motor in this fashion, it is called a *synchronous motor*.

There is no substantial difference in the design details of the direct-current machine, whether it be a motor or a generator. Although as a rule, it is not necessary that there be differences in design details of alternators and synchronous motors, frequently better synchronous motor characteristics are obtained if slight changes are made, such as in the length of air-gap, the field ampere-turns, etc. Alternators can be either of the salient- or non-salient-pole type. Synchronous motors are almost always of the salient-pole type, since this type gives greater stability, particularly when the machine loses its excitation.

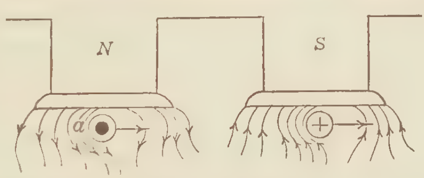


FIG. 238.—Torque developed by a synchronous motor.

153. Principles of Operation.—Figure 238 shows a conductor a under a north pole and carrying a current flowing toward the observer. By the corkscrew rule, the direction of the m.m.f. about this conductor is counter-clockwise. It is, therefore, upward on the right-hand side of the conductor and downward on the left-hand side. The direction of flux from the north pole is downward. Hence, the m.m.f. due to the current in the conductor a opposes this flux on its right-hand side and assists it

on its left-hand side. The flux density, therefore, will be decreased to the right of a and increased to the left of a. Conductor a will then tend to move from left to right (Fig. 238) (see Part I, page 257). If the current be alternating, it will reverse its direction for the next half-cycle and the flux density will be increased to the right of the conductor a and decreased to the left of conductor a. The torque then acts from right to left. The net torque, therefore, over any given number of complete cycles is zero and no continuous motion can result. This is the condition existing in a synchronous motor when at standstill. The armature conductors carry alternating current and the poles have fixed polarity, being excited with direct current. The synchronous motor, therefore, as such, develops no starting torque.

If, however, conductor a can in some manner be brought under the next pole, which is a south pole, for the half-cycle during which the current is in the reverse direction, the flux density is still greatest to the left of the conductor (Fig. 238) and the resulting torque will still be from left to right.

Hence, a tendency toward continuous motion will result. In a synchronous motor, therefore, a given conductor must move from one pole to the next in each half-cycle, if the machine is to operate continuously. This applies to the rotating-armature type of machine. If the machine is of the rotating-field type, any given conductor must be passed by one pole every half-cycle. In any event the synchronous motor must operate at *constant speed*, if the frequency is constant. There may be momentary fluctuations of speed, but if the *average* speed differs by even a small amount from this constant value, the average torque will ultimately become zero and the motor will come to a standstill. The relation of speed, number of poles, and frequency for the synchronous motor is the same as for the alternator and for the rotating field of the induction motor. That is, the speed $S = 120f/P$ r.p.m., where f is the frequency and P the number of poles (see Pars. 9 and 135, pages 14 and 232).

Example.—What is the rated-load speed and the no-load speed of a 200 kv-a., 12-pole, 2,300-volt, 60-cycle synchronous motor?

With constant frequency the speed must be the same at all loads. Hence,

$$S = \frac{120 \times 60}{12} = 600 \text{ r.p.m.} \quad \textit{Ans.}$$

154. The Synchronous Motor as an Elastic Coupling.—The stator of the synchronous motor is wound in the same manner as the stators of the alternator and the induction motor. Hence, if polyphase currents be supplied to the stator, a rotating magnetic field must result. Such a rotating field, having four

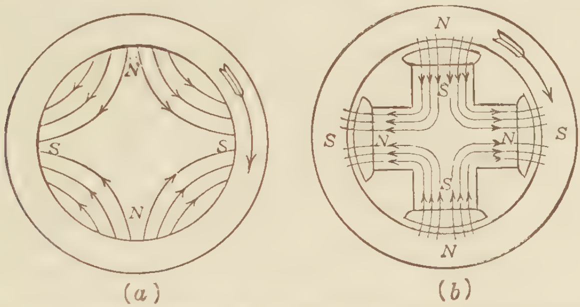


FIG. 239.—Interlocking action of salient poles with a rotating magnetic field.

poles and rotating clockwise, is shown in Fig. 239 (*a*). If a salient-pole rotor having two north and two south poles be placed in this field (Fig. 239 (*b*)), and be brought near synchronous speed, the south poles of the rotor will be strongly attracted to the north poles of the stator and the north poles of the rotor will be attracted to the south poles of the stator. The rotating field of the stator, thus being locked with the rotor poles, will drag the rotor around at synchronous speed. There is, therefore, magnetic or elastic coupling between stator and rotor not unlike the mechanical coupling (Fig. 240). Disc *A*, rotating in a counter-clockwise direction, drives disc *B* through the spring *S* which is connected between two pins, one on each disc.

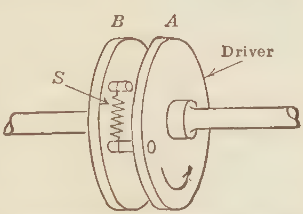


FIG. 240.—Spring coupling analogue of synchronous motor.

So long as the spring coupling remains intact, the disc *B* must rotate at the same speed as disc *A*. Likewise, the rotor of the synchronous motor (Fig. 239 (*b*)) must rotate at the speed of the rotating field so long as the magnetic coupling between stator and rotor remains intact.

155. Effect of Loading the Synchronous Motor.—If a load be applied to a direct-current shunt motor, the speed is slightly decreased. This reduces the back e.m.f., permitting more current to enter the armature and enabling the motor to carry the increase of load (see Part I, page 271).

When load is applied to a synchronous motor, its *average speed* cannot decrease since the motor *must* operate at constant speed. Its field does not change appreciably, hence its back e.m.f. remains substantially constant in magnitude. The synchronous motor, therefore, cannot draw increased current from the line because of a reduction in speed and, hence, of counter e.m.f., as the shunt motor does.

If increased load be applied to the driven disc B (Fig. 240), its speed will not change, except momentarily, since it must always run at the speed of A. An angular displacement will occur, however, between the discs B and A due to the increased elongation of the spring. When load is applied to the rotor of a synchronous motor, this same effect occurs. An angular displacement between the rotor and the rotating field occurs. Figure 241 (a) shows the motor rotating in a clockwise direction and without load. The line ab at the pole-center of the north pole N of the stator rotates synchronously with it, the rotating field being produced by the armature or stator ampere-conductors. The pole-center of the S-pole of the rotor which is locked with N is shown by the arrow c. This pole-center coincides with the line ab.

Now apply a load torque T to the shaft (Fig. 241 (b)). The position of the line ab is not changed appreciably by this load, since its position is determined by the N-pole produced by the stator ampere-conductors. The rotor, however, is pulled back an angle α from the line ab or from the position that it would occupy if load had not been applied. The stretching of the lines of force in the gap (Fig. 241 (b)) tends to pull the rotor back to its no-load position. It is obvious then, that the rotor still continues to operate at its synchronous speed, but the position of the rotor is slightly behind the position which it would have had were load not applied. That is, there is a flexible magnetic coupling between the rotor and the rotating stator field not unlike the spring coupling between the discs A and B (Fig. 240). Thus,

the synchronous motor does not run at reduced speed when load is applied to its shaft, but its rotor shifts its phase backward. When the rotor shifts its phase backward, it changes the phase of the back e.m.f. with respect to the terminal voltage, and this phase shift allows more current to enter the armature and thus to carry the load. This backward shift of the rotor is readily observed by means of a stroboscope (see page 255).

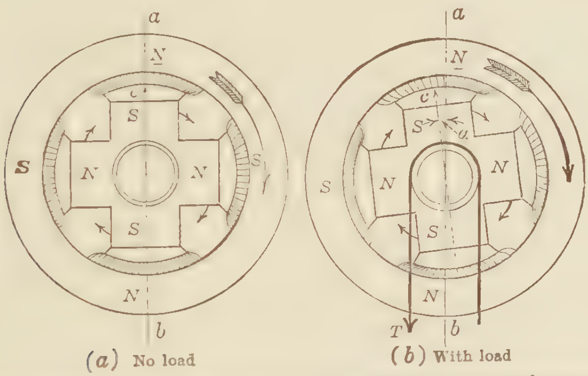


FIG. 241.—Effect of applying torque to the shaft of a synchronous motor.

156. Overexcitation.—When the field of a direct-current shunt motor is strengthened, there is a temporary increase in the armature induced e.m.f. This decreases the armature current and the torque is lowered, since the change in armature current is much greater than the corresponding change in the field. As a result, the motor slows down until its counter e.m.f. drops to a value that will allow the flow of sufficient current to carry the load.

When the field of a synchronous motor is increased, the motor cannot slow down, except momentarily, for it must run at constant average speed. Since its speed is constant, its back e.m.f. must increase when the field is strengthened. It might seem that the motor would stop, for its induced e.m.f. must apparently become greater than its terminal voltage. In the direct-current motor, an induced e.m.f. exceeding the terminal voltage would mean generator action with the result that the machine would cease to operate as a motor.

The synchronous motor, however, may operate as a motor and at the same time its back e.m.f. may exceed its terminal voltage in magnitude. Under these conditions, the motor is said to be *overexcited.* Two reactions occur which enable the motor to operate with an overexcited field.

First, the motor takes a *leading* current. A leading current in a synchronous motor *weakens* the field. This is illustrated in Fig. 242, which shows a motor coil moving from left to right. When its axis is in the position Y, shown dotted, the coil sides are under the centers of the poles and the induced e.m.f. is a maximum. By Fleming's right-hand rule the direction of the *induced* e.m.f. is inward in the coil-side under the north pole. As the *terminal* voltage is practically in phase opposition to the induced

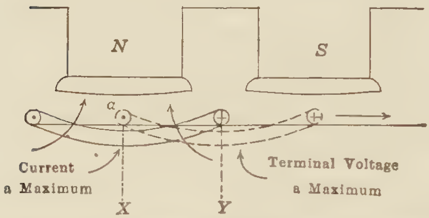


FIG. 242.—Demagnetizing effect of leading current on the field of a synchronous motor.

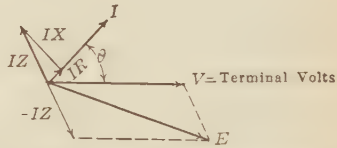


FIG. 243.—Induced armature voltage greater than terminal voltage when synchronous motor current leads terminal voltage.

e.m.f., it acts at this instant inward in the right-hand side of the coil and outward in the left-hand side, as shown.

If the current leads this terminal voltage by 90° it will reach its maximum value one-fourth of a cycle ahead of the voltage, or at a time when the axis of the coil is in position X. It will be observed that for this position of the axis, the ampere-turns of the coil act in direct opposition to those of the N-pole. The effect of the leading current, therefore, in the synchronous motor is to weaken the field. In other words, *on overexcitation the armature reaction tends to annul the effect of the increased field current.*

The second effect is illustrated by the vector diagram (Fig. 243). V is the terminal voltage and I is the armature current

leading V by an angle θ. The resistance-drop IR in the armature is laid off in phase with the current I and the reactance-drop IX in the armature is laid off at right angles to the current I and leading, in the usual manner. The impedance-drop IZ is the vector sum of IR and IX. The voltage E, necessary to balance the back e.m.f., is found by subtracting IZ vectorially from V, just as in the shunt motor the component of terminal voltage which is necessary to balance the back e.m.f. is found by subtracting the IR drop from the terminal voltage.

To subtract IZ from V, $(-IZ)$ is added to V. It will be noted that the e.m.f. E is numerically *greater* than the terminal voltage V. That is, by taking a leading current, the synchronous motor is able to operate with an induced e.m.f. *greater* numerically than the terminal voltage, a condition which is not possible with direct-current motors.

157. Underexcitation.—When the field of a direct-current shunt motor is weakened, the motor speeds up until its back e.m.f.

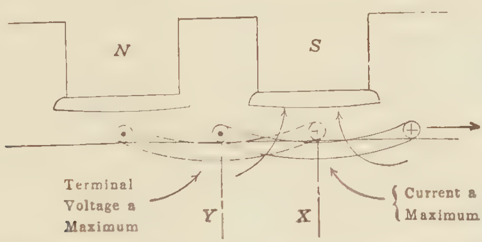


FIG. 244.—Magnetizing effect of lagging current on the poles of a synchronous motor.

reaches a value which gives the proper armature current for the particular load condition.

When the field of a synchronous motor is weakened, it cannot speed up permanently for it must run at a constant average speed. It takes a *lagging* current, however. This current has two effects.

Figure 244 shows a coil, dotted, whose axis is in position Y. In this position the coil sides are opposite the centers of the pole-faces and the back e.m.f. is, therefore, a maximum. The terminal voltage, which is nearly 180° from the back e.m.f., has its maximum value also for this position of the coil, its

direction being indicated in the dotted coil. If the current is lagging the terminal voltage by 90°, it will not reach its maximum value until the coil axis reaches position X. The current under these conditions is in such a direction as to strengthen the S-pole. In a synchronous motor, therefore, a lagging current *strengthens* the field through the effect of armature reaction. Hence, *on underexcitation the motor takes a lagging current which strengthens the field by armature reaction and tends to annul the effect of the weakening of the field.*

When the field of the motor is weakened, the motor has not sufficient excitation. It must, therefore, take some of its excitation from the alternating-current lines by means of a lagging current. This is similar to the induction motor which, however, takes *all* its excitation from the alternating-current line by means of its lagging exciting current.

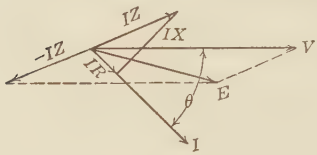


FIG. 245.—Induced armature voltage less than terminal voltage when synchronous motor current leads terminal voltage.

Figure 245 shows the vector diagram when the motor takes lagging current. The IR and IX drops are laid off with reference to the current in the usual manner and the IZ drop obtained. When $(-IZ)$ is added to V, however, E, which is opposite and equal to the back e.m.f., becomes numerically much less than V. That is, the phase shift of the IZ drop is in such a direction that the machine runs as a motor with a very considerably reduced back e.m.f.

The synchronous motor with salient poles will usually operate even if the field current is reduced to zero. If the salient-pole rotor (Fig. 239(b), page 275) be without excitation and brought near the speed of the rotating stator field, the magnetic lines from the stator will attempt to make the rotor take such a position that the magnetic reluctance is a minimum or the flux is a maximum. In order to accomplish this result, the pole-pieces of the rotor when running become locked in with the poles produced by the stator winding (Fig. 239 (b)). These rotating stator poles pull the salient poles of the rotor around with them and in this manner enable the motor to carry a limited load without direct-current excitation. Although the motor may carry a limited

load without any direct-current excitation, its power-factor will be very low and the current will be lagging, which is undesirable.

158. Power-factor of the Synchronous Motor.—The power-factor of the *induction motor* for a given load cannot be altered without changing the motor design and the ordinary induction motor always takes a lagging current. The power-factor of the *synchronous motor* can be altered at will, and the current can be changed from lagging to leading by simply changing the field excitation.

The connections for testing a 3-phase synchronous motor are shown in Fig. 246, a polyphase wattmeter being used to measure the input. If the load be kept constant, and the field current

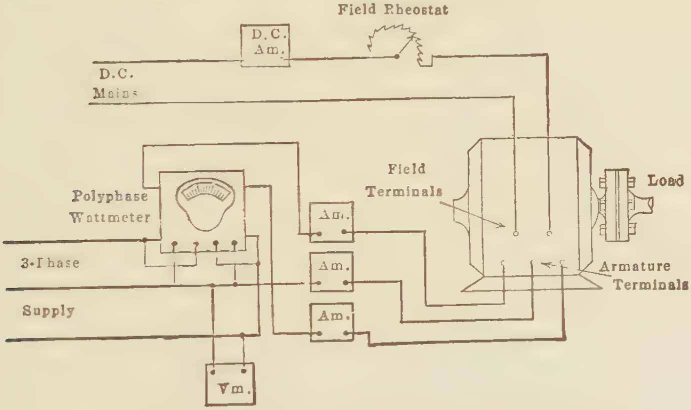


FIG. 246.—Connections for testing a synchronous motor.

be varied, it will be found that for small values of field current the motor takes a large armature current (Fig. 247) and the power-factor is low. As the field is strengthened, the armature current decreases and reaches a minimum at a. For values of field current less than the value given at a the motor is *underexcited* and takes a *lagging* current (Par. 157). At a, the minimum value of armature current, the power-factor is unity. The corresponding value of field current is the *normal* value. For values of field current exceeding a the motor is *overexcited* and takes a *leading* current (Par. 156). Thus, it is seen that with underexcitation the motor behaves like an inductance in that it takes lagging current. With overexcitation the motor behaves

like a condenser in that it takes leading current. The fact that the power-factor of the synchronous motor may be varied over wide ranges at will makes it very useful for regulating the power-factor of power systems.

The connections (Fig. 246) may also be used if a brake test of the motor is to be made. It is obvious that the torque of a synchronous motor is directly proportional to its output since the speed is constant. The current and power-factor depend on the field current. The load characteristic is often determined with the field-rheostat set to give normal field current or minimum armature current at no load.

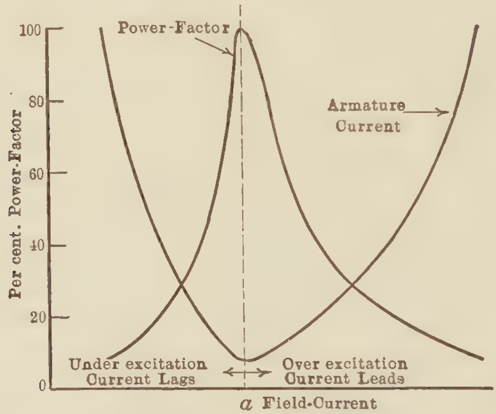


FIG. 247.—Phase characteristics of a synchronous motor.

159. The Synchronous Condenser as a Corrector of Power-factor.—The fact that the power-factor of the synchronous motor may be varied at will over wide ranges makes it useful on power systems, since by its use the system power-factor may be improved. Improved power-factor is so important that synchronous motors are often connected to power systems for the sole purpose of controlling the power-factor. That is, the motors operate without mechanical load. When the machine is so used it is called a *synchronous condenser*.

The electric power load in most factories and mills consists chiefly of induction motors. Although the power-factor of the induction motor is moderately high near its rated load, the fact

that many of the motors in a mill are usually lightly loaded may bring the system power-factor to such low values as 0.5 and 0.6 (see Fig. 213, page 239). Such low power-factors are undesirable since they cause poor regulation of the generating apparatus, transformers, and lines. Also, since low power-factor requires more current for the same power, the generators, transformers, and lines must have greater kilovolt-ampere capacity, and the system investment charges are increased.

This low power-factor which is due to the load taking lagging current, may be improved by the use of a synchronous condenser connected in parallel with the load and taking a leading current. The power-factor may even be brought to unity. A load (Fig. 248) which may consist of induction motors, takes a current I at voltage V. As the current I lags the voltage V by $\theta_1°$, the load power-factor is $\cos \theta_1$. This may be a single-phase load or it may be one phase of a 3-phase system. Resolve the load current I into two components at right angles to each other. One component I_1 is in phase with the voltage V and is called the *energy current.* The power

$$P = VI \cos \theta_1 = VI_1 \tag{62}$$

since $I_1 = I \cos \theta_1$.

The power is equal to the product of the voltage and the energy current. The component I_2 at right angles to V can contribute no power, since the net power is zero when the current and voltage are in quadrature (see page 59). The current I_2, therefore, is called the *quadrature* or *wattless* current. I_2 can contribute no power since the entire power is represented by equation (62).

It follows that if the quadrature current were eliminated or if it were neutralized by a leading quadrature current, the power-factor of the system would be raised to unity and the net power of the system would remain unchanged. By means of an over-excited synchronous condenser, connected in parallel with the load (Fig. 248), a leading quadrature current I_s is obtained equal to the lagging quadrature current I_2. (Since the synchronous condenser losses are small, the current I_s may be assumed as leading V by 90°.) The total current I_1 supplied to the system must be the vector sum of I and I_s. I_1 is in phase with V and

the power-factor of the system has been raised to unity by means of the synchronous condenser.

Example.—The power-factor of a 150-kw., 600-volt, 60-cycle single-phase load is 0.65, the current lagging. (*a*) Determine the energy current and the quadrature current of this load. (*b*) What rating of synchronous condenser is necessary to raise the system power-factor to unity?

(*a*) Cos $\theta = 0.65$ $\theta = 49.5°$ sin $\theta = 0.760$ (page 411). The total load current (see Fig. 249)

$$I = \frac{150{,}000}{600 \times 0.65} = 385 \text{ amp.}$$

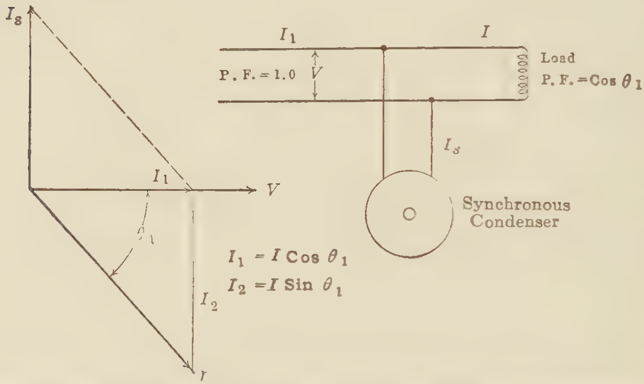


FIG. 248.—Raising power-factor to unity by means of synchronous condenser.

The energy current, $I_1 = I \cos\theta = 385 \times 0.65 = 250$ amp. Also, since the power

$$P = EI \cos\theta \text{ and } I_1 = I \cos\theta,$$

$$I_1 = \frac{P}{E} = \frac{150{,}000}{600} = 250 \text{ amp. (\textit{check}).}$$

The quadrature current

$$I_2 = 385 \sin\theta = 385 \times 0.760 = 293 \text{ amp.}$$

(*b*) The synchronous condenser must be able to take a leading current of 293 amp. at 600 volts. Hence, its rating in

$$\text{Kv-a.} = \frac{600 \times 293}{1{,}000} = 175.8. \quad \textit{Ans.}$$

The vector diagram is given in Fig. 249. The kilovolt-ampere rating of the synchronous condenser here used to obtain unity power-factor is greater than the kilowatts required by the load. It is often more economical to use a smaller unit and obtain power-factors of the order of 0.9 rather than unity. At the present

time *static condensers,* made of foil and paper immersed in oil, also are used to improve power-factor (see Part I, page 196).

It is obvious that a synchronous motor may deliver mechanical load and yet operate overexcited to improve the power-factor.

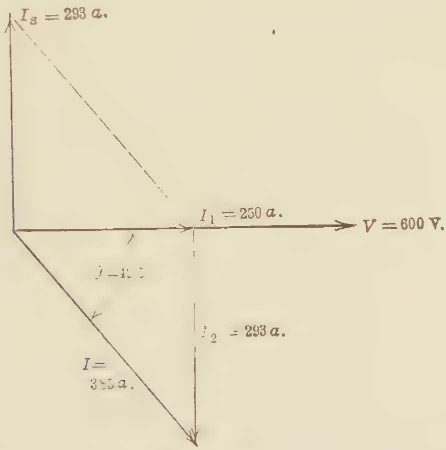


FIG. 249.—Power-factor of a 150-kw. load raised to unity with a synchronous condenser.

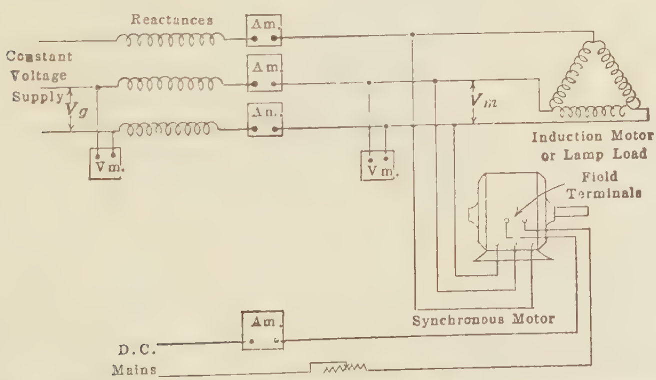


FIG. 250.—Synchronous motor for controlling voltage at end of transmission line.

160. The Synchronous Condenser as a Regulator of Voltage. If an induction motor or a lamp load (Fig. 250) be supplied from a constant-voltage supply V_g through inductive reactances

the voltage V_m at the load will be considerably less than V_g due to the excessive voltage-drop in the reactances and to the fact that this drop is nearly in phase with the voltage V_g. If, however, a synchronous motor be connected in parallel with the load, it will be found that by overexciting the motor, thus causing it to take a leading current, the voltage V_m may not only be made to equal V_g but may actually be made to exceed V_g. The synchronous motor or condenser, therefore, may be utilized to control the voltage at such a load (see page 315).

Overhead transmission lines have considerable series inductive reactance (see page 312). Therefore, by connecting synchronous motors (or condensers) at the receiving end of the line and varying the field excitation of these motors (or condensers), the voltage at the load may be controlled. The voltage at the load may readily be made to exceed the voltage at the sending end of the line, a condition which is not possible with direct current without the introduction of energy between generator and load.

161. Amortisseur or Damper Windings.—Figure 251 shows the rotating field structure of a synchronous motor, around which a squirrel-cage winding is built. The conductors of the squirrel cage are embedded in the pole-faces of the rotor. Such windings are called *amortisseur* or *damper* windings, or simply *dampers*. They assist the motor in starting and they damp out any tendency of the rotor to oscillate or "hunt." Hunting usually results from pulsations in the power supply which may be caused by the variable torque of engine-driven units or by hunting of turbines themselves due to oscillations of their governors. Moreover, dampers tend to prevent the motor falling out of synchronism on account of line disturbances caused by short-circuits, switching, etc.

The action of the damper winding involves the principle of both the induction motor and the induction generator. So long as the rotor is rotating at synchronous speed, the rotating field of the armature or stator does not cut the dampers and they have no effect. Assume that the rotor slows down momentarily. For an instant the rotating field due to the armature m.m.f. is rotating faster than the field structure. This is equivalent to the rotor slipping temporarily, and currents are induced in the dampers. That is, induction-motor action results and the cur-

rents in the dampers are, therefore, in such a direction that they tend to pull the rotor back again toward its normal position.

Again, if the field poles for some reason swing ahead of their normal position, the dampers cut the rotating field in the opposite direction or the slip becomes negative, temporarily. Induction generator action follows, putting a load on the rotor and tending to slow it down. These dampers, therefore, always tend to pull the motor back into synchronism and thus prevent hunt-

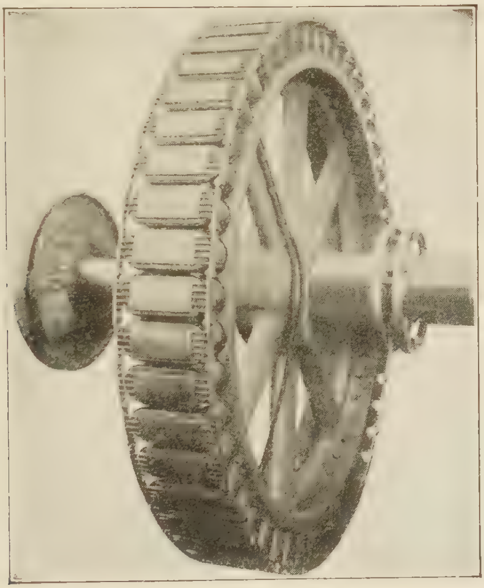


FIG. 251.—Rotor of a 600-volt, 60-cycle, 48-pole synchronous motor showing amortisseur winding.

ing. Such windings are often used on alternators, particularly of the engine-driven type, to prevent hunting when operating in parallel.

162. Starting the Synchronous Motor.—It has been pointed out that the *synchronous* motor is not self-starting. It must first be brought nearly or actually to synchronous speed before it can operate. There are several methods of accomplishing this.

The motor may be brought up to speed by external means and synchronized like an alternator, after which it operates as a

motor. The direct-current exciter is sometimes used to bring the motor up to synchronism if sufficient direct-current power is available. Likewise, if the synchronous motor is connected to a direct-current generator, this generator may be operated as a motor to bring the synchronous motor up to synchronism. After the motor is synchronized, the field of the direct-current machine is strengthened and it then acts as a generator, taking mechanical power from the synchronous motor.

The synchronous motor may start as an induction motor. First, the field circuit is opened. A polyphase alternating voltage is then impressed on its stator and a rotating field is, therefore, set up about the rotor. As a rule, it is desirable to use a compensator so that reduced voltage is applied to the stator windings. The rotating field sets up currents in the pole-faces of the rotor and in the amortisseur winding as well, if such exists. This is obviously induction-motor action. Because of the comparatively long air-gap and the imperfectly conducting paths in the pole-faces, the motor develops little starting torque with large armature current.

However, the starting torque, though small, is usually sufficient to start the machine, which then accelerates until it is at or near synchronism. Before the compensator is thrown into the running position, the field switch is usually closed, so as to minimize disturbances to the system. If the rotor is slipping slightly, it will usually pull into synchronism when the field switch is closed, the field poles locking in with the poles produced by the armature m.m.f. (Fig. 239, page 275).

The motor may pull into synchronism before the field circuit is closed. As the motor approaches synchronism, the salient poles lock in with the poles of the rotating stator field (Fig. 239).

When the field circuit is closed, it may excite the motor poles so that their polarity is opposite to that produced by the rotating field, *i.e.*, by the armature reaction (Fig. 239). The rotor is then thrown back one pole, or in other words, it slips a pole. This may cause considerable disturbance to the system and, for this reason, the field is usually closed when the compensator is in the starting position. This difficulty may be avoided by applying a weak direct-current field to the motor as it approaches synchronism. This causes the armature reaction to act in con-

junction with the direct-current field windings, and the poles then come into synchronism with the same polarity as will be produced by the direct-current excitation. After the motor has pulled into synchronism, it is necessary merely to strengthen the direct-current field to the desired value. The starting compensator may then be thrown quickly into the running position.

When voltage is first applied to the synchronous motor, there may be a very high voltage induced in the field winding. The stator acts as the primary of a transformer, a primary having comparatively few turns. The flux produced by the stator or primary cuts the field winding at synchronous speed, and as the field has a very large number of turns, a very high e.m.f. is induced in the field. This e.m.f. may be sufficiently high to puncture the field winding. The field winding, therefore, should be insulated for voltages considerably in excess of that which normal operation requires. The field is sometimes short-circuited, or is shunted by a resistance when starting, in order to decrease this high voltage. The e.m.f. induced in the field winding decreases as the rotor comes up to speed, until at synchronism it becomes zero.

163. Industrial Applications of the Synchronous Motor.—Single-phase synchronous motors are rarely used in practice. Like the single-phase induction motor, the direction in which they rotate is determined by the direction in which they are started. Unlike the polyphase synchronous motor, they will not start by induction-motor action but must be brought up to speed by other means. Polyphase synchronous motors are commonly used.

The inherent disadvantages of the synchronous motor are that it requires a direct-current supply for its excitation, its starting torque is very small, and the motor is very sensitive to system disturbances and may fall out of step when these occur. On the other hand, the ease with which its power-factor can be controlled is a distinct advantage, often outweighing all the disadvantages. The fact that its speed is constant is of little moment, since induction motors, especially in the larger sizes, have only 1 or 2 per cent. speed regulation.

The synchronous motor is used only in the larger sizes where the cost of attendance per kilovolt-ampere is low. Moreover, it

should not be used where there are sudden applications of the load, as it may drop out of step under such conditions. An important field of use is in connection with motor-generator sets where a large unit is required. A few such motors, situated at various points in a large system, may make it possible to operate the generating station and many of the transmission lines and substations at high power-factor, in spite of low power-factor of the consumers' loads. Another important use of synchronous motors is their application to the driving of compressors, an illustration of which is shown in Fig. 252.

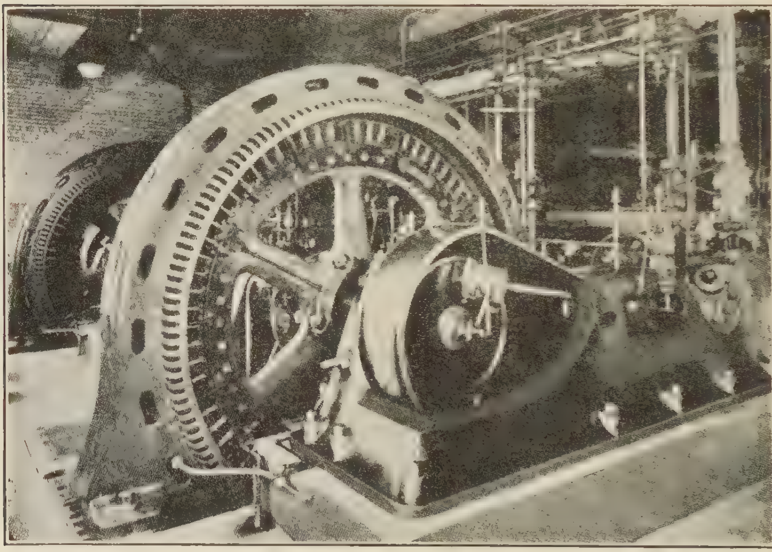


Fig. 252.—Two 160-hp., 100-r.p.m. synchronous motors, direct-connected to ammonia compressors. (Electric Machinery Mfg. Co.)

Synchronous motors are also coming into use for the electric propulsion of cargo and merchant ships. Such ships, when under way, operate at a constant speed and the constant-speed characteristic of the synchronous motor is not a disadvantage, therefore. As such motors can be operated at unity power-factor, the weight of motor, generator, and connecting leads is smaller than when induction motors are used.

164. Synchronous Motors of Very Small Size.—Because of their absolutely constant-speed characteristics, synchronous motors are very useful for driving such devices as must be held in absolute synchronism with the supply frequency. Such uses involve the measurement of slip in the induction motor (see page 255), the driving of oscillograph mirrors, stroboscopic devices, electrical clocks, mechanical rectifiers, etc.

As the power required of such motors is extremely small and the matter of low power-factor is of no moment, they are often made to operate without direct-current excitation. In Fig. 253, is shown a 4-pole motor of this type The 4-pole rotating member consists of a cruciform-shaped piece of iron with the spaces filled with wood to make it cylindrical. The stator is made up of U-shaped laminations and is excited from the alternating-current supply.

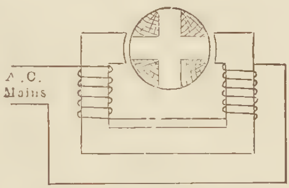


FIG. 253.— Miniature 4-pole synchronous motor.

When the rotating member is brought up to speed, two diametrically opposite armature poles are attracted to the stator poles as the flux is increasing. Because of the inertia of the rotating member, it continues to rotate when the flux is passing through zero. The next pair of poles are then attracted by the flux as it increases in the opposite direction. Such a motor will, therefore, run at constant speed, provided the frequency is constant.

SYNCHRONOUS CONVERTER

It has been stated that for economic reasons a large proportion of electrical energy is generated and transmitted as alternating current (see Chap. I). Direct current, however, is necessary for many purposes, such as charging batteries, electrolytic work, power, lighting service in populous cities, street railways, etc. While there are several methods of obtaining direct current from alternating current (see page 302), at present there are only two practicable methods of obtaining direct current from alternating current on a large scale, namely, motor-generator sets and synchronous converters. The synchronous converter has

advantages over the motor-generator set in that it is a single machine having one armature, one field, two bearings, etc.

165. Principle of the Synchronous Converter.—It has already been demonstrated that alternating current is generated in the individual armature coils of the ordinary direct-current generator. If taps be brought out properly from the armature winding to slip-rings, alternating current may be taken from this same winding and the machine becomes an alternator. If alternating current be supplied to the slip-rings the machine will obviously operate as a synchronous motor.

With the exception of slip-rings, the construction of the converter is the same as that of direct-current machines (also see

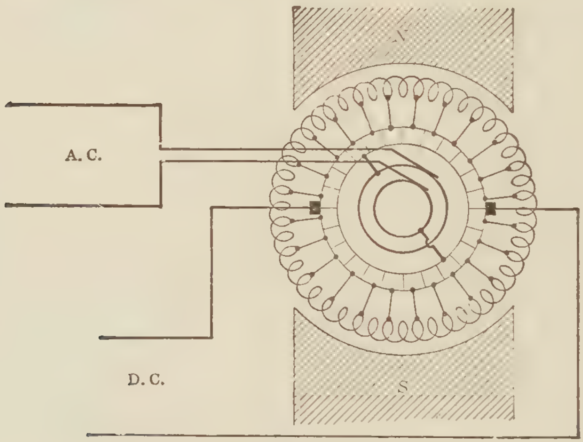


FIG. 254.—Two-pole, single-phase, synchronous converter.

page 329, Fig. 286), although the relative dimensions may differ to some extent. The converter has fixed poles, a rotating armature, a commutator, a shunt field, and usually a series field.

In the synchronous converter, as commonly used, alternating current is supplied to the slip-rings and direct current is taken from the commutator and brushes. Direct current may be supplied to the brushes and commutator and alternating current taken from the slip-rings. The machine is then said to be an *inverted converter*. If, however, the direct-current brushes be open-circuited or removed, the machine becomes, under these conditions, a synchronous motor of the rotating-armature type.

On the other hand, if direct current be supplied to the brushes and commutator, and the slip-ring brushes be disconnected, the machine becomes a shunt or compound motor.

If the machine be driven mechanically and current be taken from the slip-rings only, it becomes an alternator. On the other hand, if current be taken from the commutator only, it becomes a direct-current generator. Both alternating and direct current may be taken from it simultaneously, and it then becomes a *double-current* generator.

The connections of a 2-pole, single-phase converter are shown in Fig. 254. In a 4-pole converter it would be necessary to have two taps from opposite points in the winding to each slip-ring, making four taps in all. A 4-pole, 3-phase converter would have two taps to each slip-ring, or six taps in all, etc. The taps are not necessarily taken from the commutator segments directly, but may be taken from the coils themselves.

The converter owes its economy to the fact that a motor current and a generator current flow in the armature conductors simultaneously. Since the motor current and the generator current flow in opposite directions, the net current is small, being much less than either the direct or the alternating current alone. Hence, the commutator is proportionately larger in converters than in direct-current machines.

166. Polyphase Converters.—The rating of a given machine when operating as a converter increases much more rapidly with increase in the number of phases than with other types of machinery. This is shown by the following table:

EFFECT OF NUMBER OF PHASES AND OF POWER-FACTOR ON THE OUTPUT OF A SYNCHRONOUS CONVERTER

Number of phases	P.F. = 1.0	P.F. = 0.9
1	0.85	0.74
D. C.	1.00	1.00
3	1.33	1.09
4	1.65	1.28
6	1.93	1.45
12	2.18	1.58

It will be noted that the output when operating six phase is more than double the output when operating single phase and is 17 per cent. greater than when operating three phase. Converters, therefore, are ordinarily operated six phase, since it is a comparatively simple matter to transform from three to six phase (see page 128). Twelve-phase operation is seldom used owing to the transformer and wiring complications.

It is also to be noted in the foregoing table that the rating of converters decreases very rapidly with decrease in power-factor. The decrease in rating is very pronounced even at a power-factor no lower than 0.9. Hence, for maximum economy, synchronous converters should operate near unity power-factor.

167. Voltage and Current Ratios in Converters.—It is important to know the ratio between the voltage and current at the direct-current side of the converter to the voltages and currents at the alternating-current side.

Both the direct-current voltage and the alternating-current voltage are produced by the same conductors cutting the same flux at the same speed. Hence, *fixed ratios* exist between the *induced* direct-current voltage and the *induced* alternating-current voltage. The terminal-voltage ratio may differ somewhat from these fixed ratios, due to the impedance- and resistance-drops in the armature.

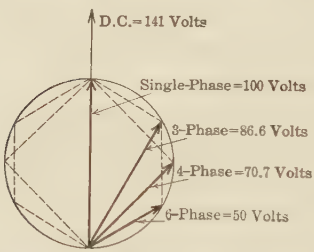


Fig. 255.—Relations existing among voltages in a synchronous-converter armature.

There are the same number of series conductors between the brushes as between single-phase slip-ring taps. The direct-current e.m.f. is the sum from instant to instant of the e.m.fs. induced in all the series-connected conductors between brushes. The alternating-current e.m.f. when it is at its maximum value is equal to the sum of the e.m.fs. induced in all the series conductors between slip-ring taps at that instant. Therefore, the maximum value of the alternating-current e.m.f. is equal to the direct-current e.m.f. Hence, the direct-current e.m.f. is equal to $\sqrt{2}$, or 1.41 times the r.m.s. single-phase e.m.f. That is, if the single-phase e.m.f. is 100 volts, the direct-current e.m.f. is 141

volts (Fig. 255). If a circle be drawn having 100 volts to scale as a diameter (Fig. 255), the 3-phase e.m.f. is given by a chord subtending 120°, the 4-phase by a chord subtending 90°, etc.

Assuming unity power-factor and no losses, the ratios of alternating voltages and currents to direct-current voltage and current are as given in the following table:

Number of slip-rings	Number of phases	Ratio E_{AC}/E_{DC}	Ratio I_{AC}/I_{DC}
2	1	0.707	1.414
3	3	0.612	0.943
4	4	0.500	0.707
6	6	0.354	0.471

Example.—Find the alternating current and voltage at rated load of a 1,000-kw., 500-r.p.m. 25-cycle, 6-phase, 250-volt synchronous converter. Assume unity power-factor and neglect losses.

The rated direct current,

$$I_{DC} = \frac{1{,}000{,}000}{250} = 4{,}000 \text{ amp.}$$

$$I_{AC} = 4{,}000 \times 0.471 = 1{,}884 \text{ amp.} \quad \textit{Ans.}$$

$$E_{AC} = 250 \times 0.354 = 88.5 \text{ volts.} \quad \textit{Ans.}$$

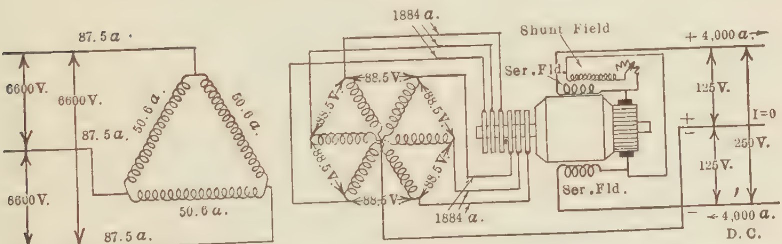


Fig. 256.—Connections of a 1,000-kw., 25-cycle, 6-phase, 250-volt synchronous converter taking energy from a 6,600-volt, 3-phase line.

These currents and voltages are shown in Fig. 256. The converter is compounded and supplies a 250-volt, 3-wire system, the neutral of which is connected to the 6-phase, star-connected transformer secondaries (see page 128). Any neutral current due to unequal loads on the sides of the 3-wire system returns to

or leaves the armature through the secondaries and slip-rings. Two series-field windings are necessary. If but one were used, on the positive side, for example, loads from negative to neutral would have no effect on the compounding. This converter is supplied from a 6 600-volt, 3-phase circuit. The primaries of the transformers are connected in delta. The currents are determined as follows:

Example.—Determine the line currents and the primary coil currents in the system (Fig. 256). Neglect losses and assume unity power-factor.

The line currents

$$I = \frac{1{,}000{,}000}{\sqrt{3} \times 6{,}600} = 87.5 \text{ amp.} \quad \textit{Ans.}$$

The transformers have a 6,600/177.0 ratio of transformation. Hence, the primary coil currents, from equation (56) (page 193).

$$I_C = 1{,}884\,\frac{177.0}{6{,}600} = 50.6 \text{ amp.} \quad \textit{Ans.}$$

$$50.6\sqrt{3} = 87.5 \text{ amp. } (\textit{check}).$$

168. Voltage Control.—*Change of Excitation.*—The ratio of the direct current e.m.f. to the alternating-current e.m.f. in a converter armature is fixed, regardless of field excitation. The ratio of *brush* voltage to *slip-ring* voltage, however, may be changed a limited amount by varying the field excitation. If the field excitation be increased, the converter, since it operates as a synchronous motor, takes a leading current and its induced e.m.f. is raised (see page 278, Fig. 243). It is further raised by leading current flowing through the series reactance of the transformers. Likewise, if the field be weakened the induced e.m.f. is decreased (see page 280, Fig. 245). With sufficient reactance, the direct-current voltage may be varied about 5 per cent. above and below its average value. By compounding (Fig. 256) the regulation is accomplished automatically. The principal objection to this method is that regulation is accompanied by a change of power-factor and this is objectionable, since converters for the best results should operate near unity power-factor.

Transformer Taps.—The converter voltage may be adjusted approximately to the desired value by taps on the transformers. Owing to the arcing and burning of sliding contacts, the use of transformer taps is not common for adjustment during operation.

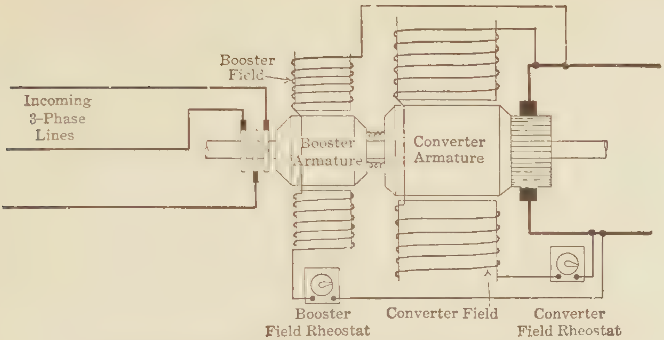


FIG. 257.—Synchronous converter with series booster.

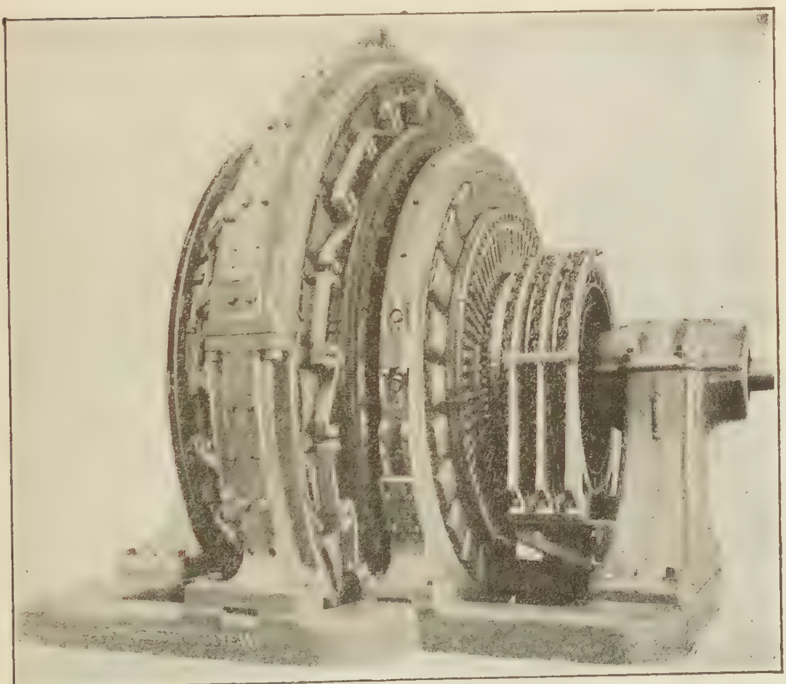


FIG. 258.—General Electric 3,500-kw., interpole-type, synchronous converter with synchronous booster.

The use of taps for fixed adjustment of voltage is, however, common.

Induction Regulator.—The induction regulator has already been described in connection with the induction motor (see page 257). This type of regulator may be connected between the transformers and the converter, and the alternating voltage impressed on the converter terminals may be raised and lowered thereby. This changes the direct-current voltage by a corresponding amount. Under these conditions the voltage may be raised independently of power-factor, but the extra equipment is an objection to the use of the induction regulator.

Series Booster.—A low-voltage alternator is often connected to the shaft of the converter. This alternator has the same number of poles as the converter. The armature of the alternator is connected in series with the alternating-current lines supplying the converter (Fig. 257). By raising the field of the alternator or booster, the alternating voltage of the converter is raised. The converter voltage may be lowered, not only by decreasing the booster field, but by reversing it as well. The advantage of this method of control is that the voltage may be varied independently of power-factor. Its use is common with large units. The objection to this type of voltage control is the additional machine. Figure 258 shows a converter having a booster-generator.

169. The Inverted Synchronous Converter.—When a converter operates from a direct-current source and delivers alternating current, it is known as an *inverted* synchronous converter. The direct-current side has characteristics very similar to those of a shunt or compound motor. The alternating-current side has characteristics very similar to those of an alternator. When operating from the alternating-current supply, the speed of the converter must be in synchronism with the supply, and hence constant. When operating from the direct-current supply, the speed is determined by the back e.m.f. and the flux, just as in any direct-current motor, and the speed may vary. An inductive load on the alternating-current side weakens the field through armature reaction, in the same manner that the field of an alternator is weakened under similar conditions. The weakening of the field increases the speed of the converter. This increased

speed causes the current to lag still more because of the increased frequency. The effect is cumulative, and may cause the armature to reach dangerous speeds. Therefore, a centrifugal device which trips the breaker when the speed exceeds the safe value is often used.

170. Starting the Synchronous Converter.—*Alternating-current Side.*—If polyphase currents are supplied to the armature of a converter, a rotating field is produced about the armature, similar to the rotating field of the induction motor, except that it is produced by a rotating armature about itself. If the armature speed is below synchronism, this field cuts the pole-faces and the damper windings (Fig. 260), and induces currents in them. A reaction results between the rotating field and these induced currents, producing rotation.

When starting the converter in this manner the armature becomes the primary of a transformer and the shunt and series fields are the secondaries (see page 289). To prevent too high a voltage being induced in the shunt field it is sectionalized (Fig. 259). The field-splitting switch, therefore, should be opened. To prevent large currents being induced in the series field, the short-circuiting switches and shunts should be opened. The rotating field about the armature cuts coils short-circuited by the brushes, so that excessive sparking occurs under the brushes, particularly if the machine has interpoles. With large machines, therefore, the brushes are raised from the commutator during starting except for one brush in a positive brush-arm and one in a negative brush-arm, these supplying the field current. The converter is usually started at reduced voltage from taps on the secondaries of its transformers. The field is closed when the

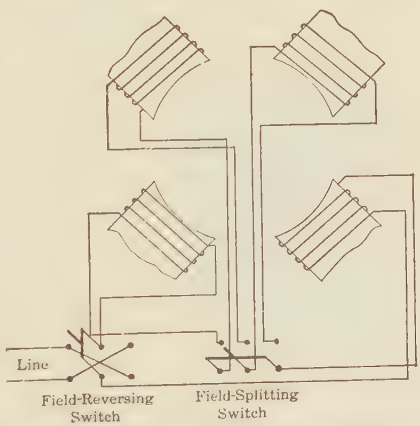


FIG. 259.—Connections of shunt field and shunt field-splitting switch.

converter is near or at synchronism, after which the converter is connected directly across the line. If the machine comes up with the wrong direct-current polarity, the field may be opened and then the line switch opened long enough for it to slip back one pole. Likewise, before connecting across the line, the armature may be made to slip a pole by throwing over the field-reversing switch (Fig. 259). Before the direct current in the field has had opportunity to reverse due to the reversed polarity, thus causing the armature to slip another pole, the field switch must be thrown back quickly to its original position. This procedure causes the armature to slip back a pole and, as a result, the direct-current polarity reverses.

Direct-current Side.—The converter may be started from the direct-current side as a shunt motor. When started in this manner, the series field should be short-circuited, as it will oppose the shunt field when the machine operates as a motor and will, therefore, reduce the starting torque. The transformer secondaries are short-circuits on the direct-current armature at starting, as the frequency is zero and their resistance is very low. The transformers, therefore, should be disconnected. The proper speed is obtained by adjusting the shunt field. As there is practically no voltage control in the simple converter when operating in this manner, it is not always possible to adjust the alternating voltage to a value equal to that of the line. To prevent any disturbance which may result from synchronizing at a voltage other than bus-bar voltage, some of the starting resistance is often left in the armature circuit until after the machine has been synchronized.

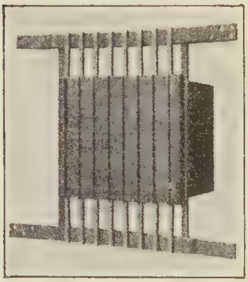


Fig. 260.—Main pole with damper winding.

171. Dampers.—The current which produces torque in the converter armature is the *difference* of the alternating and the direct current. This difference is small. Hence, a small percentage change in either direct or alternating current, such as would be caused by switching, change of load, disturbances, etc., causes a large percentage change in the net or torque current. Converters, therefore, are more sensitive even than syn-

chronous motors, "hunt" very readily and dampers (Fig. 260) are essential.

172. Experimental Determination of Voltage and Current Relations in a Converter.—An instructive laboratory experiment is carried out with a converter connected in the manner shown in Fig. 261. The series reactances may be omitted if the transformers themselves have sufficient leakage reactance. Connect instruments to measure the 3-phase input, a voltmeter to measure the transformer primary voltage, a voltmeter to measure the slip-ring voltage, ammeters to measure the currents between the transformer secondaries and the converter, direct-current instruments to measure the converter output, and a direct-current ammeter to measure the field current.

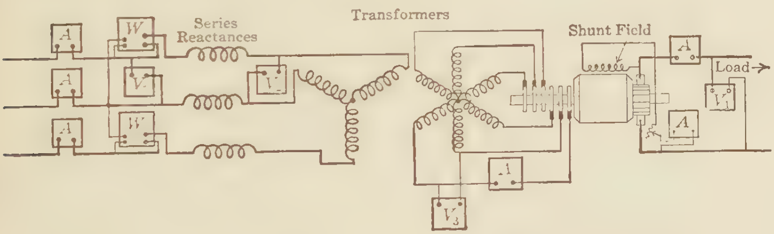


FIG. 261.—Connections for testing a synchronous converter.

Keep the load on the converter constant at 50 per cent. of its rated value. Vary its field over the maximum range of operation, reading all instruments. With field current as abscissas, plot as ordinates:

1. Voltages V_1, V_2, V_3, and V_4.
2. Efficiency of the entire unit.
3. Power-factor.

Also, check the currents by the tables of Par. 167 (page 295). Note the effect of power-factor on efficiency.

Other experiments may be performed using these same connections, such as keeping the field current constant at its normal no-load value (P.F. = 1.0), and noting the changes in efficiency and power-factor as the load is increased. Plot efficiency and power-factor as ordinates with output as abscissas.

173. Industrial Applications.—As has already been pointed out, the principal industrial application of the synchronous converter

is to convert alternating to direct current on a large scale. Units as large as 4,000 kw. at 25 cycles and 3,000 kw. at 60 cycles are common. The converter has an efficiency of 0.95 to 0.96 at rated load and unity power-factor, as compared with less than 0.9 for the motor-generator set. The converter efficiency, however, drops rapidly with decrease of power-factor (see Par. 166). Being a single machine, the converter occupies less floor space than the motor-generator set. A converter, however, always requires transformers. Up to 15,000 volts, the synchronous motor does not require transformers. Hence, up to 15,000 volts, the cost, floor space, and efficiencies of the transformers must be taken into consideration when the converter is in question. For voltages in excess of this, the motor-generator also requires transformers. With a converter alone, the voltage and power-factor are not independent of each other; if regulating apparatus, such as a booster-generator is used, the cost of the converter unit is increased and its efficiency is decreased.

Until within the last ten years, engineers were inclined to favor the motor-generator set, particularly for 60 cycles. Radical improvements in design, however, give the converter preference at the present time.

RECTIFIERS

Where comparatively small amounts of direct-current power are required, as for example, to charge small storage batteries, to operate magnetite lamps, etc., it is possible to obtain direct current from alternating-current supply without employing rotating machinery. Devices which so convert alternating to direct current are called *rectifiers*. Some of the more common types of such rectifiers are here described.

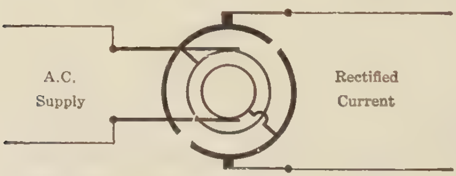


FIG. 262.—Commutating-type rectifier.

174. The Rectifying Commutator. — Rectification of alternating current may be accomplished mechanically, one method being by the use of a commutator driven by a synchronous motor. The segments are so connected that when the alternating current

reverses, the connections to the direct-current circuit are simultaneously reversed (Fig. 262). A unidirectional current is thus obtained. As the brushes cannot have zero width, it is difficult to commutate at the point of zero current and the current and voltage are rarely zero at the same time. Hence, such devices spark more or less, and so are limited to small currents and voltages.

175. The Mercury-arc Rectifier.—The principle of the mercury-arc rectifier is the valve action of mercury vapor. In order to obtain the best operation, the tube containing this vapor must be exhausted to a very high vacuum. Figure 263 shows a mercury-arc rectifier tube having four terminals. The lower terminal is the cathode, to which the current goes from the tube. The two terminals, A_1 and A_2 are the anodes from which the current enters the tube. A_3 is a starting anode, by means of which the mercury arc is established. Current then *enters* the tube from either anode A_1 or A_2, depending upon which side of the transformer secondary *ab* is positive. When the current attempts to reverse its direction, however, the mercury vapor acts as a valve and prevents any current entering the tube at the cathode. If only one anode were used, the negative half of the alternating-current wave would be eliminated in each cycle and the resultant wave would appear as shown in solid line (Fig. 264 (*a*)). This

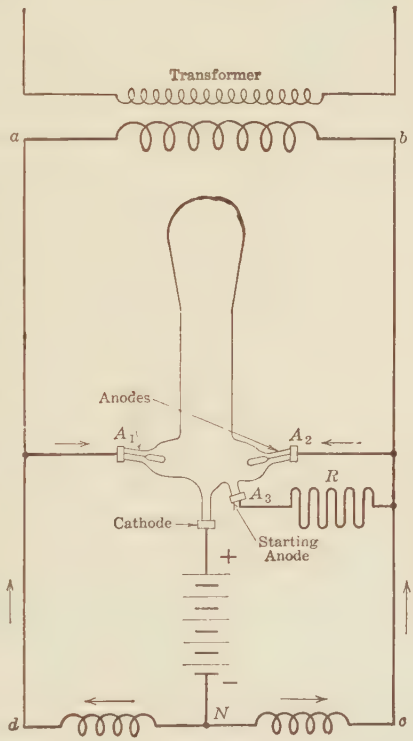
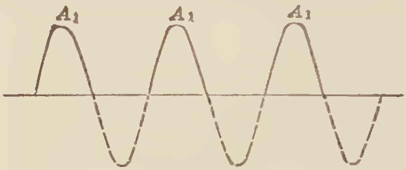


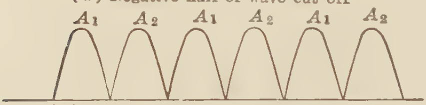
FIG. 263.—Mercury-arc rectifier for low voltages.

condition of operation could not be maintained with the mercury arc, because the arc is extinguished as soon as the current becomes zero.

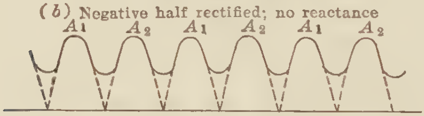
To obtain a continuous flow of current through the tube, two anodes, A_1 and A_2 are necessary, one anode being connected to each end of the transformer secondary. When one end of the transformer becomes negative, the other becomes positive, so that either one anode or the other is always positive. Current, therefore, is always entering the tube from either one anode or the other. Were there no inductance in circuit, the rectified wave under these conditions would appear as shown in Fig. 264 (*b*).



(*a*) Negative half of wave cut off



(*b*) Negative half rectified; no reactance



(*c*) Negative half rectified; reactance producing a pulsating uni-directional current

FIG. 264.—Rectified alternating-current waves.

The portions of the wave marked A_1 are due to anode A_1, and those marked A_2 to anode A_2. Each of these portions reaches the zero value twice for each cycle of current supply. This would cause the arc to be extinguished. By introducing inductance in the circuit however, the current is held over the zero point and the resulting wave is similar to that shown in Fig. 264 (*c*), being more or less pulsating in character.

The direct current leaves the cathode, enters the positive terminal of the battery to be charged (or other translating device) and flows to the neutral of the auto-transformer. It then divides, part returning through each anode.

The anode A_3 is for starting purposes only. When the tube is tilted, a conducting stream of mercury is established between A_3 and the cathode. The resulting current flow vaporizes some of this mercury and so establishes the arc. A ballast resistance R is necessary in order to limit the current at starting, since there is then a metallic path of low resistance between A_3 and the cathode.

This type of rectifier is used to charge batteries, particularly vehicle batteries. It is also used extensively for series magnetite-arc circuits to rectify the alternating current supplied from the secondaries of constant-current transformers. The current under these conditions is of the order of 6.6 amp. and the voltage is of the order of 4,000 to 5,000 volts. For low-voltage circuits, this type of rectifier has not as yet been developed in large capacities.

176. The Tungar.—The tungar is based on the following principle: An incandescent filament emits minute negative charges called *electrons* (see Chap. XIII, page 342). When the discharge of these elctrons occurs in an electrostatic field, the electrons attain considerable velocity. If a gas is present these electrons collide with the gas atoms and ionize them. That is, when an electron collides with an atom of gas, that atom is broken up into an electron and a positive ion.

The ions, being positive charges or carriers of positive electricity, move along the electrostatic lines and by carrier action cause current to flow from the positive to the negative electrode. Ions, when so moving under the influence of the electrostatic field, attain high velocity and collide with other atoms causing further ionization. Hence, the region in which this action occurs becomes ionized. Due to carrier action, ionized gas acts as a conductor of electricity.

Figure 265 (a) shows a glass bulb containing an inert gas, usually argon, at reduced pressure, and also an ordinary coiled tungsten filament. Near the filament is a graphite anode. A transformer ab steps down the supply voltage and the filament is connected across its secondary. The filament then becomes incandescent and tends to emit negative charges or electrons.

One terminal of the transformer secondary c and one end of the filament are connected to the transformer primary at h.

The filament is then at practically the same potential as that of the power-supply line $b'b$. The voltage of the battery being charged is somewhat less than the voltage between line $a'a$ and line $b'b$. The potential of the graphite anode, therefore, is different from the potential of point c, usually by approximately 5 or 6 volts. Consequently, during one half-cycle the potential of the filament is negative with respect to that of the anode and during the next half-cycle its potential is positive with respect to that of the anode.

When the filament is negative, the negative charges or electrons are repelled by it, because like charges repel each other. These electrons attain a considerable velocity and break up the gas

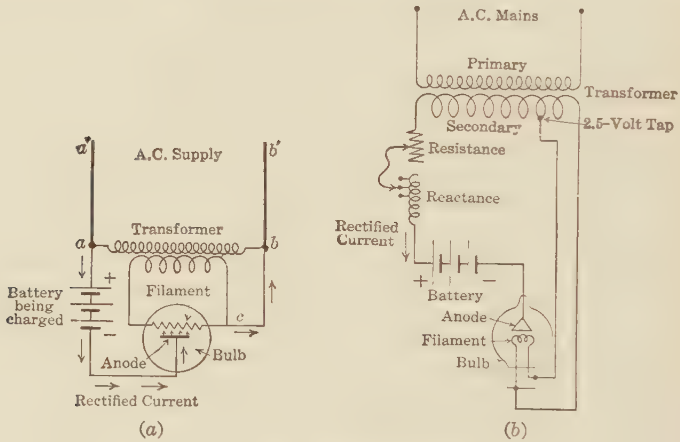


Fig. 265.—Tungar rectifier.

particles into ions. The region between the filament and the anode becomes conducting and as a result current flows from a into the positive terminal of the battery, through the battery to the anode, to c and then to b.

When the filament is positive, the electrons or negative charges which it tends to emit due to its incandescence are attracted toward the filament, since positive and negative charges attract each other. Consequently, the electrons which produce the ionizing action are withdrawn from the region between the filament and the anode. As a result, the gas is no longer ionized and it

ceases to be a conductor. No current can flow, therefore, during this half-cycle. The current can flow only in one direction, therefore, from the graphite to the filament, and the device acts as a rectifier.

Figure 265 (*b*) shows the connections for one commercial type of low-voltage tungar, the switches and cut-outs being omitted. Both the current to be rectified and the current for heating the filament are supplied by the transformer secondary, the filament being connected between a 2.5-volt tap and one end of the secondary. Current regulation may be obtained by adjusting the resistance and the reactance. Where electrical connection between load and primary mains is permissible, an auto-transformer with taps may replace the transformer shown in the diagram.

The efficiency of the tungar rectifier is from 35 per cent. in the smaller sizes to 75 per cent. in the larger sizes. The capacities at present are not much in excess of 750 watts.

177. Electrolytic Rectifiers.—Electrolytic rectifiers are based on the following principle: If a lead plate and an aluminum plate be immersed in a sodium- or ammonium-phosphate solution, current can pass from the solution to the aluminum. As soon as the current attempts to reverse and pass from the aluminum to the solution, a thin insulating film of aluminum oxide is instantly formed over the aluminum plate, and acts as an insulator up to about 150 volts. This prevents the current flowing from aluminum to solution and such a device may be used, therefore, as a rectifier. Figure 266 shows such a simple rectifier, giving a continuous pulsating current like that shown in Fig. 264 (*b*).

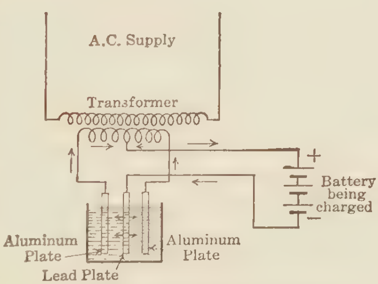


FIG. 266.—Electrolytic rectifier.

Such rectifiers are of low efficiency, 60 per cent. and lower, and are of small capacity. They are used primarily for charging low-voltage batteries from alternating-current supply. Their advantage lies in their simplicity and cheapness.

CHAPTER XI

TRANSMISSION AND DISTRIBUTION OF ELECTRICAL ENERGY

178. Typical Transmission and Distribution System.—It was pointed out in Chap. I that for economic reasons electrical energy is ordinarily generated on a large scale in central stations or hydroelectric plants and is *transmitted* to the points where it is utilized. The fact that electrical energy can be transmitted economically over long distances permits the generating station to be located where conditions are most favorable for the generation of energy, for example, such as being near an adequate supply of condensing water, near tide water so that transportation costs of coal are lowered, etc.

Hydraulic energy, which would otherwise not be available due to its remoteness from centers of population, becomes available owing to the fact that electrical energy is readily transmitted.

Ordinarily, electrical energy is generated at moderate voltages, is transmitted at high voltages, and is utilized at comparatively low voltages. A typical modern system is shown in Fig. 267, although no attempt is made to show switches, instruments, etc. The energy is generated at 6,600 volts in a vertical type of water-wheel generator and is stepped up from 6,600 to 110,000 volts for transmission by a delta-Y transformer bank. It is transmitted over a 110,000-volt line on steel towers to a distance of approximately 100 miles. A line at this voltage requires a private right of way, owing to possible dangers from falling wires, and cannot be carried to the sub-stations which are located in populous territory.

It is, therefore, necessary to step down the voltage to a value that will permit the use of underground cables. Cables cannot be operated at the high voltages which are possible with open-wire lines. Three-conductor cables, as a rule, do not operate at

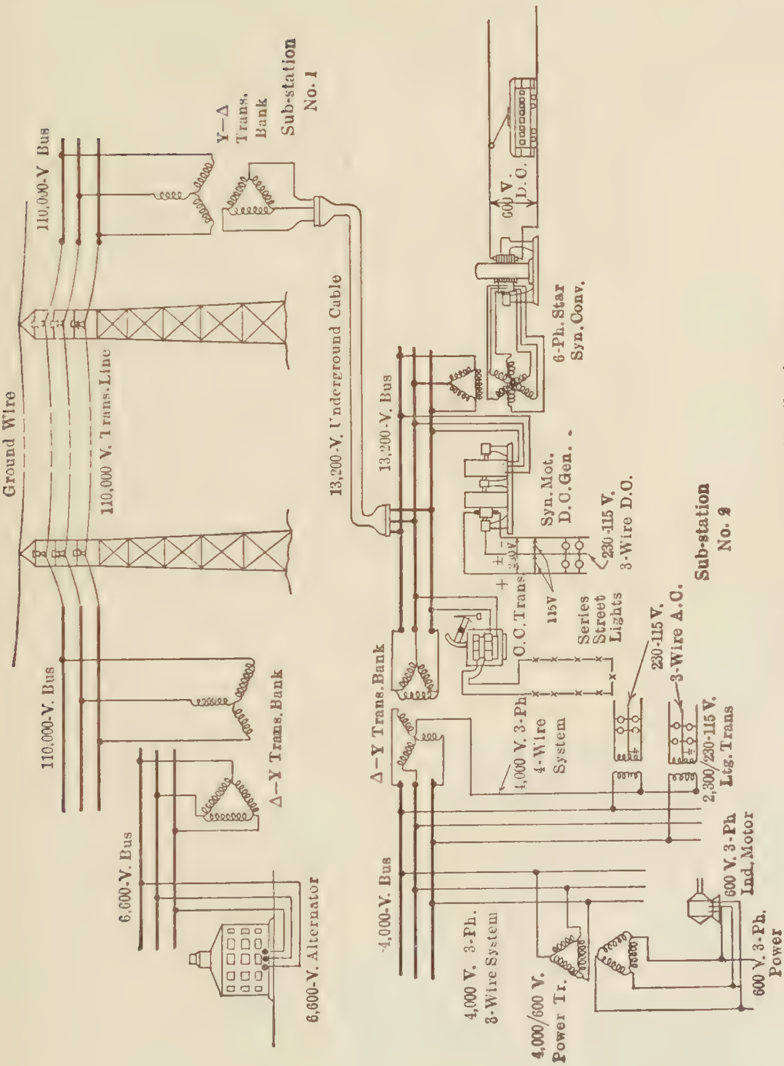


FIG. 267.—Typical transmission and distribution system.

voltages much higher than 33,000 volts between conductors. 13,200-volt cables are utilized in the installation (Fig. 267). Hence, the energy is stepped down from 110,000 to 13,200 volts by a Y-delta transformer bank in sub-station No. 1. It is then carried underground to sub-station No. 2, near the load center and to other similar sub-stations. Direct-current energy at 600 volts for street railways is obtained from the 13,200-volt bus-bars by means of a synchronous converter taking its energy through a delta-star transformer bank with 6-phase secondaries. If $^{230}\!/_{115}$-volt, direct-current service is desired, either another synchronous converter or a synchronous motor-generator set (Fig. 267) may be used. The synchronous motor operates directly from the 13,200-volt bus-bars.

If series street lights are to be supplied, constant-current transformers may be operated directly from the 13,200-volt bus-bars. For general alternating-current distribution, a 4,000-volt, three-phase system is used (Fig. 267), the 4,000-volt bus-bars being supplied from the 13,200-volt bus-bars through a delta-Y transformer bank. To eliminate the flicker of lamps due to motors being connected across the line, separate feeders are used for power and for lighting. Lighting loads are supplied from one 4,000-volt, 3-phase, 4-wire system, which gives $4{,}000/\sqrt{3}$, or 2,310 volts to neutral. Lighting transformers having a 10:1 ratio are connected to neutral and $^{230}\!/_{115}$-volt, 3-wire systems obtained from their secondaries (also see page 216). The power feeders also run from the 4,000-volt bus-bars. The 600-volt, 3-phase service for machine shops, factories, mills, etc., is obtained directly from the 4,000-volt lines by the use of either V-connected, or delta-connected transformers (usually of the 3-phase type) located at the premises and stepping down the voltage to 600 volts (Fig. 267).

The foregoing shows the manner in which the several classes of power and lighting service secure their respective voltages, currents and frequencies, through various transformations, from a hydroelectric plant over 100 miles away, generating at 6,600 volts, 3-phase.

179. Voltage and Weight of Conductor.—Further discussion of transmission problems is better understood if the following principle be kept in mind: *The weight of conductor varies inversely*

as the square of the voltage, when the power transmitted, the distance, and the loss are fixed.

This becomes obvious when it is remembered that power loss in a given conductor varies as the current *squared*. If the voltage of a system is doubled, the current is halved for the same power. With half the current, the transmission loss is *quartered* if the same size conductors are used. If the transmission loss is to remain unchanged, conductors of one-fourth the original cross-section can be used.

This is illustrated by the following example, which, for simplicity, is given for a direct-current case, but which applies equally well to alternating current, since with alternating current the power loss is also equal to I^2R.

Example.—A load of 120 kw. at 110 volts is delivered at a distance of 400 ft. from the power station over a 1,000,000-C.M. feeder, each conductor of which has a resistance of 0.0043 ohm. Find: (*a*) the power loss in the feeder; (*b*) the weight of copper; (*c*) repeat (*a*) for the same load and distance, but employing 220 volts at the load and using a 250,000-C.M. feeder, each conductor of which has a resistance of 0.0172 ohm.; (*d*) repeat (*b*) for the 250,000-C.M. feeder in (*c*). (1,000,000-C.M. cable weighs 3,090 lb. per 1,000 ft.)

(*a*) The current

$$I_1 = \frac{120{,}000}{110} = 1{,}090 \text{ amp.}$$

The total feeder loss

$$P_1 = (1{,}090)^2\, 0.0086 = 10{,}220 \text{ watts.} \quad \textit{Ans.}$$

(*b*) $\dfrac{800}{1{,}000} \times 3{,}090 = 2{,}472$ lb. *Ans.*

(*c*) $I_2 = \dfrac{120{,}000}{220} = 545$ amp. $P_2 = (545)^2\, 0.0344 = 10{,}220$ watts. *Ans.*

(*d*) Obviously, the weight of the 250,000-C.M. cable is one-fourth that of the 1,000,000-C.M., or 618 lb. *Ans.*

That is by doubling the voltage, the weight of copper is reduced from 2,472 to 618 lb. If 550 volts were used, the weight of copper for these same conditions would be only 2,472/25, or 99 lb.

It is obvious then, why very high transmission voltages are used when the energy is transmitted over long distances. For example, at 150,000 volts one ten-thousandth (1/10,000) as much copper is required to transmit a given amount of energy, a given distance with a given loss as would be required if 1,500 volts were used.

From the point of view of investment in copper it is desirable that transmission and distribution voltages be as *high* as possible. On the other hand, other factors, both economic and operating, act to limit the voltage. The cost of insulators increases rapidly with the voltage. The higher voltages necessitate greater spacings of conductors and this requires more costly transmission structures, stations, etc. Transformers, switches, lightning arresters, etc. increase rapidly in cost with increase in voltage. There is one best voltage which gives the minimum cost of the transmission system. In general, this works out to be approximately 1,000 volts for each transmission mile. For example, a 100-mile line would operate at 100,000 volts.

180. Transmission-line Reactance.—When considering the transmission of power with direct current, the line resistance only need be considered. With alternating current, the line *reactance* must ordinarily be taken into consideration; if the line is long and the voltage is high, the line *capacitance* must also be considered. Cables have large capacitance which ordinarily cannot be neglected.

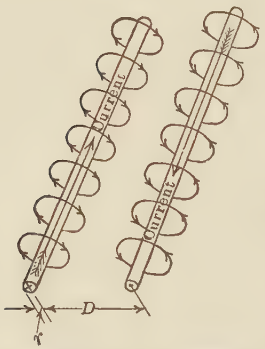


FIG. 268.—Flux linkages in a single-phase line.

The reactance of the line is due to the fact that the lines of induction which are due to the current in the conductors, *link* the loop formed by the line. The flux linking a portion of such a line is shown in Fig. 268. At the instant shown, the current in the left-hand conductor is inward and the current in the right-hand conductor is outward. The flux due to the left-hand conductor, the distribution of which is circular about the conductor, is, by the corkscrew rule, clockwise in direction. Since the current in the right-hand conductor is opposite in direction to that in the left-hand conductor, the flux about the right-hand conductor must be counter-clockwise in direction. Both fluxes, however, combine to act downward in the single-turn loop formed by the conductors. These fluxes both link the loop and inductance must result.

It can be shown that the inductance of such a loop is

$$L = 2l\left(0.080 + 0.741 \log_{10}\frac{D}{r}\right) \text{ milhenrys,} \tag{63}$$

where D is the distance between conductor centers, and r is the radius of each conductor, both expressed in the same units. l is the length of the line in miles. The reactance of the loop is

$$X = 2\pi fL = 4\pi fl\left(0.080 + 0.741 \log_{10}\frac{D}{r}\right)10^{-3} \text{ ohms} \tag{64}$$

where f is the frequency.

181. Transmission-line Capacitance.—If a *direct-current* voltage be applied to a transmission line under no-load conditions, no current flows after the first few moments, except the almost negligible leakage current. If an *alternating* voltage be applied to a transmission line, considerable current may flow, even if there be no appreciable leakage and no connected load. This current is the *charging current* of the line, and leads the voltage by almost 90°. The line acts as a condenser, the conductors being the plates and the air the dielectric. Each conductor becomes charged, first positively and then negatively, which results in an alternating current.

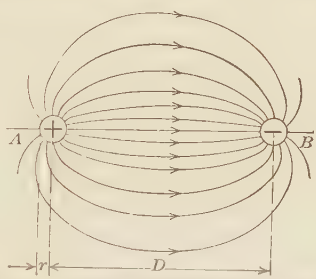


FIG. 269.—Electrostatic flux between line conductors.

This is illustrated by Fig. 269, which shows conductors A and B of a single-phase line. At the instant shown, conductor A is positive and conductor B is negative. The electrostatic flux existing in the field between A and B is shown. The capacitance *between conductors* of such a line can be shown to be approximately

$$C = \frac{0.0194}{\log_{10}\frac{D}{r}} \mu\text{f. per mile,} \tag{65}$$

where D is the distance between conductor centers and r is the radius of each conductor, both expressed in the same units.

182. Transmission-line Calculations.—Figure 270 (a) shows diagrammatically a single-phase transmission line having a resistance of $R/2$ ohms per conductor and an inductive reactance of $X/2$ ohms per conductor. The

resistance of the entire loop is R ohms and the reactance of the entire loop is X ohms ($X = 2\pi fL$, equation (64)). The voltage at the load, or receiver is E_R and the voltage at the sending end, or generator is E_G. The load takes a current I at the voltage E_R, and at power-factor $\cos\theta$. Given the load voltage E_R, the load current I, and the load power-factor $\cos\theta$, let it be required to determine the sending-end voltage E_G.

In Fig. 270 (*b*) the current I is shown lagging the voltage E_R by the angle θ. The resistance-drop in the line IR must be in phase with the current. The reactance-drop IX must lead the current by 90°. The vector sum of IR and IX gives the impedance-drop IZ. The sending-end voltage E_G must be equal to the receiving-end voltage plus the impedance-drop IZ. Hence, IZ is added vectorially to E_R to find E_G (Fig. 270 (*b*)). The same result may be obtained by adding IR and IX separately to the end of vector E_R (Fig.

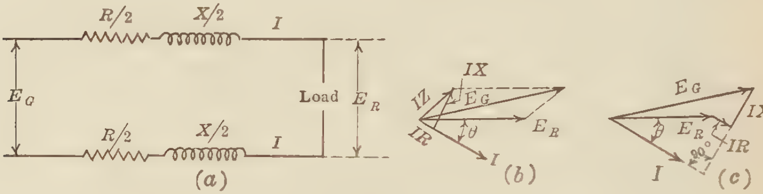


FIG. 270.—Vector diagrams for single-phase transmission line.

270 (*c*)). This figure is similar to Fig. 154 (page 168). Its solution is also the same as that given for Fig. 154.

$$E_G = \sqrt{(E_R \cos\theta + IR)^2 + (E_R \sin\theta + IX)^2}. \tag{66}$$

With a leading current,

$$E_G = \sqrt{(E_R \cos\theta + IR)^2 + (E_R \sin\theta - IX)^2} \text{ (see Fig. 155, page 170).} \tag{67}$$

Example.—A sub-station located 6 miles from a central station requires 1,000 kw. at 6,600 volts, 0.8 power-factor, lagging current and 60 cycles. The single-phase line consists of 300,000-C.M. stranded copper conductors spaced 4 ft. apart. Find: (*a*) the voltage at the central station; (*b*) the efficiency of transmission.

(*a*) From Appendix G (page 416) the resistance per mile of 300,000-C.M. stranded copper conductor is 0.190 ohm. Hence, the total resistance

$$R = 12 \times 0.190 = 2.28 \text{ ohms.}$$

From Appendix H (page 417) the reactance per mile of 300,000-C.M. cable spaced 48 in. is 0.650 ohm. Hence, the total reactance

$$X = 12 \times 0.650 = 7.80 \text{ ohms.}$$

The current

$$I = \frac{1,000,000}{6,600 \times 0.8} = 190 \text{ amp.}$$

$$IR = 190 \times 2.28 = 433 \text{ volts.}$$
$$IX = 190 \times 7.80 = 1,480 \text{ volts.}$$

Using equation (66),

$$\cos\theta = 0.8,\ \theta = 36.9°,\ \sin 36.9° = 0.6.$$

$$E_G = \sqrt{(6{,}600 \times 0.8 + 433)^2 + (6{,}600 \times 0.6 + 1{,}480)^2}$$
$$= \sqrt{32{,}640{,}000 + 29{,}600{,}000} = \sqrt{62{,}240{,}000} = 7{,}890 \text{ volts.} \quad \textit{Ans.}$$

(*b*) The line loss $= I^2R$

$$= (190)^2 \times 2.28 = 82{,}300 \text{ watts or } 82.3 \text{ kw.}$$

The efficiency

$$= \frac{\text{output}}{\text{output} + \text{losses}} = \frac{1{,}000}{1{,}000 + 82.3} = \frac{1{,}000}{1{,}082} = 0.925 \text{ or } 92.5 \text{ per cent.} \quad \textit{Ans.}$$

The foregoing amount of power would seldom be transmitted single phase. The same method of calculation would apply, however, to each phase *to neutral* of a 3-phase line.

It has already been stated that if a synchronous motor takes a leading current through a reactance, a *rise* of voltage occurs. The reactance of transmission lines is in series with the load. If a load be made to take a leading current by means of synchronous motors or otherwise, this current, in virtue of its flowing through the line reactance, may cause the load voltage to be even higher than the sending-end voltage. In the preceding example, assume that the power-factor is 0.8, *leading* current, using equation (67),

$$E_G = \sqrt{(6{,}600 \times 0.8 + 433)^2 + (6{,}600 \times 0.6 - 1{,}480)^2}$$
$$= \sqrt{32{,}640{,}000 + 6{,}150{,}000} = 6{,}230 \text{ volts.} \quad \textit{Ans.}$$

That is, there is a *rise* of voltage from the generator to the load without the introduction of energy along the line. This condition is impossible with direct current. Hence, within limits, a synchronous motor or condenser may be made to control the voltage at its end of a transmission line.

183. Transmission-line Structures.—For relatively light lines operating at comparatively low voltage, wooden poles are used as supports. Since the insulators for higher-voltage lines become more expensive and are also more frequent sources of trouble, it is desirable to use fewer supports, and longer spans are, therefore, necessary. Wooden poles have not the requisite height and torsional strength for the long spans and wide spacing which must be used. Steel poles, which ordinarily consist of steel channels held together by latticework are frequently used where the right of way is narrow, as along a railroad, but such poles are comparatively expensive for their strength. Steel towers, of the windmill type (Fig. 271) give the greatest strength for a given amount of material and are, therefore, common supports for high-voltage lines of considerable power.

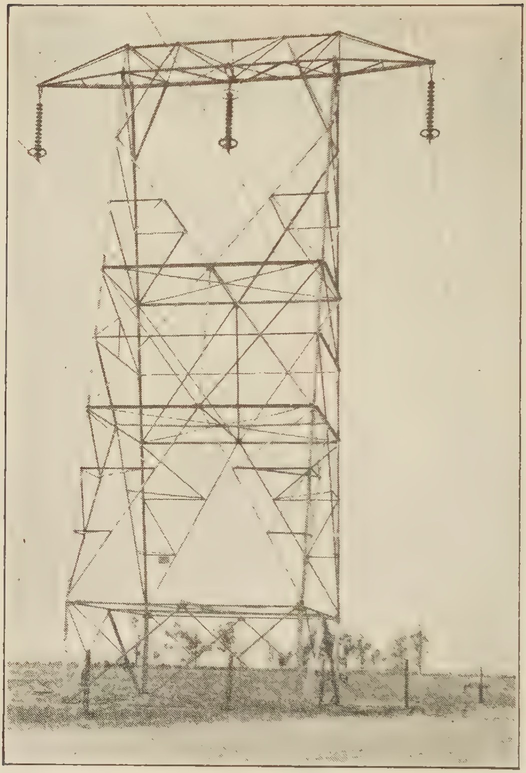


Fig. 271.—220,000-volt transmission tower. (Southern California Edison Co.)

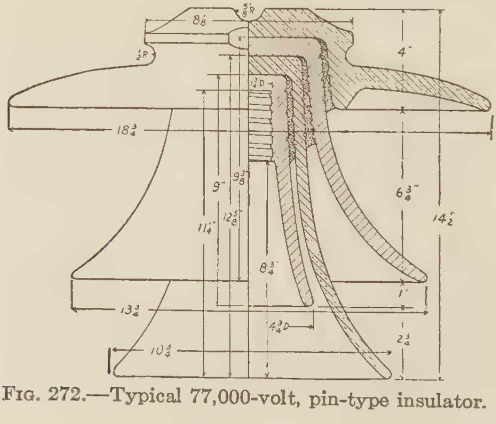


Fig. 272.—Typical 77,000-volt, pin-type insulator.

184. Insulators.—The most satisfactory material for high-voltage line insulators is glazed porcelain. This material has high dielectric strength and it can withstand high mechanical stresses, high dielectric stresses, and weathering, all simultaneously. For low-voltage lines the pin-type insulator is satisfactory. Pin-type insulators (Fig. 272) can be used for line potentials as high as 77,000 volts, but at such voltages they are bulky, expensive, and produce large torsional stresses in the cross-arms.

These disadvantages are eliminated by the use of suspension insulators (Fig. 273). The suspension insulator string is composed of a number of porcelain units suspended in series. By employing a sufficiently large number of units, a suspension string can be used for the highest voltages. Obviously, tension is the only force that the insulator string can sustain. Hence, the horizontal stresses in adjacent spans should be equal in order that the insulator strings may hang vertical.

Fig. 273.—Dry flash-over of a 7-unit suspension insulator string

This type of insulator is used for voltages as low as 25,000 volts, since the number of units can be readily adapted to line conditions. The electrical factor of safety is readily and economically increased by adding one or more units to each string. If the transmission voltage is raised, it is merely necessary to connect additional units in the string. Suspension strings are also used for strain insulators, the insulator string then becoming a part of the span.

185. Lightning Arresters.—Electric power lines are frequently subjected to overvoltage usually of a transient nature, due to lightning, to disturbances caused by short-circuits, by switching,

etc. It is desirable to relieve the line of these voltage surges as soon as possible, since they may flash over insulators (Fig. 273), destroying them and shutting down the line. It is economically impossible to provide protection to each insulator string, although arcing rods and rings which hold the arc away from the string are often used (see Fig. 271). Surges would, however, cause great damage to apparatus like transformers, generators, etc. Hence, lightning arresters should be connected near such apparatus. The function of lightning arresters is to relieve the line of these overvoltages by passing the charge to ground. At the same time the arrester must suppress the dynamic arc which the normal line voltage tends to maintain after the transient discharge has ceased.

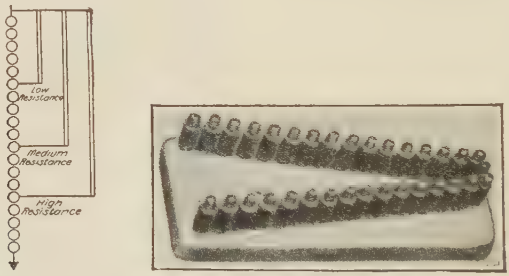


Fig. 274.—General Electric multigap lightning arrester.

One type of arrester for low voltage is shown in Fig. 274. A number of cylinders made of non-arcing metal are connected between the line and ground. There is a small air-gap between adjacent cylinders. A carbon rod of high resistance is shunted from the conductor across approximately three-fourths of these cylinders, a medium resistance across approximately one-half the cylinders, and a comparatively low resistance across a little over one-quarter of the cylinders. The cylinder spacing is such that the full line pressure which exists across the last five cylinders cannot jump the series gaps. Any considerable increase of line voltage, however, causes a discharge through the resistance, across these five gaps, and thence to ground. If the discharge becomes sufficiently heavy, the next four gaps break down and assist in the discharge. A very heavy discharge passes to ground through the entire series of gaps. When the line returns to nor-

mal voltage, the cooling effect of the large number of cylinders, combined with the rectifying property of their metallic vapors, tends to prevent the dynamic arc being sustained. Such arresters are suited only for low voltages (up to 5,000 volts), and can absorb only small amounts of energy.

The *oxide-film* lightning arrester has been developed to protect power lines of considerable kilowatt capacity. It consists (Fig. 275) of a porcelain annulus, about 7½ in. (19 cm.) in diameter and ⅝ in. (1.59 cm.) thick, over which two circular metal discs are crimped. The inner surfaces of the discs are coated with oxide films which puncture at about 300 volts. The space between the discs is filled with lead peroxide. A number of such discs are connected in series, the number depending on the voltage of the line. When the voltage per disc exceeds 300 volts, the film punctures and the discharge flows to ground through the peroxide. The heat developed at the point of puncture causes the lead peroxide to change to litharge and red lead, which are both fairly good insulators. After the voltage has dropped to normal, these lead compounds seal the puncture. Hence, in time, the film is replaced by litharge and red lead. This gradually raises the puncture voltage per unit and in time the units must be replaced. Ordinarily, however, it is not necessary to replace the units until after several years of service.

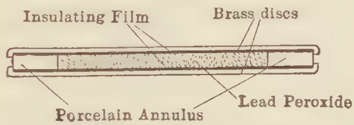


FIG. 275.—Oxide-film lightning arrester unit.

Since such arresters have considerable electrostatic capacitance they would take charging current continuously if connected directly across the line. Hence, a short sphere gap is connected between arrester and line (Fig. 276).

Lightning-arrester stacks are connected as shown in Fig. 276. A choke-coil is connected between the incoming line and the station bus-bar. In a three-phase system each lightning arrester stack is connected between its line, outside the choke-coil, and a common junction forming a neutral. This neutral is connected to ground through a fourth stack called the ground stack. When an incoming surge, which ordinarily consists for the most part of high frequencies, reaches the station, it has the choice of two paths, the inductive path through the choke-coil to the bus-bar

and the condensive path through the arrester to ground. Obviously, the condensive path offers by far the lesser impedance to the high frequencies and the discharge breaks down the oxide film and goes to ground.

DIRECT-CURRENT DISTRIBUTION

Because of the difficulty in obtaining high direct-current voltages, direct-current transmission is not used in this country. In congested city districts, however, direct current is often more desirable than alternating current because of the absence of

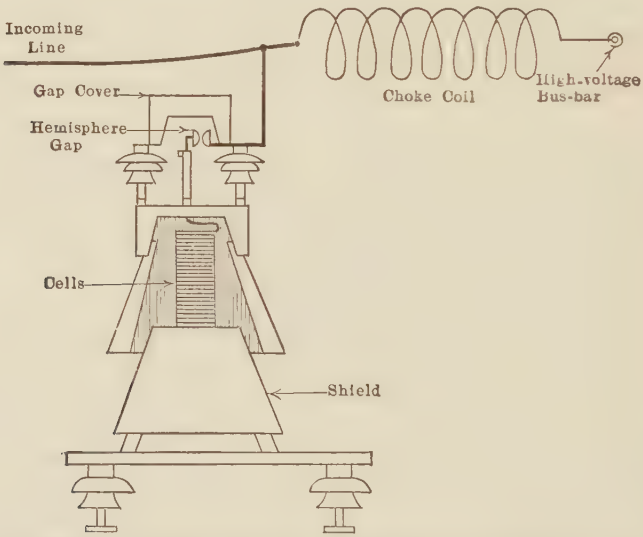


FIG. 276.—Oxide-film lightning arrester with front partially removed.

capacitive and skin effects in cables, the fact that a storage-battery reserve is possible, etc. Also, direct-current motors are preferable for elevators and printing presses, types of load which are common in large cities. Direct current is essential for street railways, since no satisfactory alternating-current motor for street-railway work has as yet been developed.

186. Mains.—Services or loads to individual customers are taken directly from *mains*. These loads are ordinarily concentrated at various points along the mains. If the mains are not too long, single conductors of uniform cross-sections may be used

(Fig. 277 (*a*)). Where the mains are of considerable length, it may be more economical to reduce the cross-section (Fig. 277 (*b*)) as the current decreases. A good rule to remember is that the current *density* in each section should be approximately the same. For example, the first section may consist of a 250,000-

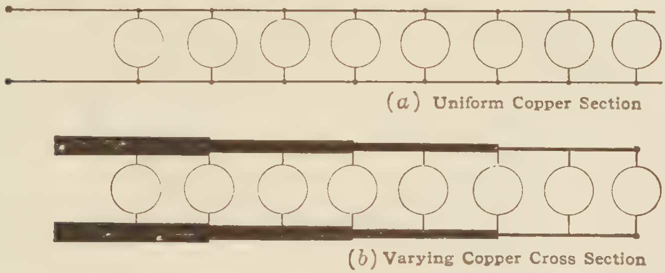


FIG. 277.—Copper cross-section of distributing mains.

C.M. conductor, carrying 200 amp.; assume that the second section carries 150 amp.; it should be a $^{150}\!/_{200} \times 250{,}000 =$ 190,000-C.M. conductor. Ordinarily 4/0 (211,600-C.M.) wire would be used for this second section.

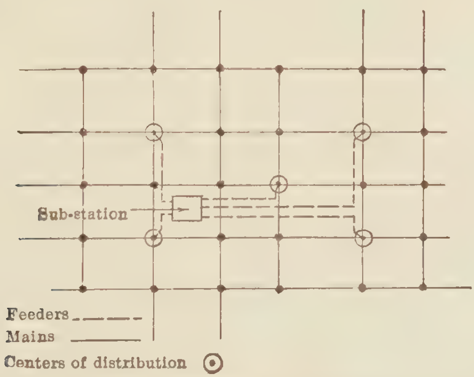


FIG. 278.—Typical feeder and main distribution system.

In a city a direct-current distribution system consists ordinarily of a network of mains all connected together (Fig. 278). The mains are fed at various points called *centers of distribution*, by *feeders* running directly from sub-stations. The centers of distribution are all maintained at approximately the same potential

and the entire system is held to within 1 or 2 per cent. of the same potential.

187. Feeders.—Feeders are conductors, each feeder consisting usually of a positive and a negative conductor (except trolley feeders), which run directly from the station bus-bars to the centers of distribution (Fig. 278). In distinction to mains, services are not taken from feeders. In most instances the only load on the feeders is the single load at its end or the load taken at the center of distribution. When the ladder system is used, trolley feeders often feed the trolley at regular intervals (see page 323).

Since there are no intermediate loads on a feeder, the end may be maintained at the potential to which it is desired to hold the center of distribution.

The potential at the center of distribution is determined by means of two *pressure wires*, which are small conductors, about No. 14 A.W.G., in the cable, and are connected from the center of distribution to a station voltmeter. The potential at the centers may be changed by raising and lowering the bus-bar voltage. If a station feeds several centers, three of four sets of bus-bars, maintained at different voltages, are often used. The feeder is connected to the bus-bars which give the proper voltage at the center of distribution.

Example.—It is desired to maintain the potential at 234 volts at a center of distribution. A 1,000,000-C.M. cable, 800 ft. long, connects the center with the station bus-bars. If the feeder current is 900 amp., to what voltage must the feeder be connected at the station?

The resistance of 1 C.M.-ft. of copper at 20°C. is 10.37 ohms (see Part I, page 41). The resistance per conductor of the cable is

$$R' = 10.37 \frac{800}{1,000,000} = 0.00830 \text{ ohm.}$$

The voltage-drop in each conductor

$$IR' = 900 \times 0.00830 = 7.47 \text{ volts.}$$

The total voltage-drop

$$IR = 2 \times 7.47 = 14.94 \text{ volts.}$$

It would be necessary to have 234 + 15, or 249 volts across the station end of the feeder. *Ans.*

The feeder would probably be connected across 250-volt bus-bars.

188. Electric-railway Distribution.—Electric-railway generators are generally compounded, the series field being on the negative side. The negative terminal is usually connected directly to ground or to the rail through a switch. The positive terminal feeds the trolley through an ammeter, a switch, and a circuit breaker. (See Fig. 279.)

On short lines, with light traffic, the trolley alone may suffice to carry the current to the car. Except in small installations, the trolley is of insufficient cross-section to supply the required

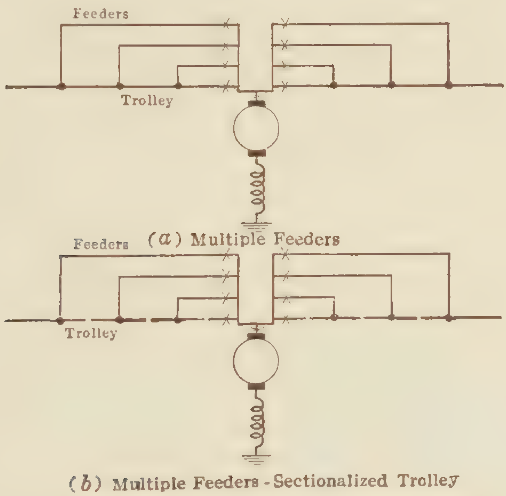


FIG. 279.—Methods of feeding a trolley system.

power. As the size of the trolley wire is limited by the trolley wheel, it cannot be conveniently increased. The same effect as increasing the size of the trolley may be obtained by running a feeder in parallel with the trolley and connecting the feeder to the trolley at short intervals. This is called the *ladder system* of feeding. The trolley and feeder together may be considered as forming a single conductor.

Where the density of traffic requires several feeders, the best results are obtained by connecting the feeders in the manner shown in Fig. 279 (*a*). Each feeder is protected by a circuit breaker.

The objections to the preceding methods of feeding are that trouble, due to a ground, for example, at any point on the trolley, involves the entire system. In cities where traffic is particularly dense, it is not permissible to take chances of having the entire system shut down due to a ground at one point only. The trolley, therefore, is sectionalized (Fig. 279 (*b*)). In this method the trolley is divided into insulated sections, each of which is supplied by a separate feeder. Trouble in one section is not readily communicated to the other sections. This increased reliability is obtained at the expense of a less efficient use of the copper, as the feeders are unable to assist one another.

189. Electrolysis.—Most trolley systems use the track as the return conductor for the current taken by the car. The return currents not only pass through the tracks themselves, but seek the paths of least resistance by which they may return to the

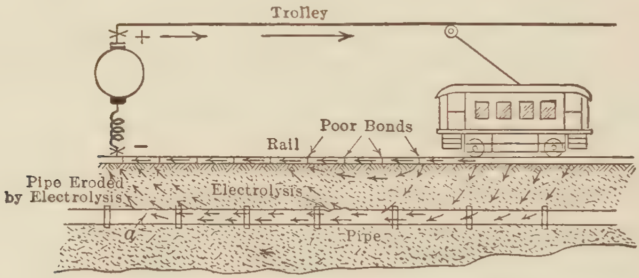


FIG. 280.—Electrolysis by earth currents.

negative terminal of the station generator. Such currents in spreading through the earth follow such low-resistance conductors as water pipes, gas pipes, cable sheaths, etc. (Fig. 280). The fact that the current *enters* and flows along these conductors in itself does no harm. It is obvious, however, that such currents must ultimately leave these pipes as at (*a*) (Fig. 280). In so doing they tend to carry the metal of the pipe into electrolytic solution, which ultimately results in the pipe being eaten away. To decrease the effects of electrolysis, several expedients have been devised. The two most successful methods are the following: (*a*) Provide as good a return path through the track as is practicable. This is done by good bonding and by using insu-

lated negative feeders, that is, heavy copper feeders that are run back to the negative bus from various points along the track. Figure 280 shows how poor rail bonds may cause the current to leave the track and enter the pipe. In some cities the total permissible drop in the ground return circuit must not exceed from 10 to 15 volts. (*b*) Discourage the entering of the current into the pipes by inserting occasional insulating joints in the pipes.

190. Series-parallel System.—Lighting and household service has become standardized at 110-115 volts. (Small isolated plants requiring storage batteries, like farm-lighting systems, may operate at 32 volts.)

From the point of view of the amount of copper, the service voltage should be as high as safety permits, since the amount of copper in mains, etc., varies inversely as the *square* of the voltage. From the point of view of incandescent lamps a low voltage is desirable since the thicker, shorter filaments which result are less fragile and such lamps are more efficient. The service voltage must be a compromise between these conflicting factors and 110–115 volts has been chosen as being the voltage which gives the greatest overall economy. Moreover, it is not so high as to be dangerous to life.

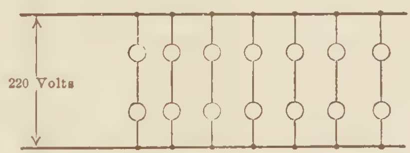


FIG. 281.—Series-parallel system.

If service mains could be operated at 220 volts rather than 110 volts, the weight of copper would be reduced to one-fourth its initial value. With lamps, this saving in copper may be effected by connecting the lamps so that two are always in series (Fig. 281).

The obvious disadvantages of this series-parallel system are that lamps can only be switched in groups of two and if one lamp burns out, the lamp to which it is connected ceases to operate. Also, both of the lamps in series must be of the same rating.

191. The Edison 3-wire System.—The objections to the series-parallel system may be eliminated by running a third wire, called a *neutral*, between the two outer wires. This neutral maintains all the lamps at approximately 110 volts. The advantage of a

higher voltage in reducing the weight of copper is obtained by the use of this system. If there were no neutral wire, the 220-volt system would require one-fourth the copper of an equivalent 110-volt system. If it be assumed that the neutral of the Edison 3-wire system is of the same cross-section as the two outer wires, the total copper for the 3-wire system is 3/8 or 37½ per cent. of that for a 110-volt system of the same kilowatt capacity. The saving in copper, therefore, is 62½ per cent. In practice, the neutral can be made smaller than the two outer wires so that the saving in copper is even greater than 62½ per cent.

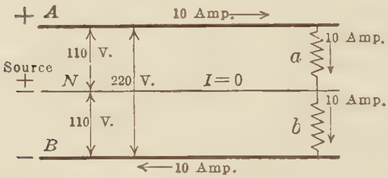


FIG. 282.—Edison 3-wire system—balanced loads.

The general plan of the system is shown in Fig. 282. Two wires A and B have 220 volts maintained between them, A being the positive and B the negative. A third wire N is maintained at a difference of potential of 110 volts from each of the other two wires. N, therefore, must be negative with respect to A and positive with respect to B. That is, current tends to flow from A to N, and from N to B.

Figure 282 shows the conditions which exist when the loads on each side of the system are equal. Each of the loads a and b

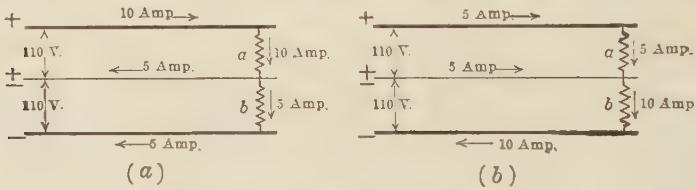


FIG. 283.—Unbalanced 3-wire systems.

takes 10 amp. The 10 amp. taken by load a passes through to load b and then back through wire B to the source. This is equivalent to a series-parallel system as both loads are equal and in series. Under these conditions the current in the neutral wire is zero and the loads are said to be balanced.

Figure 283 (a) shows the conditions existing when the load a on the positive side of the system is 10 amp., and the load b on

the negative side is 5 amp. Under these conditions the extra 5 amp. taken by load *a* must *flow back* through the neutral to the generator or source. There are 5 amp., therefore, in the neutral returning to the generator. In Fig. 283 (*b*) the load *b* is now 10 amp. and load *a* is 5 amp. Under these conditions the extra 5 amp. must *flow out* to the load through the neutral. It will be observed that the current in the neutral may flow in either direction, depending upon which load is the greater. If an ammeter, therefore, is used in a neutral it should be of the zero-center type. Moreover, it will be observed that the neutral carries the *difference* of the currents taken by the two loads. In practice the loads are usually so disposed that they are nearly balanced. Twenty-five per cent. unbalancing (that is, a neutral current which is 25 per cent. of that in the outer wires) is usually allowed.

If the neutral of the 3-wire system is opened and the loads are unbalanced, the voltage across the lesser load, which obviously has the higher resistance, will exceed that across the greater load. This may result in lamps being burnt out, etc.

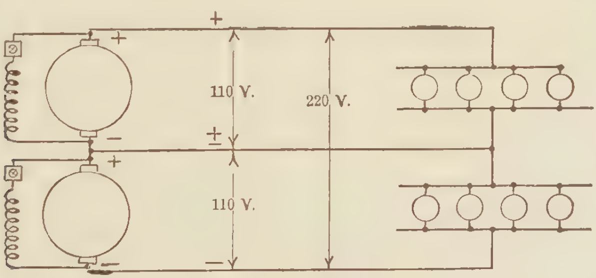


FIG. 284.—Two generators supplying a 3-wire system.

192. Methods of Obtaining 3-wire System.—*Two-generator Method.*—Two shunt generators may be connected in series as shown in Fig. 284. The positive terminal of one should be connected to the negative terminal of the other, that is, the generators are in series between the outers. Both generators may be driven by the same prime mover. When connected in this manner, each machine supplies only the load on its own side of the line. The obvious objection to this method is that two separate machines are required.

Balancer Set.—A balancer set is a very common method of obtaining the neutral. This set consists of a motor and a generator mechanically coupled together. They are connected in series across the outer wires and the neutral is brought to their common terminal, as shown in Fig. 285. Either machine may act as a motor and either machine may act as a generator, depending on which side of the system has the greater load. The action of this set is as follows: If the system is in balance, both machines operate as two shunt motors connected in series across the outer conductors. They take just enough current to supply their losses If the loads become unbalanced, as shown in Fig. 285,

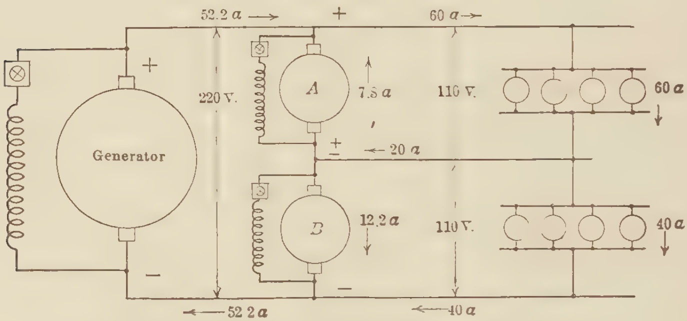


FIG. 285.—Balancer set giving neutral in a 3-wire system.

where a load of 60 amp. is shown across the positive side of the system and a load of 40 amp. is shown across the negative side of the system, machine *A* across the side having the greater load acts as generator and the other machine *B* acts as motor. The motor *B* takes *some* of the current returned by the neutral and utilizes the power represented by the product of this current and the voltage across its side of the system, to drive the machine *A* as generator. The generator *A* then causes the remainder of the neutral current to be returned to the positive line. In Fig. 285, each machine is assumed to have an efficiency of 80 per cent., giving 64 per cent. as the efficiency of the set. The system voltages are also assumed to be in balance. The motor requires 12.2 amp. and the generator delivers only 0.64 of this current or 7.8 amp. to the positive conductor. If the balancer set were 100 per

cent. efficient, the motor would take 10 amp. and the generator would deliver 10 amp.

With shunt machines, better voltage balance is obtained if the shunt field of the machine on the negative side of the system is connected from neutral to positive, and the shunt field of the machine on the positive side of the system is connected from neutral to negative In practice each machine has a series winding which is cumulative when the machine acts as generator and differential when it acts as a motor. This accentuates both the motor and the generator action without the voltages of the system becoming too greatly unbalanced.

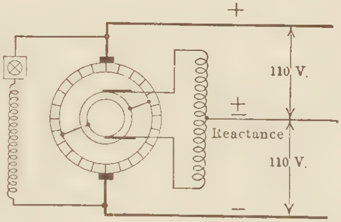


FIG. 286.—3-wire generator connections (Dobrowolsky method).

In practice the capacity of the balancer set is of the order of only, 5 or 10 per cent. of the generating capacity, depending on the degree of unbalancing.

Three-wire Generator.—The 3-wire generator or Dobrowolsky method is a very efficient method of obtaining a neutral. The method was mentioned in connection with the synchronous converter (see page 295). In its simplest form, the ordinary 3-wire shunt or compound generator has two slip-rings in addition to its commutator (Fig. 286) and across these rings an alternating e.m.f. must obviously be generated. In fact, the generator operates as a double-current generator, delivering practically all its power from the d.-c. or commutator end. The slip-rings deliver only sufficient current to excite the reactance coil which is connected across them. This coil has low resistance but very high reactance. Direct current, therefore, can flow through it very readily but it can take but a very small alternating current from the slip-rings. The center of the coil is at the center of gravity of the voltages generated within the armature. If the 3-wire neutral be connected to the center of this coil, therefore, the voltage to either brush from the neutral will be the same. Moreover, any current coming back through the neutral can readily flow into the armature through this reactance, since reactance has no effect on the flow of a steady direct current. The neutral current divides, half

flowing each way through the reactance. Occasionally the reactance is placed within the armature. This arrangement requires but one slip-ring, but increases the weight of the armature.

193. Storage Batteries.—Theoretically, storage batteries can be used to equalize the load on direct-current systems, relieving the generating equipment on the heavy loads or "peaks" and being charged during light loads or "valleys." Practically, it is more economical, except in special instances, to operate extra machinery on the peak loads, because of the high maintenance cost of storage batteries. In places where a complete shutdown would be disastrous, however, as in a large city, batteries are frequently installed to carry the load in emergencies. They are kept charged, ready to put in service, or are even kept "floating" across the bus-bars, enabling them to take the load automatically in case of shutdown of the generating equipment.

The most common method of controlling the discharge of such batteries is to have an excess of cells, called end-cells (Fig. 287), which may be cut in or out, according to the load the battery is called upon to deliver. It is essential to cut cells in and out without opening the circuit. For this purpose an end-cell switch is used similar to that shown in Fig. 287. The main contact is connected to the auxiliary contact by a resistance R. When sliding from one battery contact to the next, the auxiliary contact maintains the circuit connections through the resistance R. Were there zero resistance between the main contact and its auxiliary contact, the individual cells would be dead short-circuited during the transition period. The resistance R is usually so chosen as to allow the normal battery current to flow during the transition period. The end-cell switches become rather massive in large battery installations and are often operated by a motor-driven worm. This also permits remote control.

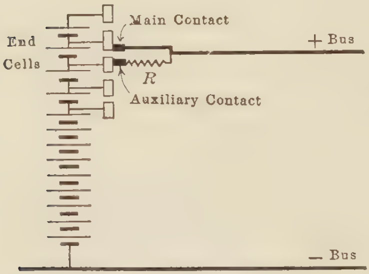


FIG. 287.—End cell control of storage battery (transition period).

The end-cells, not being in continuous service, are discharged to a lesser degree than the others. They require, therefore individual attention on charging.

194. Series Distribution.—Series distribution is used where the translating devices all take the same current and where the resulting high voltage is not objectionable. The advantages of such a system are the large saving in copper and the simplicity of the circuit. Practically the only systems which fulfil the foregoing requirements are street-lighting systems, where the lamps all take the same current, operate at the same time, and since the installation is out of doors, the high voltage is not objectionable.

The series system differs from the parallel system in the manner of removing loads. If a load is open-circuited in the constant-potential system, the other loads are not affected except perhaps by a slight change in voltage. In the series system, the loads are all in series with one another so that the same current passes through each. If, therefore, the circuit of any one load be opened, the current to all the other loads will be interrupted. As this is not permissible in practice, a load must be *short-circuited* when it is desired to remove it from service.

Direct current is required for magnetite arc-lamps and is obtained either from a series direct-current generator or from a constant-current transformer operating in conjunction with the mercury-arc rectifier (see page 303). Alternating current is supplied by a constant-current transformer without the rectifier. These methods all tend to maintain constant current under all conditions of load. If, therefore, the circuit be opened and a very high resistance thus introduced, a constant current is maintained across a high resistance and a very high voltage results. For this reason the lamps used on a constant-current system are protected by having a thin disc of paper between the lamp terminals (film cut-out). If the lamp burns out, the high voltage across this paper punctures it and so prevents the circuit being opened.

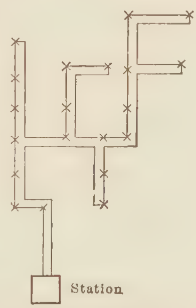


FIG. 288.—Parallel-loop series circuit.

The parallel-loop system (Fig. 288) is the most common system of series distribution.

CHAPTER XII

ILLUMINATION AND PHOTOMETRY

Light is a form of radiant energy and is probably due to vibrations of very short wave length set up in the ether by luminous bodies. It has the property of producing the sensation of vision on the retina of the eye and so enables objects to be seen and distinguished.

Illumination means specifically the light incident on a surface or object, but in a broader sense it has come to signify that branch of engineering having to do with the distribution and utilization of light. The measurement of light and light distribution is called *photometry*.

195. Candlepower.—The brightness of a light source is called its *luminous* intensity. The luminous intensity of a body is measured in terms of the light intensity in a horizontal direction given by a standard candle, and is called candlepower. Candlepower is denoted by I.[1] That is, if a light source, such as an incandescent lamp, were replaced by 14 standard candles without altering either the total light emitted or its distribution, the incandescent lamp would have a luminous intensity in a *horizontal* direction of 14 cp.

Because of its inherent inaccuracies, the candle has long since been abandoned as a working standard of luminous intensity. At the present time, lamps, calibrated at some known voltage by the Bureau of Standards, are used as working standards of luminous intensity.

Mean spherical candlepower is the average luminous intensity of an illuminant taken in all directions in space. Light sources seldom have equal luminous intensity in every direction in space. Ordinarily, the luminous intensity in the horizontal zone is greater

[1] Photometry symbols will often be found to duplicate electrical symbols. For example, I = candlepower and in electrical units I = current. Photometric and electric units are not of the same character.

than the mean spherical candlepower. The ratio of mean spherical to mean horizontal candlepower is called the *spherical reduction factor.* The spherical reduction factor is ordinarily less than unity.

Example.—A 60-watt Mazda lamp has a mean horizontal luminous intensity of 51 cp. and a mean spherical luminous intensity of 41 cp. What is its spherical reduction factor?

$$\text{Spherical reduction factor} = \frac{41}{51} = 0.804. \quad \textit{Ans}$$

196. Law of Inverse Squares.—In Fig. 289 is shown a light source and also one cone of light flux which it emits, the cross-section of the cone being so small that the light intensity over it is substantially uniform. Consider two cross-sections A_1 and A_2 at right angles to the axis of the cone and at distances D_1 and D_2 from the light source. Since light waves, in a homogeneous medium, travel in straight lines, no light enters or leaves the cone through its surface. Hence, the same light flux must cross both sections A_1 and A_2. Since the diameters of both A_1 and A_2 are proportional to their distances D_1 and D_2 from the light source at the apex of the cone, their areas must be proportional to the *squares* of the distances D_1 and D_2. Thus,

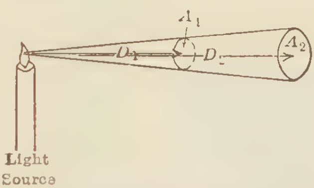


FIG. 289.—Illustration of the law of inverse squares.

$$\frac{A_1}{A_2} = \frac{D_1^2}{D_2^2}.$$

As the same light flux crosses each section, the ratios of the luminous intensities must be inversely as the areas. That is, the ratio of intensities

$$\frac{E_1}{E_2} = \frac{A_2}{A_1} = \frac{D_2^2}{D_1^2}. \tag{68}$$

The intensity of illumination from a point source, therefore, varies inversely as the square of the distance from the source.

This law of inverse squares is strictly true only when the light source is a point. It is impossible to obtain a point source in practice. With the usual light sources, no great error is intro-

duced in assuming a point source, unless the distance from the source is small.

Intensity of illumination is expressed in *foot-candles*. A surface 1 ft. from a light source having a luminous intensity of 1 cp. and normal to the direction of the light rays has an illumination of 1-foot-candle.

It follows from (68) that the intensity of illumination in foot-candles is equal to the candlepower of the source divided by the square of the distance in feet.

Example.—A Mazda C lamp suspended from the ceiling has a luminous intensity of 75 cp. in a vertically downward direction. (*a*) What is the illumination on a horizontal surface 1 ft. beneath this lamp? (*b*) What is the illumination on the surface of a table 3 ft. beneath this lamp? (*c*) If the lamp is raised 1 ft., what does the intensity of illumination on the table become?

(*a*) $E_1 = \frac{75}{1^2} = 75$ foot-candles. *Ans.*

(*b*) $E_2 = \frac{75}{(3)^2} = \frac{75}{9} = 8.33$ foot-candles. *Ans.*

(*c*) $E_3 = \frac{75}{(4)^2} = \frac{75}{16} = 4.69$ foot-candles. *Ans.*

ELECTRICAL ILLUMINANTS

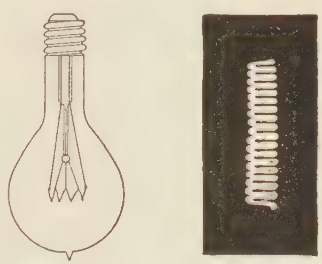
197. Incandescence and Luminescence.—Light is emitted by bodies under two conditions, incandescence and luminescence. Incandescence is produced by raising the body to a high temperature. The radiation increases very rapidly with increase of temperature, so that a small increase in temperature produces a large increase in light intensity Only those substances which can operate at very high temperatures make efficient illuminants. Luminescent substances, however, can emit light at moderate and even very low temperatures. Examples of luminescent substances are the mercury-arc, vacuum tubes such as the Moore tube, and the firefly.

INCANDESCENT LAMPS

198. Carbon Lamps.—For a number of years the only successful incandescent lamp was the carbon-filament lamp invented by Edison. The filament was made of carbonized bamboo and was operated in a vacuum. Although carbon has a very high melting point, it also has a high vapor pressure.

That is, carbon vapor is given off very readily at comparatively low temperatures. Hence, this type of lamp could not operate at such temperatures as would give it high efficiency because the carbon would evaporate readily and the life of the lamp would shortened. Carbon lamps with untreated filaments had an be efficiency of 3.0 watts per horizontal candle and "G.E.M. metallized"- filament lamps, whose filaments consisted of carbon flashed in a gas rich in hydrocarbons, had an efficiency of 2.5 watts per horizontal candle.

199. Mazda B Lamps.—The metal tungsten has a high melting temperature, a low vapor pressure, and a high resistivity. These properties make it ideal for a lamp filament. In 1911 the process of drawing tungsten wire was perfected so that tungsten filaments, which had hitherto been fragile, could be made tough and, therefore, had long life. In the Mazda B lamp the filament operates in a vacuum as did the carbon filament, but since it operates at a much higher temperature its efficiency is greater. The efficiency is about 1.2 watts per mean horizontal candlepower. The lamp could be operated even at a much higher temperature, hence at better efficiency, were it not for the rapid evaporation of the tungsten and the resulting decrease of life and the blackening of the bulb.



(a) (b)

Fig. 290.—Gas-filled lamp (300 watt) and section of filament.

200. Mazda C Lamps.—In the Mazda C lamp the filament is operated in an atmosphere of nitrogen and helium whose pressure is slightly greater than atmospheric, rather than in a vacuum. The pressure of the gas reduces evaporation and permits operation at higher temperature and hence greater efficiency. Since the gas increases the convection losses, it is necessary to coil the filament (Fig. 290 (*b*)) so as to reduce the surface of the filament exposed to the gases. The bulbs are made with a long neck (Fig. 290 (*a*)), in which the heated gases in rising deposit the tungsten vapor. The neck is, therefore, blackened by the condensed vapor, but it cuts off but a negligible portion of the light

flux. It is desirable, therefore, to burn the lamp with the neck upward. The Mazda C lamps have an efficiency of approximately 0.85 watt per mean horizontal candlepower in the smaller sizes and 0.60 watt per mean horizontal candlepower in the larger sizes.

Incandescent lamps are usually designed to operate at a temperature which will give them a life of approximately 1,000 hrs.

ARC LAMPS

201. Arc Lamps.—Arc lamps were the first devices to be used as electric illuminants, the arc being maintained between carbon electrodes by the electric current. Practically all the light from such lamps comes from the incandescent crater of the positive electrode. Such lamps when operated from constant-potential mains require "ballast" or series resistance with direct current and series reactance with alternating current. The object of the ballast is to steady the arc. The resistance of the arc itself tends to decrease as the current increases, and the arc, therefore, automatically takes more and more current. Each increase in current still further decreases the resistance, until finally, short-circuit results. Carbon arcs, with the exception of searchlights and projection machines, are practically obsolete. The metallic electrode or magnetite lamp has taken its place as an illuminant.

202. The Metallic Electrode or Magnetite Lamp.—The metallic-electrode arc lamp or luminous arc or magnetite lamp (Fig. 291) as it is often called, differs from other arc lamps in that it employs metallic electrodes and the light is derived from the arc itself, being due to the luminescence of the vapor which comes from the cathode or negative electrode. The positive electrode is a large cylinder of solid copper. The copper being an excellent conductor of heat tends to keep moderately cool. The negative electrode is made of an iron oxide containing titanium to give a white light. Other ingredients are added to give the electrode desirable burning characteristics.

The arc stream consists of negatively charged gas particles from the negative electrode, which are attracted toward the positive electrode. Hence, the copper must always be positive. If it becomes negative, the arc will then consist of luminescent copper vapor and be green in color.

As the electrodes are comparatively cool under operating conditions, there is not sufficient heat to maintain the arc if the electric power is interrupted even for an instant. Hence, outside the question of the greenish arc resulting from the copper operating as cathode, this type of lamp cannot be used with alternating current.

With other types of arc lamp the electrodes are touching when not in service. The application of current causes solenoids to pull the electrodes apart and hence draw out the arc. This type of feed cannot be used with the magnetite lamp since the hot

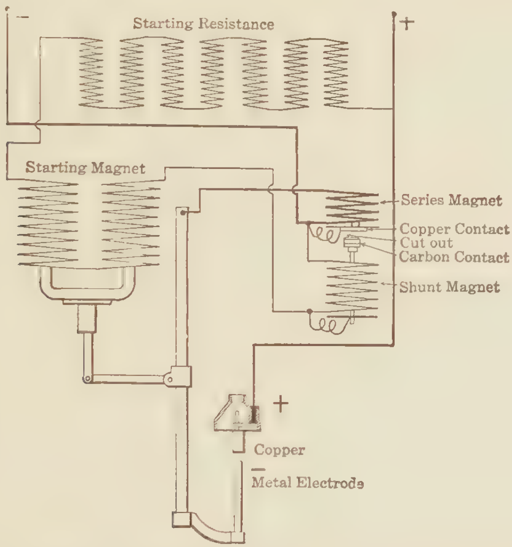


FIG. 291.—Mechanism and connections of luminous arc lamp, series type.

metal of the cathode or negative electrode would weld to the copper and "freeze" when the current was turned off. Therefore, the mechanism is so designed that the feed is intermittent, the arc being maintained by restriking. When the lamp is out of circuit its electrodes are separated by a gap. When current flows, the starting magnet (Fig. 291) brings the lower electrode into contact with the upper electrode, striking a sharp blow. This operation allows the current to flow through the series mag-

net which, acting in conjunction with the remainder of the feeding mechanism, allows the lower electrode to fall, drawing out the arc. When the arc becomes too long, the regulating mechanism causes the lower electrode again to strike the upper electrode a sharp blow, bringing about a shorter arc once more.

The intense white light of this type of lamp makes it very attractive, particularly when the lamp is mounted on ornamental poles. In addition to general street lighting, it is used to a considerable extent for "white way" and boulevard lighting.

PHOTOMETRY

Photometry is the measurement of light. Light measurements are nearly all made by direct comparison. The chief source of inaccuracy in making such comparisons is the question of color. Unless the lamps under comparison are of almost exactly the same color, only an approximate photometric balance can be obtained. This balance varies with different persons owing to the effect of different colors upon the eye.

203. The Bunsen Photometer.—The Bunsen photometer (Fig. 292) is the simplest type of photometric measuring device. Assume that it is desired to measure the candlepower of the incandescent lamp *L*, using the candle *C* as a standard for comparison. The two lights are placed some 10 to 20 ft. (3 to 6 m.) apart and a movable screen *S* is placed between them. The screen *S* consists of a piece of paper or parchment with a grease spot in the center. The grease spot on the screen is translucent and so allows light to pass. If viewed from the side which is illuminated, the spot will appear darker than the rest of the screen, owing to the fact that light passes through the translucent grease spot more readily than through the surrounding part of the screen.

On the other hand, if this same screen be viewed from the non-illuminated side, the grease spot will appear brighter than the rest of the screen, since it is more translucent than the rest of the screen. If both sides of the screen receive equal illumination, the grease spot will look the same in comparison with the surrounding portion of the disc, when viewed from either side. When this occurs, a photometric balance is obtained.

In order that the observer may view both sides of the screen simultaneously, two mirrors M (Fig. 292) are set at an angle and reflect the light in the manner shown by the dotted lines.

The screen S is moved until the screen looks the same on both sides, and the distances l and l_1 are read. Let E be the candlepower of the candle or standard and E_1 the candlepower of the test lamp. Remembering that the intensity of illumination varies inversely as the square of the distance, the luminous

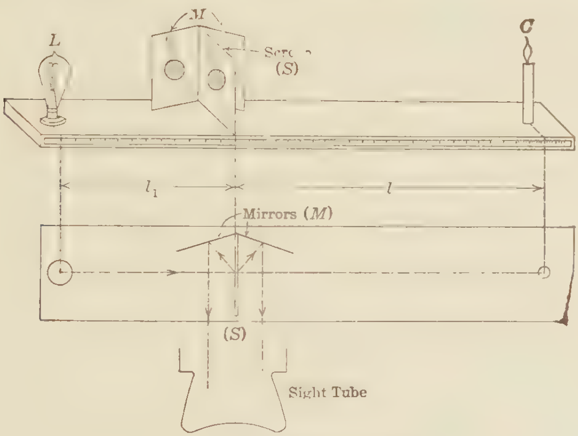


FIG. 292.—Bunsen photometer.

intensities of the source must be proportional to the squares of their distances from the screen.

$$\frac{E_1}{E} = \frac{l_1^2}{l^2}.$$

The candlepower of the test lamp

$$E_1 = E\frac{l_1^2}{l^2}.$$

If a standard candle is used, E equals 1.0.

If the two lights have different color, the two sides of the screen will never appear alike and only an approximate balance can be obtained. The position of balance is to a considerable extent determined by the personal equation of the observer.

Because of the unreliability of candles, standard incandescent lamps are used. These may be obtained from the Bureau of

Standards at Washington. Such lamps, when used at the voltage at which they are calibrated, are very accurate standards. It is customary to use the standard lamp only to calibrate a working standard so that the candlepower of the ultimate standard will not change due to its being in too constant use.

As the candlepower of lamps is very sensitive to changes in voltage, the connections are often made as shown in Fig. 293. Both lights are fed through an adjustable resistance R_2. An adjustable resistance R is in series with the standard lamp and another adjustable resistance R_1 is in series with the test lamp. A voltmeter is in parallel with each lamp. Both lamps are brought to the desired values of voltage by adjusting roughly

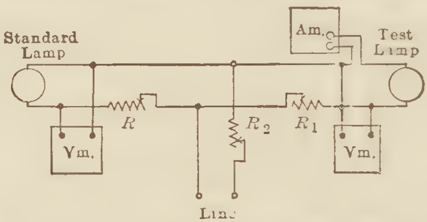


Fig. 293.—Connections for photometric test.

R_2, and then by separate adjustments of R and R_1. Any fluctuations of line voltage can be taken care of by the resistance R_2, which affects both lamps. Further, any unnoticed change of line voltage affects both lamps approximately to the same degree if the lamps have similar voltage-candlepower characteristics.

204. The Lummer-Brodhun Photometer.—In the Lummer-Brodhun screen (Fig. 294) the field of view consists of two clearly defined elliptical areas. When a photometric balance is obtained, the two ellipses merge into one if the lights have the same color. The operation of this screen is as follows: S_1, S_2 is a white, opaque screen. The light coming from one source falls on the side S_1 and that coming from the other source falls on the side S_2. The brightness of S_1 and of S_2 depends on the intensity of the source which illuminates each. The light from S_1 and S_2 is reflected by the plane mirrors M_1 and M_2 to the total reflecting prisms P_1 and P_2. The hypotenuses of the two prisms are in contact over a circular area only. The light striking this area of

contact can pass through; other light is totally reflected. That is, only the central beam, shown dotted, from the mirror M_1, passes through to the eyepiece. The remainder of the light is turned away. On the other hand, the central beam, shown by a solid line, from M_2 passes through the center circle and is absorbed by the walls of the box. The remaining light is reflected to the eyepiece The observer sees two distinct ellipses if the photometer is out of balance. (The circle of contact of

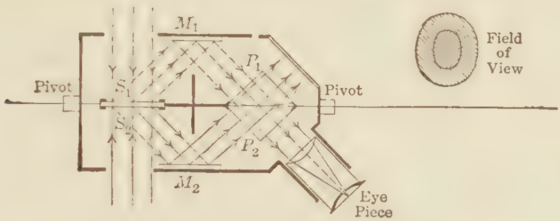


FIG. 294.—Lummer-Brodhun photometer.

the two prisms appears as an ellipse to the observer because he is viewing the circle at an angle.) When the transmitted light from M_1 (center dotted line) is equal in intensity to the reflected light from M_2 (two outside solid lines) the two ellipses have the same appearance. That is, when the two ellipses blend, the same illumination is coming from each source and the photometer is in balance. To eliminate any errors due to differences in the two sides of the screen S_1 and S_2, the photometer screen should be reversed.

CHAPTER XIII

ELECTRON TUBES

205. Electrons.—Electron tubes, vacuum tubes, or thermionic valves, as they are called, depend for their operation on the fact that, according to the latest investigations, electricity is atomic. That is, electricity is composed of extremely small *negative* charges called *electrons*. The charge of each electron is 1.59×10^{-19} coulomb, and its mass is 1/1,845 of that of the hydrogen atom or the extremely small mass 9.0×10^{-28} gram. An atom of matter consists of a positively charged nucleus, called the proton, with which the electrons are associated, the number of electrons depending on the substance. In a *neutral atom* the number of electrons associated with the proton is such that the resultant charge of the atom is zero. In non-conductors of electricity, the electrons are very closely associated with the proton and it is difficult to remove an electron from the atom. In the metals, which are conductors, a small proportion of the electrons appear to be free in the sense that they are able to pass easily from one atom to the next. But even so, the density of these free electrons in a metal is extremely large, being approximately 16,000 coulombs per cubic centimeter.

These free electrons in a metal are supposedly in constant motion and are continually colliding with one another and with the atoms of the metal, which are also in motion. Their motion is similar to that of the atoms of a gas in a confined space. As with the atoms of a gas, the velocities of the individual electrons at any instant differ widely, but their velocity as a whole gives an average velocity which is determined by the temperature of the metal.

206. Emission.—The surface of the metal is a boundary surface for the free electrons and is impervious to most of them. A force of repulsion is exerted at this surface on the electrons in contact with it, which turns back all those whose velocities fall

below a certain critical value, while allowing those having velocities greater than this critical value to pass through. If the space outside the metal is evacuated, it will gradually become filled with electrons. These electrons collide among themselves, however, causing some of them to return to the metal through the surface. This process is accelerated by the *space-charge* effect of the electrons. That is, the electrons in the space outside the metal, all being negatively charged, mutually repel one another and drive some of the electrons back into the metal. This effect is aided by the fact that the withdrawal of negative charges from the metal leaves it charged positively, which in turn tends to attract the electrons back to itself. A condition of equilibrium is reached when the number of electrons leaving the metal is equal to the number returning to it in any given time.

207. Critical Velocity.—The critical velocity which an electron must have in order to escape from a metal is of the order of magnitude of 10^8 cm. per second. It varies somewhat for the different metals. It is more usually expressed in terms of the difference of potential through which an electron must fall in order to acquire this velocity, because this difference of potential is the quantity actually measured. The energy of an electron carrying a charge e and having a mass m when it has fallen freely through a difference of potential V and acquired a final velocity v is

$$Ve = \tfrac{1}{2} mv^2. \tag{69}$$

The difference of potential corresponding to a velocity of 10^8 cm. per second is about 4 volts.

The emission of electrons from a metal is analogous to evaporation from a liquid. The surface tension of the liquid corresponds to the apparent repulsive force at the surface of the metal. The latent heat of evaporation corresponds to the work done by the electrons against the repulsive force at the surface.

At room temperature, the number of electrons emitted from a metal over any ordinary interval of time is very small, because the average velocity of the electrons within the metals is low and only occasionally does one attain sufficient velocity to escape from the surface. The emission may be increased by increasing the average velocity of the electrons by raising the temperature. The

emission increases very rapidly with increase of temperature. The emission may also be increased by diminishing the surface repulsive force with the action of ultra-violet light or x-rays (photoelectric effect), and by the impinging on the surface of high-speed electrons.

Richardson, in 1901, showed that the emission per unit area is an exponential function of the temperature and is also a property of the material. Hence, the emission increases very rapidly with temperature (Fig. 297). This relation is known as *Richardson's law.* Barium oxide has better emission characteristics than thorium, and thorium has much better emission characteristics than tungsten. Hence, oxide-coated filaments (platinum coated with barium and strontium oxide) emit more electrons at a given temperature than tungsten, and require less energy.

208. Operating Temperature.—The maximum temperature at which filaments may be operated depends on their melting points and their physical dimensions. The temperature is usually chosen to give a life of approximately 1,000 hr. This corresponds to current practice with incandescent lamps (see page 336). The operating temperature for tungsten filaments 0.01 inch in diameter is approximately 2,200°C. and for platinum filaments coated with barium and strontium oxides is about 1,300°C. Thorium operates at an intermediate value. At these temperatures, their emissions are approximately the same.

209. Thermionic Efficiency.—In practice the metal from which emission takes place is in the form of a long filament of small diameter which is heated by the passage of an electric current. The energy necessary to maintain this filament at a given high temperature is determined mainly by the heat radiation from it. The energy radiated varies as the fourth power of the absolute temperature, which is the Stefan-Boltzmann law of thermal radiation. Thermionic efficiency is defined as the ratio of the current emitted to the power input, both for the same surface area. For tungsten at 2,200°C. this efficiency is approximately 0.01 amp. per watt, and for oxide-coated platinum at 1,300°C. approximately 0.06 amp. per watt.

When a small fraction of thorium is alloyed with tungsten, the emission of electrons is greatly increased. A surface layer of thorium is apparently formed on the tungsten filament. This

lowers the safe operating temperature to around 1,600°C. and brings the thermionic efficiency up to about 0.05 amp. per watt. This filament is referred to as thoriated tungsten. Cerium is also being alloyed with tungsten. Its electron emission and thermionic efficiency are much greater than for thoriated tungsten or even for the oxide coating.

210. Space Charge.—It was shown in Par. 206 that when a hot filament which emits electrons is placed in an evacuated chamber, the space becomes filled with electrons. The space is then called an *electron gas* or *space charge.* The density of the charge is not uniform but obviously is greatest next to the hot filament. Equilibrium is attained when in a given time as many electrons fall back into the filament as are emitted by the filament.

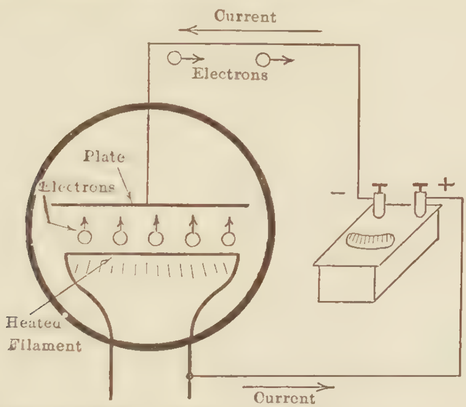


FIG. 295.—Emission of electrons from a heated filament with no plate battery.

211. Two-electrode Tube.—If a cold electrode, called the anode or plate (Fig. 295) is inserted in the evacuated chamber containing a heated filament, some electrons will reach it and it will assume the potential of the space it occupies, which will be slightly negative with respect to the filament. Let it be assumed for the present that the filament is heated in such a way that there is no fall of potential along it. If now the anode or plate is connected through a sensitive galvanometer back to the filament, a small current will flow. In the usual or conventional sense its direction will be from the filament through the galvanometer to the plate (Fig. 295). Actually, the motion of the electrons,

which are negative charges, constitutes the current. This motion is in the opposite direction or from the filament to the plate inside the tube.

212. Child's $\frac{3}{2}$ Power Law.—When a voltage is applied between plate and filament, making the plate positive with respect to the filament, the positive plate will attract the negative electrons and a much larger current will flow than when the applied voltage is zero. This may be determined experimentally by connecting the tube in the manner shown in Fig. 296. The plate is made positive with respect to the filament by means of the battery B. The voltage E between the filament and plate may

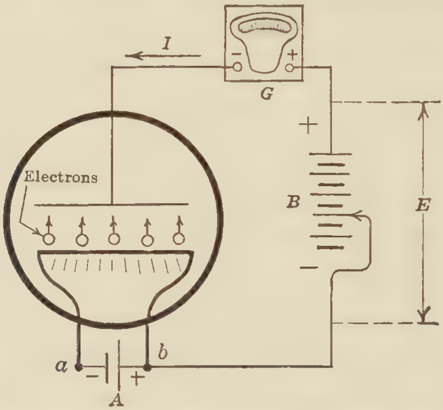


FIG. 296.—Emission of electrons from a heated filament with a plate battery.

be varied to E', E'', etc., by the battery tap, as shown. A sensitive galvanometer or microammeter G is connected in circuit to measure the plate current. The manner in which the plate current varies with the temperature of the filament for different applied plate voltages is shown in Fig. 297. First apply E' volts between plate and filament. For low temperatures and the correspondingly small emissions, the voltage E' is sufficiently large so that *all* the emitted electrons are attracted to the plate, and the current increases rapidly with the temperature along the curve *oa*. As the temperature and, hence, the emission increase, the density of the electron cloud between filament and plate also increases, and, hence, the repulsive force on the electrons leaving the filament, due to this negative space charge, increases. Ulti-

mately, this repulsive force becomes greater than the attractive force due to the plate voltage E'. When this condition is reached, the rate at which the electrons return to the filament is equal to the rate at which they are emitted and the plate current becomes constant as at b (Fig. 297). This is called *space-charge saturation*. If the plate voltage is raised from E' to E'' (Fig. 297) its attractive effect on the electrons increases. Obviously, it will require a greater space charge than before to drive the electrons back into the filament at the same rate as the filament is emitting them. Hence, *space-charge saturation* now occurs at c (Fig. 297), corresponding to a greater value of electron emission current.

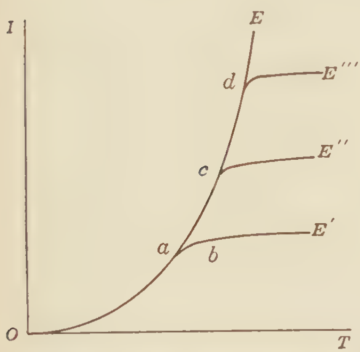


Fig. 297.—Plate current as a function of temperature for different voltages.

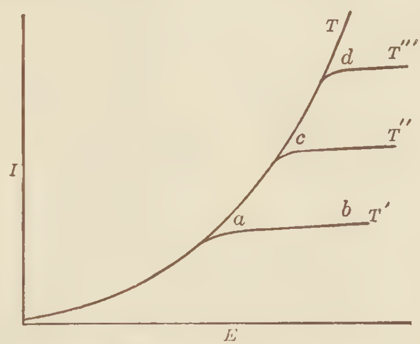


Fig. 298.—Plate current as a function of voltage for different temperatures.

The plate current may also be considered a function of plate voltage for different filament temperatures T', T'', etc., as shown in Fig. 298. At low plate voltages the electron emission or current is limited by space charge and increases as the voltage to the $\frac{3}{2}$ power. This is called *Child's $\frac{3}{2}$ Power Law*. At a given filament temperature, such as T', the voltage ultimately reaches such a value that its attractive force at the filament becomes larger than the mutual repulsive forces due to the space charge and all of the electrons emitted are drawn to the plate. The current becomes constant and is, therefore, independent of the plate voltage. This condition for temperature T' is represented

by the part *ab* of the curve. This is the true filament saturation. As the filament temperature is increased, the emission increases and filament saturation occurs at higher values of plate current as at *c*, and *d* (Fig. 298).

From the foregoing, it is obvious that the plate current is a function of two quantities or parameters, plate voltage and filament temperature.

213. Edison Effect.—When the filament is heated by passing a current through it, there is a fall of potential along the filament because of the resistance-drop. The various parts of the filament, therefore, will have different potentials with respect to the plate.

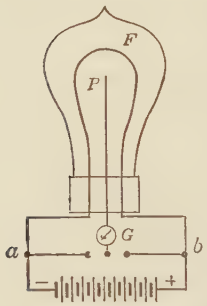


Fig. 299.—Edison effect.

For example, in Fig. 296 the left-hand end of the filament is connected to the negative terminal *a* of the filament battery *A* and the right-hand end of the filament is connected to the positive terminal *b* of the filament battery *A*. Hence, the right-hand end of the filament is at a higher potential than the left-hand end. For example, if the e.m.f. of the *A* battery is 6 volts, the potential of the right-hand end of the filament is 6 volts higher than that of the left-hand end. If the *B* battery (Fig. 296) be removed as in Fig. 295 and the plate be connected to the positive end of the filament, its potential above the negative end of the filament will be the voltage of the *A* battery. Its potential above the various parts of the filament diminishes toward the positive end of the filament. If the plate be connected to the negative end of the filament (Fig. 296), no current flows in the plate circuit as the plate is negative to all other parts of the filament. Edison first noticed this effect in 1883 when he was developing the incandescent lamp. If he connected a plate *P* sealed in the bulb near the filament *F*, through a sensitive galvanometer *G* to the negative terminal *a* of the filament *F* (Fig. 299), no current flowed through the circuit *PGaF*. If, however, he connected the plate to the positive terminal *b* of the filament *F*, a very appreciable current flowed through the circuit *PFbG*. At that time nothing was known of electrons and this current was referred to merely as the *Edison Effect*. A study of the phenomenon was made by

Fleming in 1896, but its true significance was not understood until the work of J. J. Thomson and O. W. Richardson in 1899 and 1901.

214. Fleming Valve.—The most valuable property of the 2-electrode tube is its characteristic of unilateral conduction. When the plate is positive with respect to the filament, it draws electrons from the filament and a current flows from plate to filament which is roughly proportional to the $\frac{3}{2}$ power of their voltage difference, and the device has a finite although variable resistance. When the plate is negative with respect to the filament, the electrons are all driven back into the filament and no current whatever flows. The resistance of the device becomes infinite. If an alternating voltage be applied to a 2-electrode

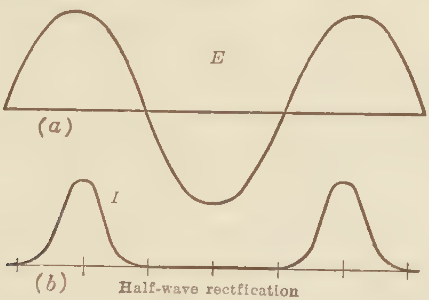


FIG. 300.—Half-wave rectification with Fleming valve.

tube, the resultant current is pulsating but unidirectional (Fig. 300 (*b*)). The negative loop is entirely suppressed. The positive loop is somewhat distorted because of the variation of the resistance of the device with the current. The foregoing is called *half-wave* rectification. This rectifying action is identical with that of the mercury-arc rectifier, and the tungar rectifier described in Chap. X. Fleming in 1905 was the first to recognize this rectifying property of a 2-electrode tube. He obtained a patent on its use as a detector of high-frequency oscillations which became one of the fundamental patents in electron-tube development.

215. Kenotron.—When used as a rectifier, the 2-electrode tube is usually called a *kenotron*. Both loops of voltage may be rectified by using two kenotrons, as shown in Fig. 301. The resulting current through the load is shown in Fig. 302. This is

called *full-wave* rectification. If the load is of high resistance, the voltage impressed on it may be smoothed out as shown by the full line in Fig. 303, by connecting a large condenser across the output circuit.

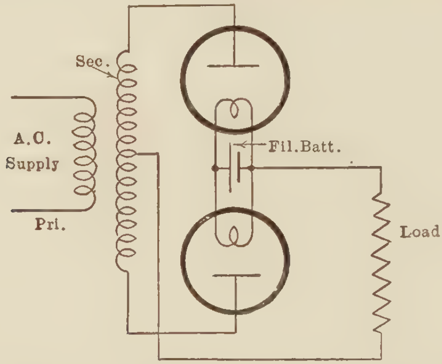


FIG. 301.—Kenotron connections which give full-wave rectification.

Kenotron rectifiers are used for the production of high-voltage direct current for high-voltage (2,000 to 15,000 volts) 3-

FIG. 302.—Full-wave rectification with kenotron.

FIG. 303.—Full-wave voltage rectification with a condenser across kenotron.

electrode oscillators, x-ray tubes, and for obtaining high voltages for insulation-testing purposes.

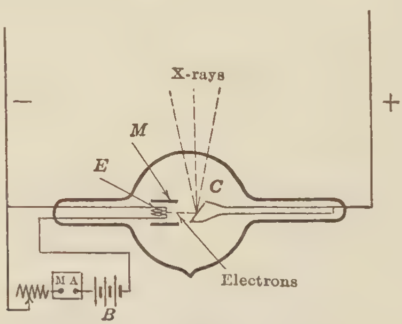


FIG. 304.—Coolidge x-ray tube.

216. X-ray Tubes.—The Coolidge hot-cathode, x-ray tube (Fig. 304) is a special kenotron, designed for the production of

x-rays. The filament E heated by the low-voltage battery B is concentrated and electrostatically shielded by the molybdenum tube M, so that the electrons are emitted in a fine pencil. The plate or anticathode C is a massive block of tungsten which serves as a target for the electrons. X-rays are given off by the target at the point where the electrons strike. The targets are frequently water cooled to allow the use of very high voltages and large currents, as, for example, 100 kv. and 0.5 amp. As is well known, x-rays have the property of penetrating substances which are impervious to ordinary light. They are used to a very large extent in medical work in the study of fractured bones, the roots of teeth; in industry to locate flaws in castings; in chemistry to determine the crystalline structure of substances, etc.

217. Three-electrode Tube.—The addition of a third electrode to control the plate current was made by DeForest in 1907 in a tube which he called the *audion*. His patent, issued in that year, became as fundamental as that of Fleming in electron-tube development. He placed a lattice or grid, as it is now called, between the filament and plate (Fig. 305). The electrons, in passing from filament to plate, must now pass through this grid. If the grid is positive with respect to the filament, it will assist the plate in drawing electrons from the filament, and hence will increase the plate current. If the grid is negative with respect to the filament, it will act in the same manner as the space charge (page 343) and will repel negative charges or electrons toward the filament, and hence will decrease the plate current. The plate current is now, therefore, a function not only of the filament temperature T and the plate voltage E_p, but also of the grid voltage E_g. As the grid is much nearer the filament than the plate, a given change in the grid voltage E_g will have a much

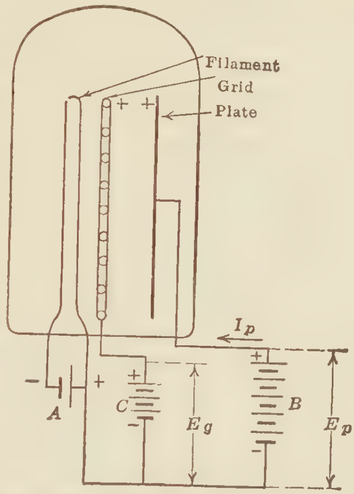


FIG. 305.—Three-electrode vacuum tube, grid positive.

greater effect on the plate current than a given change in the plate voltage E_p. Hence, a very small amount of energy applied to the grid will control a very much larger amount of energy passing from filament to plate. It is this characteristic of the tube which makes it so useful as an amplifier and as an oscillator.

218. Static Characteristics of the Three-electrode Tube.—Figure 306 shows the connections which may be used to obtain the static or direct-current characteristics of the 3-electrode tube. The voltage E_p applied to the plate by the B or plate battery depends on the type of tube being tested, but it should be well within the safe value which is used when the tube is

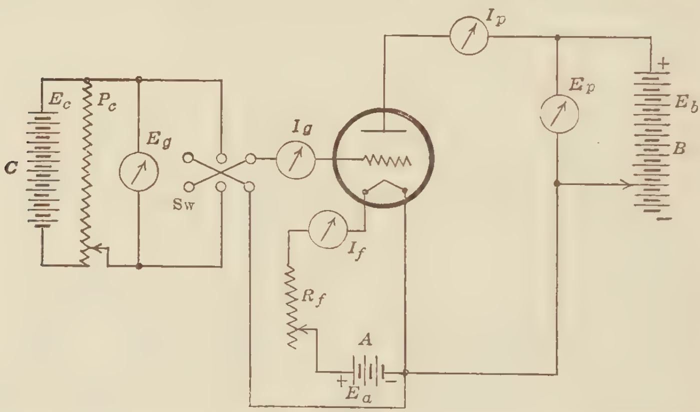


Fig. 306.—Connections for determining the static characteristics of the 3-electrode tube.

oscillating (see Table I, page 354). With a plate voltage which is too high, the slight traces of gas in the tube become ionized. thus increasing the plate current and ultimately causing the tube to burn out. The voltage applied to the grid may be readily obtained from dry cells in series (C), a high resistance drop-wire P_c being used to vary the voltage, and a reversing switch S_W to give the desired polarity. The range of grid voltage is ordinarily ± 10 volts. The plate current is of the order of 2 to 25 milliamp. The grid itself takes current of the order of 2 milliamp. Hence, very sensitive ammeters are necessary.

If the filament current and the plate voltage are held constant at some value as a plate voltage E_1 (Fig. 307) and the grid voltage

is varied, a curve *abc* is obtained. When the grid voltage is zero, current *ob* flows to the plate since the plate itself attracts electrons from the filament. In order to stop the flow of plate current, the grid potential must be negative and of a sufficient value *oa* to neutralize the effect of the plate voltage. The rate of increase of plate current decreases as the grid voltage is increased, due to filament saturation. If the plate voltage is increased to E_2, then for a given grid voltage, more current will flow in the plate

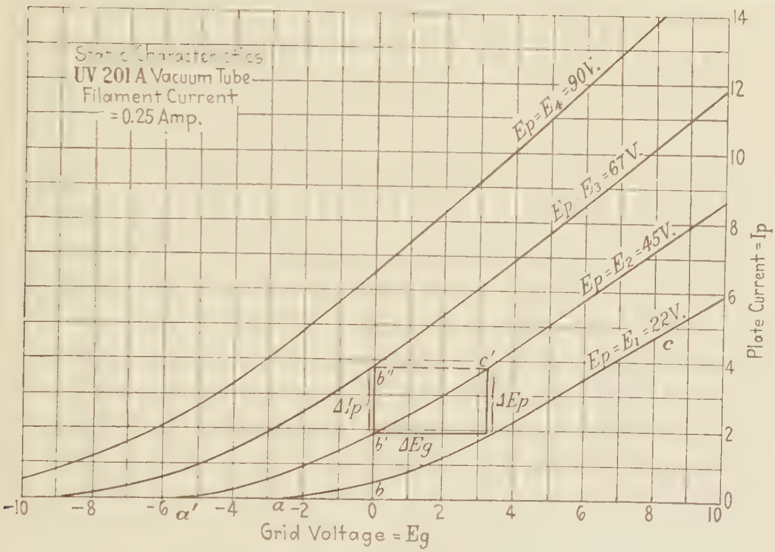


FIG. 307.—Static characteristics of a U.V. 201A vacuum tube.

circuit, and curve $a'b'c'$ is obtained. Curves for still greater plate voltages E_3 and E_4 are also given.

Assume (Fig. 307) that when $E_p = E_2$ the grid voltage E_g increases by an amount ΔE_g. This increases the plate current by an amount $b'b''$ or ΔI_p. If the grid voltage be held constant as at o, the plate voltage must be increased ΔE_p or from E_2 to E_3 in order to increase the plate current by ΔI_p. The ratio of ΔE_p to ΔE_g is the *amplification constant* and is denoted by μ. Since the plate-voltage scale is much larger than the grid-voltage scale, the amplification factor is larger than the ratio of the actual lengths of the lines $b''b'$ and $b''c'$. With ordinary detecting and amplify-

ing tubes $\mu = 6$ to 8 (see Table I below); with some special tubes $\mu = 30$ to 40. If a change of plate voltage ΔE_p produces a change of plate current ΔI_p, the factor

$$r_p = \frac{\Delta E_p}{\Delta I_p} \tag{70}$$

is called the *plate resistance.*

Below are given data for a few of the more common types of tubes.

TABLE I.—CONSTANTS OF TYPICAL ELECTRON TUBES

Model	Filament		Plate		μ	r_p kilo-ohms
	Volts	Amperes	Volts	Milli-amperes		
U.V. 199	3.0	0.06	20–80	1–2	6	15–20
U.V. 200	5.0	1.0	20	0.5–2		
U.V. 201	5.0	1.0	20–80	1–2	6	15–20
U.V. 201A	5.0	0.25	40–100	2–5	8	10–12

The filaments of the tubes U.V. 200 and 201 are pure tungsten while U.V. 199 and 201A have thoriated tungsten. U.V. 200 is a detecting tube, not an amplifier, and contains an appreciable amount of residual gas.

219. Amplification.—Since the amplification constant μ of a 3-electrode tube is considerably greater than unity, the tube may be used to amplify small voltages. Figure 308 shows a tube having a steady potential E_p connected in the plate circuit and another steady potential E_g in the grid circuit. E_g gives the grid a negative potential with respect to the filament (Fig. 309). These voltages give a definite operating point P on the plate-current—grid-voltage characteristic (Fig. 309). Now let a small alternating voltage e_g be introduced into the grid circuit. This variation of grid voltage must produce a variation of plate current. The actual plate current may be determined by projecting the e_g curve vertically on the $I_p - E_g$ curve (Fig. 309). Thus, an alternating current i_p, superposed on the steady plate current, is produced in the plate circuit. This current may be considered as being produced by a voltage μe_g acting through the constant plate resistance r_p and the effective impedance of the transformer.

The steady voltages and currents in the plate and grid circuits have no effect in determining the alternating voltages and currents, except in so far as they define the portions of the static characteristics over which the tube operates. For distortionless

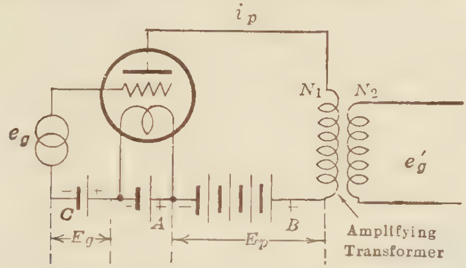


Fig. 308.—The vacuum tube as an amplifier.

reproduction, the voltages E_g and E_p should be so chosen that the alternating e.m.f. *is operating on the straight part of the characteristic* (Fig. 309).

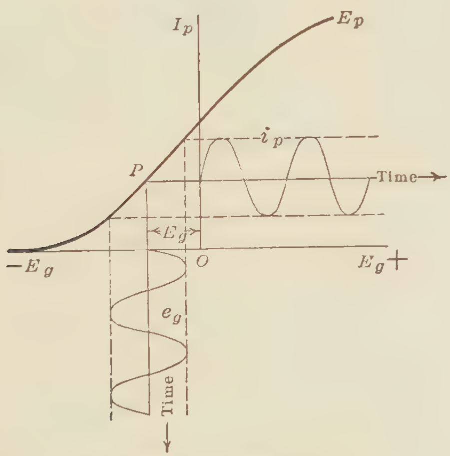
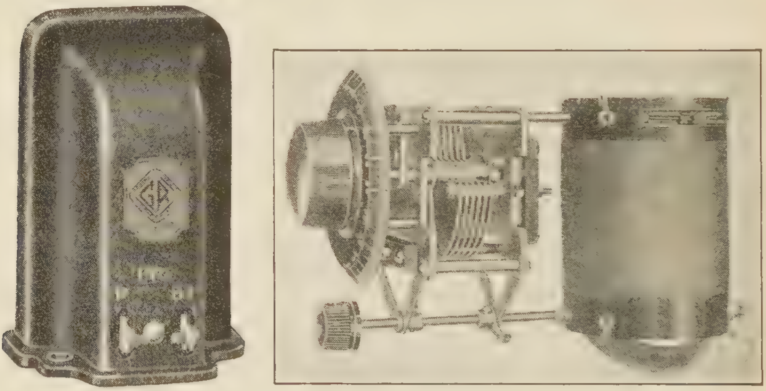


Fig. 309.—Amplification characteristics of a vacuum tube.

The total plate current flows through the primary N_1 of the amplifying transformer (Fig. 308). The steady component of this current has no effect on the secondary N_2, but the alternating component i_p causes an e.m.f. e_g' to be induced in the secondary

turns, N_2. If the turns of the transformer are properly chosen e_g' may be from 20 to 50 times the e.m.f. e_g, thus effecting *amplification*. The e.m.f. e_g' may be impressed on the grid of a second tube, similar to this first tube, and a second stage of amplification obtained. Thus, several stages of amplification may be obtained, the number being limited by the tendency of the amplifier to feed back through capacitance to the grids in the first stages, producing a tendency to oscillate at an audible frequency or to "howl."

If audio frequencies, 100 to 5,000 cycles per second, are amplified, audio-frequency (a.-f.) transformers, having laminated iron



(a) (b)

Fig. 310.—(a) General Radio Co. audio-frequency transformer. (b) National Co. radio-frequency transformer with an associated variable condenser.

cores, are used. For radio frequencies, of the order of 1,000 kilocycles per second, the use of iron is not permissible because of the high losses. Better results are obtained with radio frequencies, if a variable condenser is shunted across the transformer secondary and the entire circuit is tuned. A typical radio-frequency (r.-f.) transformer and an audio-frequency (a.-f.) transformer are shown in Fig. 310.

220. Regeneration.—It was shown in Par. 219 that in the amplifier any change of the voltage applied to the grid causes not only a change of current in the plate circuit but also a change of energy in the plate circuit which is several times greater than the change of energy at the grid. If the proper phase relation

between the plate current and the voltage in the grid circuit is obtained, it is possible to feed a portion of this energy of the plate circuit *back* into the grid circuit, and hence reinforce the effect of the grid. The grid, in turn, increases the plate current, which again reacts on the grid. This feed back of energy is called *regeneration*. The connections for one method of regeneration are shown in Fig. 311. The grid is polarized negatively with a grid battery to a potential of E_g volts. An inductance L_g is connected between the impressed alternating grid voltage e_g and the grid. A variable condenser C_g is connected between the grid and the negative terminal of the grid battery. A coil L_p, having mutual inductance M with L_g and connected in the plate circuit, serves to couple inductively the plate and grid circuits.

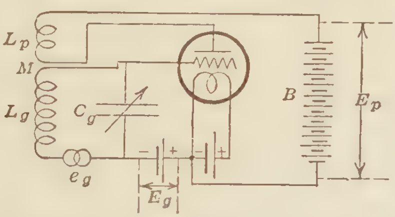


FIG. 311.—Connections for regeneration with tuned grid circuit.

If the polarity of L_p is correct, the alternating component of the plate current induces an e.m.f. in L_g which reinforces the voltage e_g, provided this induced e.m.f. is not in quadrature with e_g. Ordinarily, the induced e.m.f. would be nearly in quadrature with e_g, and hence would have little regenerative effect. If, however, the grid circuit is tuned by means of the variable condenser C_g, the induced e.m.f. in L_g and the voltage e_g may be brought substantially into phase with each other, and hence the effect of e_g is increased. The same effect may be produced by connecting a similar tuned circuit, consisting of inductance and capacitance in parallel with each other, in the plate circuit (see Fig. 312).

The effect of regeneration is to introduce a *negative* resistance into the tuned circuit. That is, this tuned circuit now becomes a generator of energy, this energy being obtained from the plate or B battery. If the mutual inductance M between L_p and L_g be made sufficiently large, the total resistance of the tuned circuit may be reduced almost to zero, where the limit of regeneration

is reached. At this limit of regeneration, the plate current has been found to be constant and independent of the magnitudes of the impressed voltage e_g, and the initial resistance of the tuned circuit, its value being determined only by the characteristics of the tube. In practice this maximum theoretical limit cannot be reached, since small disturbances such as slight mechanical vibrations of the inductances, condensers, and of the tube itself may cause the total resistance of the plate circuit to become negative momentarily and cause the tube to begin oscillating or howling (see the next paragraph). The plate current is no longer constant but decreases if the impressed voltage happens to be small and the resistance of the tuned circuit is large. That is, if regeneration is carried too far, instability occurs.

OSCILLATORS

221. Oscillation.—When the mutual inductance between the plate and grid circuit is increased to such a value that the resistance of the tuned circuit becomes zero or negative, sustained oscillations, independent of the impressed voltage e_g, are set up in the system. In fact, the impressed voltage e_g may be removed entirely without affecting the oscillations. These sustained oscillations will start even in the absence of any impressed voltage. For example, slight mechanical disturbances to parts of the system (see the previous paragraph), small electrical disturbances such as occur when the plate circuit is closed, etc., are sufficient to start the tube into oscillation. Under these conditions the tube is said to be an *oscillator*. It behaves like an alternating-current generator, converting the energy of the plate battery into alternating-current energy in the tuned circuit. The frequency of the alternating current generated is practically the natural frequency of the tuned circuit

$$f = \frac{1}{2\pi\sqrt{LC}}$$

where L and C are the inductance and capacitance of the tuned circuit (see equation (36), page 74).

This type of oscillating circuit (Fig. 311) is the type which is used in most receivers in which oscillating tubes are used, such

as continuous-wave, carrier-frequency, and superheterodyne receivers (see Pars. 234 and 239).

So far as sustained oscillations are concerned, the tuned circuit may equally well be placed in the plate circuit, as shown in Fig. 312 (*a*), the tuned circuit being inductively coupled to the grid by the mutual inductance M. This type of circuit is used in most power oscillators where the tube acts as an alternating-current generator. The grid polarizing battery C may also be replaced by a grid condenser and grid leak (Par. 230). The resistance R is not a resistance introduced purposely into the oscillating circuit but is equal to the total inevitable resistance in the tuned circuit

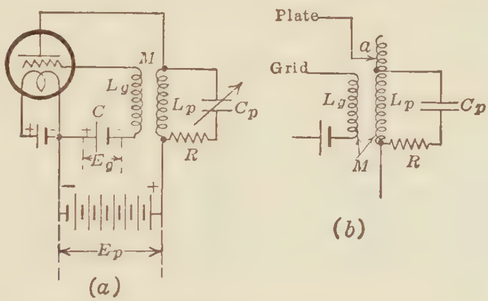


FIG. 312.—Vacuum-tube power oscillator.

and the equivalent resistance of the load on the oscillator, such as the equivalent antenna resistance, etc.

For maximum tube output there is a definite relation between L_p and C_p of the tuned circuit, this relation depending on the values of R and of the plate-circuit resistance. With sinusoidal current, the maximum output occurs when the maximum instantaneous voltage across the oscillating circuit is equal to the plate voltage E_p. It may happen that unavoidable conditions imposed by L_p, C_p, and the load prevent the oscillator from delivering more than a small portion of its total power capacity. This limitation may be overcome by connecting the plate tap (Fig. 312(*b*)), to a point *a* on the winding of L_p, point *a* being outside of the point at which the condenser C_p is connected to the coil. At times, conditions may be such that the output is increased by moving the plate connection within the point at which the condenser is connected to the coil.

The efficiencies of such oscillators when they deliver maximum output are between 50 and 80 per cent., dependent on operating conditions.

222. Meissner Circuit.—In this circuit (Fig. 313) the tuned circuit is inductively coupled to both the grid and plate circuits. No restriction is placed on the ratio of L to C. Direct coupling between grid and plate should be avoided as it is likely to cause spurious oscillations of the natural frequency of either the grid or the plate coil. This circuit is frequently used where the constants of the tuned circuit, such as an antenna system, are fixed

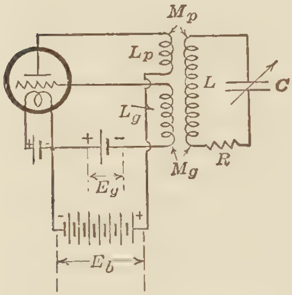


FIG. 313.—The Meissner circuit.

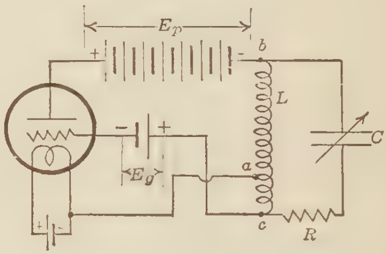


FIG. 314.—The Hartley circuit.

223. Hartley Circuit.—In this circuit (Fig. 314) both the grid and plate are connected directly to the tuned circuit. The filament should divide the inductance L of the tuned circuit so that the inductance from the filament connection a to the plate connection b is approximately μ (amplification factor) times the inductance from the filament connection a to the grid connection c.

As shown in Fig. 314, the three batteries are all at different potentials. Because of the mutual capacitances between these batteries, large losses and possibly oscillations would occur. By using a leak resistance and a by-pass condenser (see Par. 230) it is possible to bring all the condensers to the same alternating-current potential, as in the other connections. The advantage of this circuit is its simplicity, since the inductance L is merely a single coil with a single tap a.

224. Power Tubes.—All the tubes (Table I, page 354) with the possible exception of U.V. 200, will function as oscillators.

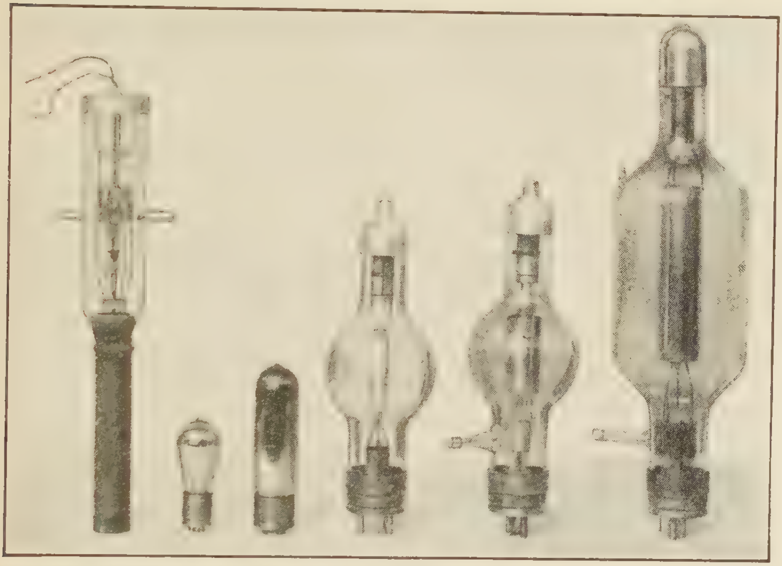


FIG. 315.—Pliotron power tubes models (left to right 207, 202, 203, 204, 206, 208). (M'f'd by General Electric Co.)

Higher power tubes are shown in Fig. 315. Below are the electrical characteristics of a few typical tubes.

TABLE II.—POWER-TUBE CHARACTERISTICS

Model	Filament		Plate		μ	Output, watts
	Volts	Amperes	Volts	Milli-amperes		
U.V. 202......	7.5	2.35	350	50	7.5	5
U.V. 203A[1]....	10.0	3.25	1,000	125	25	50
U.V. 204......	11.0	14.75	2,000	250	20	250
U.V. 208......	22.0	24.5	15,000	450	300	5,000
U.V. 207[2].....	22.0	52.0	15,000	1,800	40	20,000

[1] Thoriated tungsten, all others pure tungsten filaments.
[2] Water-cooled plate.

MODULATION

225. Modulation.—Electrical communication over wires in its simplest form employs alternating currents of audio frequencies only, either singly or in combination. These currents may be amplified (see Par. 219) but only a single communication can be conducted over a single effective circuit at one time. In order to open new channels of communication over any given effective wire circuit, *carrier* wire telephony and telegraphy are employed. Alternating currents having superaudio frequencies (3,000 to 33,000 cycles per second[1]) are used as carriers or vehicles for the audio-frequency currents. In radio telephony and telegraphy electro-magnetic waves are used as carriers having frequencies of from 10,000 to 30,000,000 cycles per second. Of themselves, these superaudio frequencies could not transmit signals, being for the most part beyond the range of audibility of the ear, and they can transmit only very small amounts of power. By the superposition of audio-frequency currents on these carrier currents, however, it is possible to transmit several messages simultaneously over a given effective communication circuit. The superposition of audio-frequency on carrier-frequency waves is called *modulation.*

With the usual type of modulation, the original constant amplitude of the carrier-frequency alternating current is made to vary according to the amplitude of the superposed audio-frequency current. For example, in Fig. 316 is shown the constant-frequency carrier wave having a constant amplitude A and a frequency of a cycles per second. An audio-frequency current having an amplitude B and a frequency of b cycles per second is superposed on the carrier frequency. The resulting current is shown in Fig. 316 (b). The superposed audio-frequency wave is the *envelope* of the amplitudes of both the positive and negative halves of this modulated carrier wave, as will be recognized.

It may be shown that the modulated carrier wave (Fig. 316 (b)) actually consists of three sinusoidal currents, one having the frequency a, the frequency of the original carrier wave; another having a frequency $(a - b)$ the *difference* between the carrier-

[1] The ear may be sensitive to frequencies as high as 15,000 cycles per second but most conversational frequencies do not exceed 2,500 cycles per second.

wave frequency and the audio frequency; and a third having a frequency $(a + b)$ the *sum* of the carrier-wave frequency and the audio frequency. The currents having frequencies of $(a + b)$ and $(a - b)$ are called side frequencies. This is illustrated in Fig. 317,

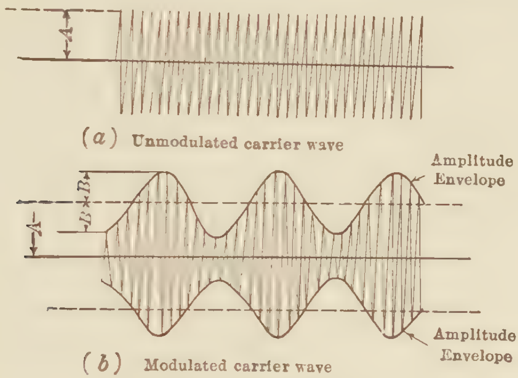


FIG. 316.—Modulation.

which shows a portion of the frequency spectrum, the abscissas being frequencies and the ordinates the amplitudes of the waves. With the more complex audio-frequency waves, such as would be produced by the voice, the resultant modulated wave is quite

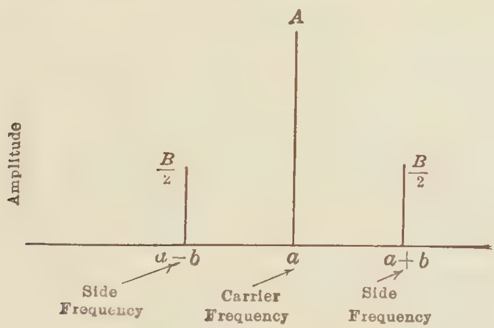


FIG. 317.—Frequency spectrum.

complex, and the side frequencies widen out into *sidebands*. The carrier frequency, however, is always sufficiently high so that the side-band frequencies are near it in the frequency spectrum and all are transmitted essentially as a single frequency.

Methods of Modulation

226. Plate-circuit Modulation.—One method of modulation is to introduce into the plate circuit of a tube, oscillating at carrier frequency, an additional voltage which is proportional to the audio-frequency current, and whose peak value is somewhat less than the steady plate voltage E_p. This causes the output current I in the tuned circuit to have an amplitude envelope (Fig. 316 (*b*)) proportional to the audio-frequency current. The connections for this method of modulation are shown in Fig. 318. The tube oscillates at carrier frequency due to the tuned circuit L_p and C_p, which is inductively coupled to the

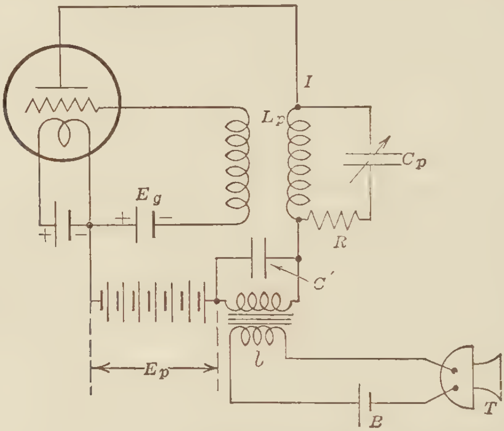


FIG. 318.—Connections for plate-circuit modulation.

grid circuit. An audio-frequency transformer is introduced into the plate circuit at *b*. The primary current of this transformer is shown as coming from a microphone circuit consisting of a battery B in series with a telephone transmitter or microphone T. Ordinarily, there is not sufficient power in the microphone circuit to give sufficient modulation, hence an amplifier between the microphone circuit and the amplifier-frequency transformer is necessary (see Fig. 308, page 355). The secondary of the transformer introduces the modulation e.m.f. into the plate circuit. The carrier- or radio-frequency current is unable to flow through the high inductance of the transformer secondary, hence a by-pass condenser C' is necessary. If it were desired to

broadcast with this circuit, an antenna would be inductively coupled to L_p, one end of the coupling winding being grounded and the other connected to the antenna.

227. Heising Constant-current Method.—The connections for this method are shown in Fig. 319. The plate battery B is connected through the audio-frequency choke-coil L_b directly to the plate of the modulator tube M and through the radio-frequency choke-coil L_a to the plate of the oscillator O. The plate circuit of the modulator tube M is in parallel with the tuned carrier-frequency circuit L_p, C_p, rather than in series with it as in Fig. 318. The condenser C_a is necessary in order to prevent the inductance L_p of the tuned circuit shunting the plate battery B. The radio-frequency choke-coil L_a prevents the modulator M being a shunt on the oscillator O.

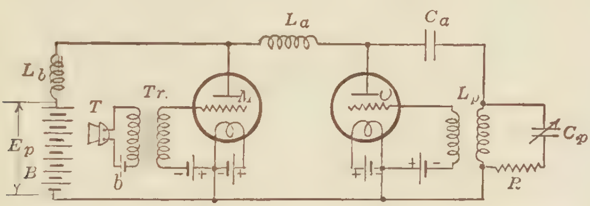


FIG. 319.—The Heising constant-current method of modulation.

Because of the audio-frequency choke-coil L_b, the current from the plate battery is practically constant. Hence, the voltage across the oscillator and modulator tubes is proportional to the plate resistance of the modulator tube. The plate resistance of the modulator tube is proportional to the audio-frequency voltage impressed on its grid. The audio-frequency e.m.f. is applied to the grid by the secondary of the audio-frequency transformer Tr, in the primary of which is connected the microphone T in series with battery b. As in the system shown in Fig. 318, the power output of the modulator tube must be as large as that of the oscillator.

DETECTION

A modulated high-frequency current, such as is shown in Fig. 316 (b), can have no effect on any ordinary sound-producing devices, since such devices are unable to respond to such high frequencies. Neither can this high-frequency current produce

any effect on the human ear, because its frequency is far beyond audible frequencies. It is, therefore, necessary to demodulate such currents in order that the receiving devices may be actuated by audio-frequency currents similar to those used for modulating. This process of demodulation is called *detection*.

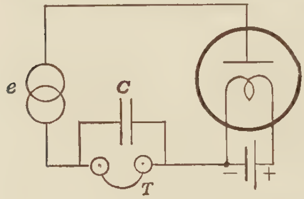


FIG. 320.—Two-electrode tube used as a detector.

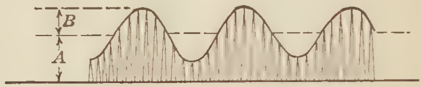


FIG. 321.—Rectified carrier wave.

228. Simple Rectification with Two-electrode Tubes.—Detection may be performed by any rectifying tube such, for example, as the 2-electrode tube (Fig. 320). The tube will eliminate the negative loops (Fig. 316 (*b*), page 363) leaving a pulsating, unidirectional current (Fig. 321) consisting of a unidirectional current, an audio-frequency current and a radio-frequency current. The unidirectional current and the audio-frequency currents will flow through the telephones *T* and reproduce the initial audio-

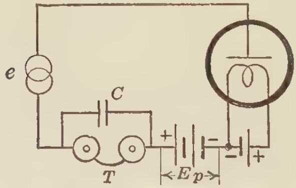


FIG. 322.—Two-electrode tube with polarizing voltage used as a detector.

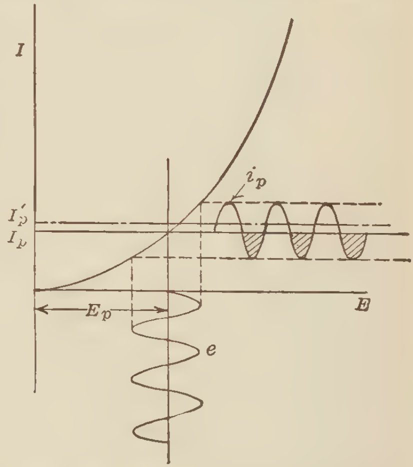


FIG. 323.—Detection with polarized two-electrode tube.

frequency current. The high frequency component will be by-passed through the condenser C.

Although the 2-electrode tube is a perfect rectifier, if used in this manner as a detector, it is very insensitive because of its

excessively high resistance when operated at low voltage. This may be seen in Fig. 323, where the current I for small values of voltage E is extremely small. This difficulty is in part overcome by inserting a positive polarizing voltage E_p in series with the tube (Fig. 322). Thus, in Fig. 323, the steady polarizing voltage E_p produces a steady current I_p in the tube circuit. Hence, an alternating e.m.f. e impressed on the tube is no longer perfectly rectified but produces an alternating current i_p. Owing to the curvature of the characteristic, this current i_p is dissymmetrical, the positive current being larger than the negative current. The negative current is shown shaded. Hence, the average current is increased from I_p to I_p' and thus the existence of the impressed e.m.f. is detected. The *change* in current $I_p' - I_p$ is greater than the current which would flow for zero polarizing voltage and has its maximum value when the polarizing voltage corresponds to the point of greatest curvature of the characteristic. When the impressed voltage is modulated this change in plate current will follow the variations of the modulating current.

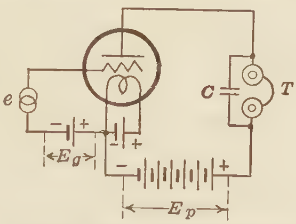


FIG. 324.—Three-electrode tube with polarized grid used as a detector.

229. The Three-electrode Tube with Polarized Grid as Detector.—The 3-electrode tube may detect in a manner very similar to the 2-electrode tube, rectification depending on operating the tube at a point of curvature on its plate-current—grid-voltage characteristic. The connections for operating a tube as a detector are shown in Fig. 324. The grid is polarized negatively with a voltage E_g by the grid or "C" battery. E_g is sufficiently large to cause the impressed grid voltage e to operate on a point of curvature of the $I_p - E_g$ characteristic (Fig. 325). As with the 2-electrode tube, a sinusoidal e.m.f. e impressed on the grid produces a nalternating current i_p in the plate circuit, the axis of i_p being I_p. Owing to the curvature of the characteristic, the negative portions of i_p, shown shaded, are less in magnitude than the positive portions, and the average current increases from I_p to I_p'. When the impressed voltage e is modulated this change in plate current $I_p' - I_p$ will follow the variations of the modulating current. The radio-frequency plate current I_p' is by-passed

through the condenser C (Fig. 324). For maximum sensitivity, the polarizing voltage E_g should be such that detection occurs at the point of maximum curvature of the characteristic.

Since the characteristic for positive values of E_g also has a curvature due to filament saturation, detection in a similar manner may occur at point a. At this point, the positive rather than the negative portion of the alternating component of plate current will be reduced in magnitude and the average plate current will *decrease*. Detection with the grid polarized positively is not so efficient ordinarily as with the grid polarized negatively, since, when positive, the grid takes current. In detection, the

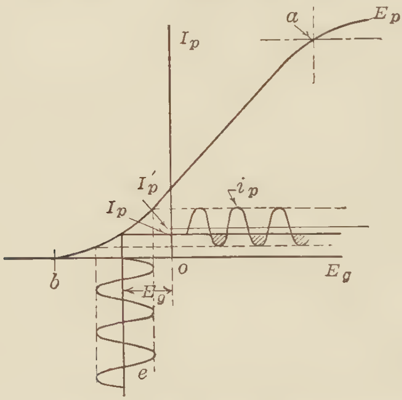


Fig. 325.—Detection with polarized three-electrode tube.

e.m.f. e impressed on the grid is usually obtained through a transformer. Losses must inevitably occur in such transformers, reducing the sensitivity of detection. The detecting action in a 3-electrode tube is greater than that of the 2-electrode tube because of its lower effective plate resistance and because of its amplifying action.

Tubes similar to the UV-200, which contain slight traces of gas, are very good detectors, because the presence of gas tends to neutralize the space charge and causes the $I_p - E_g$ characteristic to be unusually steep and so to have a large curvature at its lower bend. Since this action is due to the ionization of the residual gas, it is quite critical as regards the values of plate and grid voltages.

230. The Three-electrode Tube with Grid Resistance as Detector.—The 3-electrode tube also detects in a manner which is quite different from the foregoing, and, in modern tubes, is far more efficient. The connections are shown in Fig. 326. A small

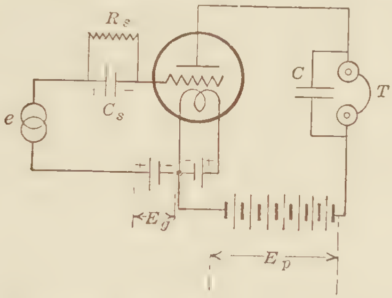


FIG. 326.—Three-electrode tube with grid resistance used as a detector.

condenser C_s, whose capacitance is between 50 and 200 $\mu\mu f$, is in series with and adjacent to the grid. This condenser is shunted by a high resistance R_s of from 1 to 5 megohms. The

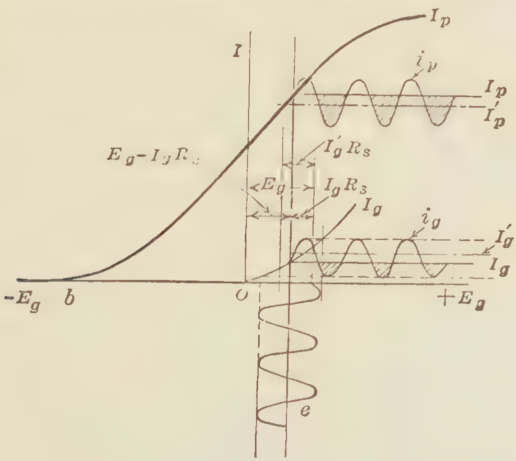


FIG. 327.—Detecting action with a grid resistance.

grid is polarized positively by the voltage E_g so that a current I_g flows in the grid circuit, as shown in Fig. 327. This current flowing through the high resistance R_s produces in it a voltage drop I_gR_s, so that the effective polarization of the grid is E_g −

$I_g R_s$. The corresponding plate current is I_p. An alternating voltage e in the grid circuit will produce an alternating current i_g in the grid circuit whose negative portions, shown shaded, are less in magnitude than its positive portions. Hence, the average grid current is increased from I_g to I_g'. This decreases the polarization of the grid from $E_g - I_g R_s$ to $E_g - I_g' R_s$. The alternating component i_g is by-passed through the condenser C_s. The average plate current is decreased from I_p to I_p' with a superposed alternating current i_p. With reference to I_p as an axis, the positive portions of i_p are less in magnitude than the negative portions shown shaded. When the impressed voltage e is modulated, this change in plate current $I_p - I_p'$, will follow the variations of the modulating current. The radio-frequency plate current i_p is by-passed through the condenser C (Fig. 326).

The large curvature of the grid-current characteristic, the large slope of the plate-current characteristic, and the fact that the high resistance R_s may be made very large all combine to make this type of detection the most sensitive of all the methods so far discussed.

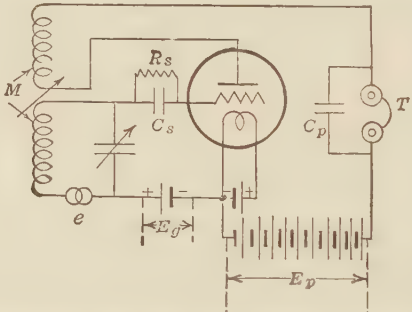
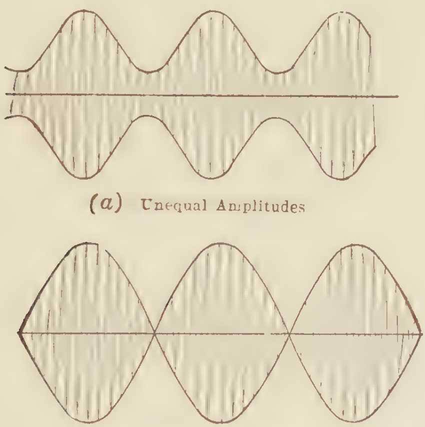


FIG. 328.—Three-electrode tube with grid resistance and regeneration used as a detector.

231. Detection and Regeneration.—The foregoing three types of detection may be operated with a tuned circuit combined with regeneration. A very efficient circuit of this character, having a tuned grid circuit, grid resistance, and grid condenser, is shown in Fig. 328. The incoming signal e is detected and a portion of the resulting energy of the plate circuit is fed back into the grid circuit through the coupling M. The condenser C_p shunts the high-frequency currents around the telephone receivers T.

232. Beat or Heterodyne Reception.—A high-frequency alternating current may have its frequency a lowered by superposing on it a second current of somewhat lower or higher frequency a'. The resulting current may be shown to be a modulated current having a frequency equal to the average of the two frequencies. For example, if the original frequency is 50,000 cycles per second and the superposed frequency is 49,000 cycles per second, the resulting frequency is

$$\frac{50{,}000 + 49{,}000}{2} = 49{,}500$$



(*a*) Unequal Amplitudes

(*b*) Equal Amplitudes

FIG. 329.—Beat-frequency envelopes.

cycles per second. Further, the amplitude *envelope* of this resultant frequency has a frequency $a - a'$, or the *difference* of the two impressed frequencies. Figure 329 (*a*) shows the resulting current curve and the resulting amplitude envelope for the general case; that is, when the amplitudes of the two currents are unequal. Figure 329 (*b*) shows the resulting current curve and the resulting amplitude envelope when the amplitudes of the two currents are equal. In neither case is the envelope sinusoidal.

A detecting tube will separate the envelope frequency from the high frequency, thus giving a current having the envelope

frequency of $a-a'$ cycles per second (see Par. 228). This frequency is called the *beat-note* frequency.

The superposed frequency may be obtained from an oscillating tube (Fig. 330) whose grid circuit is inductively coupled to the grid circuit of the detector through the mutual inductance M'. This method of reception is called *beat reception* or *heterodyne reception.*

In radio telegraphy where the high frequency or carrier current is modulated by the dots and dashes of the Morse code, the frequency of the beat note is made so low as to be audible and the dots and dashes are heard at that frequency. In radio telephony

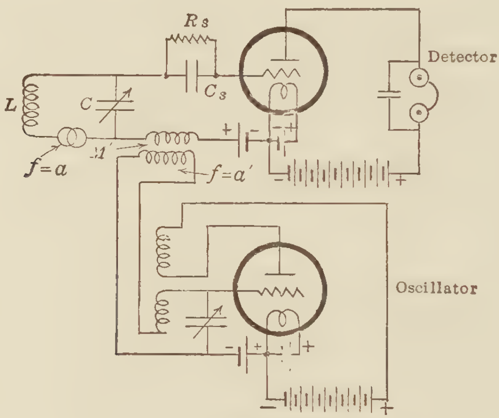


Fig. 330.—Separate heterodyne reception.

(or broadcasting), where the high-frequency current is modulated by speech or music, the frequency of the beat note (that is, the amplitude *envelope* of the frequency $(a - a')$) is made so high as to be above audibility.

For example, if the incoming modulated frequency is 1,000,000 cycles per second and the superposed frequency is 970,000 cycles per second, the resulting frequency is

$$\frac{1,000,000 + 970,000}{2} = 985,000$$

cycles per second and the frequency of the amplitude envelope, or the beat frequency, is 1,000,000 − 970,000, or 30,000 cycles per second.

By means of the detector (Fig. 330) this amplitude envelope of beat frequency is converted into a carrier-frequency current of this same beat frequency and modulated by the original audio-frequency currents, such as those produced by speech and music. This current again must be detected, amplified, etc. in the ordinary manner (see Fig. 336, page 378). This is the principle of the superheterodyne, invented by E. H. Armstrong.

The functions of oscillator and detector may be combined in one tube. This method is called *self-heterodyne* or *autodyne* reception. The connections are exactly those of Fig. 328, with perhaps the omission of the grid condenser and leak and with the grid polarizing battery reversed. In the autodyne the tube is already oscillating, which tends to increase its sensitivity, but this effect is frequently more than offset by the fact that the grid circuit is tuned to the frequency of the oscillating current and not to that of the incoming signal. If these frequencies differ by very large amounts, further amplification is necessary to make autodyne reception equal to separate heterodyne reception.

It is possible to detect a speech-modulated, high-frequency current with either the separate heterodyne or the autodyne method by making the beat note of zero frequency. This greatly increases the detecting action, but serious distortion is likely to be introduced because of the difficulty of maintaining zero-beat frequency.

RECEIVERS

233. Receiving Circuits.—In modern broadcasting, radio telephony uses the speech-modulated, high-frequency current such as is produced by the transmitters shown in Figs. 318 and 319 (pages 364 and 365) except that tuned circuit L_p, C_p, R consists of the antenna system, so that the output of the oscillator is converted into an outgoing electromagnetic wave. This wave may be received anywhere by an antenna or loop, converted into audio-frequency current by a detector, and then into sound waves by telephones or a loud speaker.

234. Single-tube Receiver.—The simplest vacuum tube receiver contains a single tube which acts as a detector. Figure 331 (*a*) shows such a receiver connected directly to an antenna. It is called a single-circuit regenerative receiver. There are two

controls, the tuning condenser C and the regenerative inductance M_p. In Fig. 331 (b) the tube is connected to the antenna by means of coupled circuits. The number of controls is increased to four, primary and secondary tuning condensers C_1 and C_2, coupling inductance M, and regenerative inductance M_p. This type of circuit is much more selective and, with regeneration, much more sensitive than the single circuit. Figure 331 (c) uses a semi-coupled

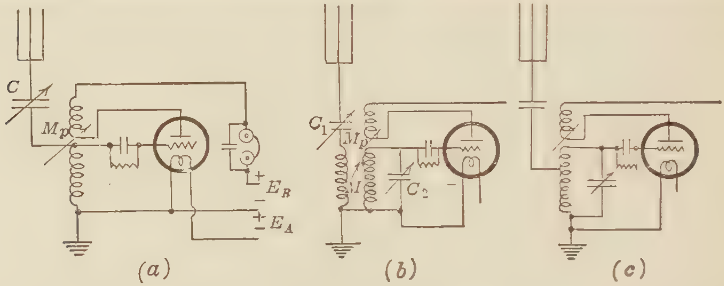


FIG. 331.—Single tube receivers.

circuit which retains some of the advantages of coupled circuits while using only two controls. The plate voltage E_B is from 20 to 40 volts and the plate battery is connected to the positive end of the filament battery whose voltage is thus added to it. The grid battery is usually omitted for the vacuum tubes which operate on dry cells and the grid return connected to the positive end of the filament.

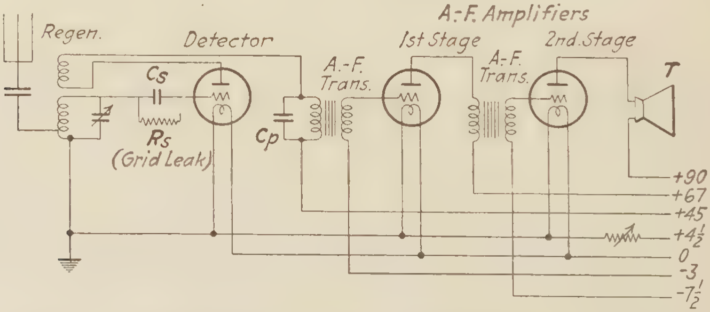


FIG. 332.—Regenerative detector with two stages of audio-frequency amplification.

235. Audio-frequency Amplification (see Figs. 308 and 309).—A 3-tube receiver, using a regenerative detector, two stages of audio-frequency amplification and a loud speaker, is shown in

Fig. 332. The various batteries, plate, filament, and grid are connected in series, and bus-bars are brought out at the proper points. An amplifying tube generally requires a plate voltage of about 60 volts and a negative grid voltage of 2 or 3 volts. If it is to operate a loud speaker, the plate voltage should be increased to 90 or 100 volts and the negative grid voltage to 6 or 7 volts, in order to prevent distortion.

236. Radio-frequency Amplification.—Radio-frequency amplification is used to increase the sensitivity and, when tuned, the selectivity of a receiver. A 4-tube receiver, having one stage of tuned radio-frequency amplification, a regenerative detector, and two stages of audio-frequency amplification, is shown in Fig. 333.

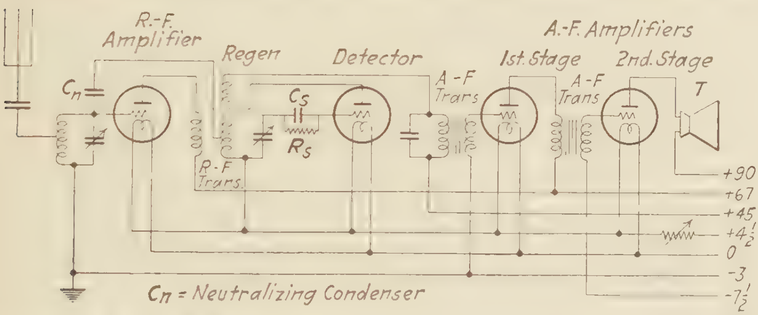


FIG. 333.—Four-tube receiver with one stage of r.-f. amplification, detector, and two stages of a.-f. amplification. (Browning-Drake receiver).

This is essentially the Browning-Drake receiver. It makes use of the neutralizing condenser C_n invented by Hazeltine. There is always a small capacitance of the order of a few micro-microfarads between the grid and plate of a tube. This produces a regenerative action when there is inductance in the plate circuit. This effect may be neutralized by connecting a suitable neutralizing condenser C_n to a part of the secondary of the radio-frequency transformer whose inductance is of the order of that of the primary. For this use the primary and secondary must be wound in the opposite sense. The two coils of the radio-frequency tuned circuits are set at right angles so that they may have no mutual inductance.

237. Neutrodyne Receiver.—The neutrodyne receiver, as developed by Hazeltine, uses two or more stages of tuned radio-

frequency amplification. A 5-tube receiver, having two radio-frequency stages, is shown in Fig. 334. The three radio-frequency coils are disposed in such a way as to minimize their mutual inductances, and two neutralizing condensers are used. The two radio-frequency transformers are not designed for maximum amplification, and regeneration in the detector is omitted. Frequently the grids of the radio-frequency tubes are polarized positively rather than negatively. These precautions are necessary to prevent oscillation in the radio-frequency stages. The amplification and selectivity in each stage of radio frequency are much less than for the receiver shown in Fig. 333.

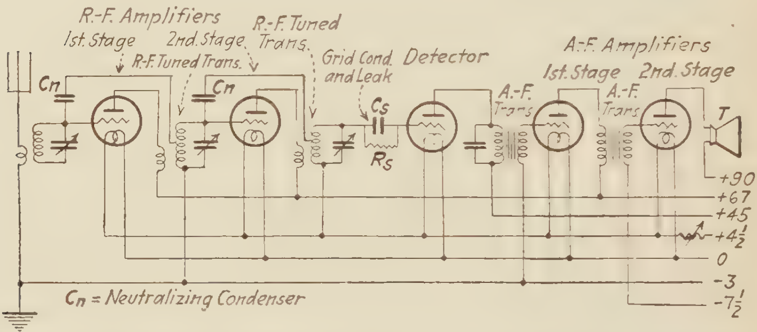


FIG. 334.—Neutrodyne receiver having two stages of radio-frequency amplification, detector, and two stages of audio-frequency amplification.

238. Reflex Circuits.—The number of tubes in a receiver having both radio- and audio-frequency amplification may be reduced by using the same tube in both stages. This is called *reflexing.* The tube operating a loud speaker should not be reflexed, because of the distortion which would be introduced into the radio-frequency circuits. It is quite usual to replace the detector tube by a crystal and to use untuned radio-frequency transformers. A reflex receiver operating on a loop, using four tubes and a crystal detector, is shown in Fig. 335. It provides three untuned radio-frequency stages, a detector, and three audio-frequency stages, and thus is equivalent to a 7-tube receiver of the conventional type. This is essentially the Acme reflex receiver. Its amplification and selectivity per radio-frequency stage is, however, much

less than obtains for either of the receivers shown in Figs. 333 and 334.

As the number of stages in any receiver increases, the decrease in amplification and selectivity per radio-frequency stage is caused by the precautions which are necessary in order that the stages as a whole may not oscillate. There results essentially an upper limit of amplication at any one radio frequency, which is independent of the number of stages. This limit does not differ much from the limit of fractional regeneration for a single tube (see Par. 220). The corresponding selectivity for a single tube

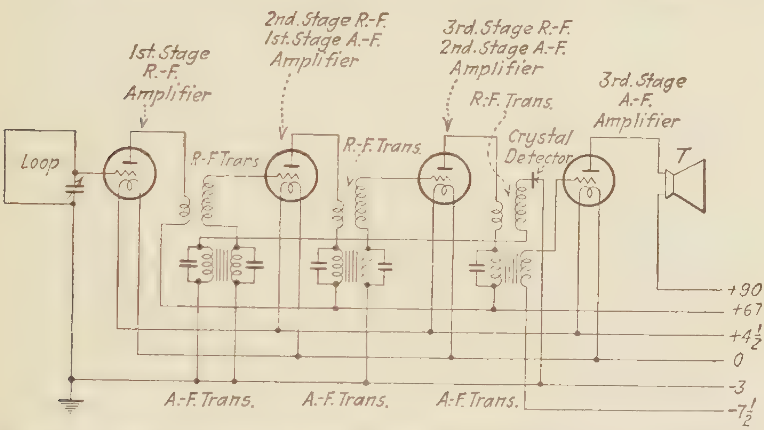


FIG. 335.—Reflex receiver having three stages of radio-frequency amplification a crystal detector, and three stages of audio-frequency amplification.

defines a resonance curve which is much too sharp to embrace both the two sidebands and the carrier frequency, so that serious distortion results. Hence, for a large amplification it is necessary to use many stages and keep the selectivity per stage low.

239. Superheterodyne Receiver.—The superheterodyne method of reception (see Par. 232) was suggested by Armstrong to overcome the limitation on amplification discussed in the previous paragraph. The new frequency introduced allows new stages of amplification which are more effective than those at the radio-frequency, both because the new frequency is constant and because it is lower. A 9-tube superheterodyne receiver is shown in Fig. 336. It has a radio-frequency amplifier, oscillator, and

detector, three intermediate-frequency stages, an intermediate-frequency detector, and two audio-frequency stages. This receiver has such a large overall amplification that, even when

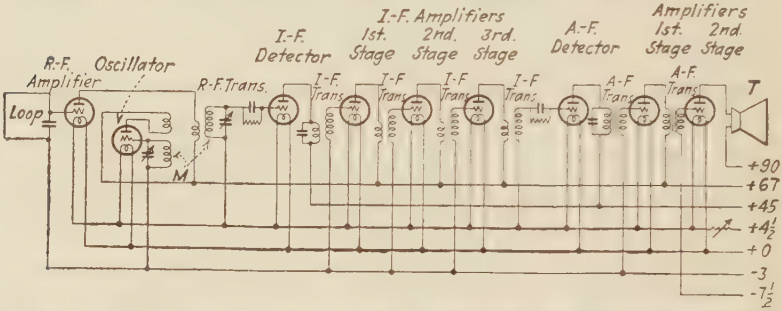


FIG. 336.—Superheterodyne receiver having one stage of radio-frequency amplification, three stages of intermediate frequency amplification and two stages of audio-frequency amplification.

receiving on a loop, any signal which is above the level of atmospheric disturbances will produce satisfactory volume on the loud speaker. This transfers the limiting factor in reception from receiver sensitivity to the ratio of signal to static energy.

CHAPTER XIV

INTERIOR WIRING

Although interior wiring in most localities must conform to the requirements of the inspection departments of the city or town, or local power company, or both, the local regulations are based, for the most part, on the "National Electrical Code" or Regulations of the National Board of Fire Underwriters. These national regulations are recommended by the National Fire Protection Association, on whose Electrical Committee are representatives of the various national electrical organizations, including associations of the manufacturers of electrical supplies, electrical inspectors, electrical supply jobbers, and the power companies.

In the majority of cases where inspections are made, the local regulations are more particular about certain features than the "Code." For example, when old or completed houses are wired, some cities require that armored conductors be used for wiring where the "Code" permits the use of non-metallic, flexible tubing (see page 393). In many places, buildings under construction must be wired with armored cables instead of permitting concealed knob and tube work. Again, some inspectors require that the bathroom fixtures be out of reach of grounded material and controlled by a switch located outside of the room. In all cases, however, the local requirements must be complied with rather than the "Code," since the code itself is legal only when made so by the local authorities.

240. Signaling Systems.—The wiring for systems in which the difference of potential between lines is approximately 20 volts or under is not governed by the " Code" or the inspection departments, except where the wiring is likely to come in contact with electric light or power circuits. An electric arc between most conductors cannot be maintained with less than 20 volts, since it acts as though there were a counter e.m.f. of from 14 to 20 volts in

the arc stream. Unless there is sufficient voltage to overcome this counter e.m.f. and also to supply the resistance-drop in the arc stream, the arc cannot be maintained. Again, the resistance of the small-sized conductors, together with the low voltage of the signaling systems, limits the current to such moderate values when short-circuits, grounds, etc. occur that there is little chance of developing sufficient heat to cause a fire.

In dry places, the conductors for telephones, annunciators, low-voltage electric clocks, bells, and other low-voltage devices may be either annunciator wire or No. 18 dampproof (DP.) office wire secured with insulated staples. The annunciator wire is cheaper and is used only for the less important installations. Where subject to dampness, rubber-covered (R.) No. 16 or 18 A.W.G. wire should be used.

When the dampproof office wire is used in damp places, there is always the possibility of high-resistance leaks between conductors or to ground. Although the leakage current may be very small, it may discharge the batteries in a short time, since the leakage current usually is flowing for 24 hr. a day. As a precaution against this leakage, the telephone companies use rubber-covered (R.) wire in wiring to the subscribers' sets.

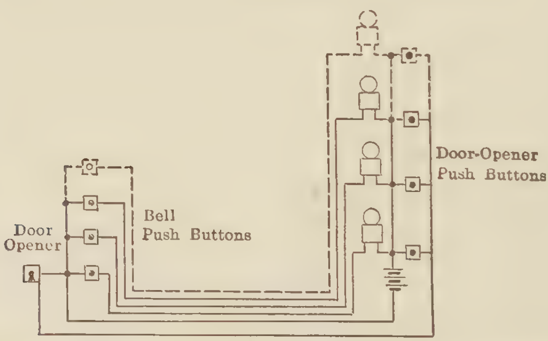
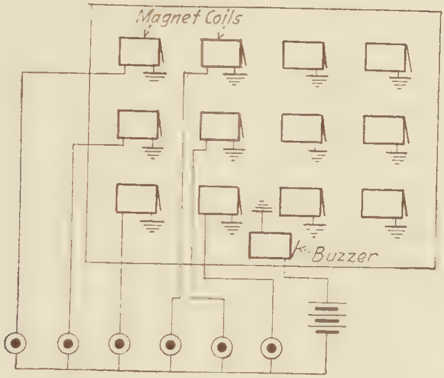


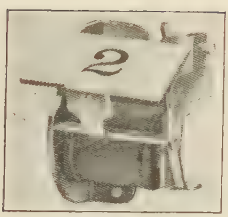
Fig. 337.—Wiring diagram of bells and door opener for an apartment house.

241. Bells and Door Openers.—Figure 337 shows the connections for the wiring of bells and a door opener for an apartment house. The number of push buttons and bells can obviously be changed to agree with the number of apartments. When the number of devices is small, the length of wire is reduced to a

minimum by using a common wire for two or more circuits. But in the more complicated systems, the amount of wire is sacrificed to simplicity in wiring, to facilitating the locating of trouble, and to the possibility of making extensions to the system after it has been completed.



(a)



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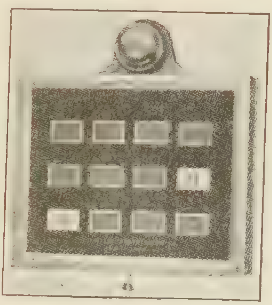


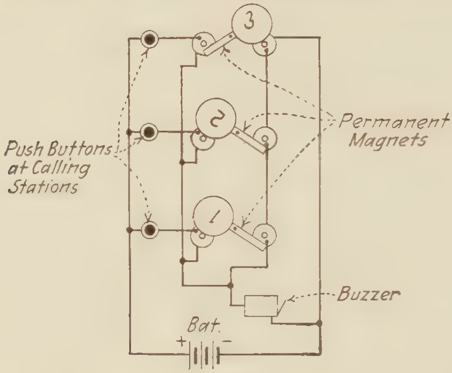
(c)

FIG. 338.—Gravity drop, mechanical reset.

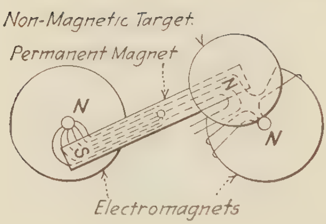
242. Annunciators.—Figures 338 and 339 show two common types of annunciator drops used in the various call systems. The number on the gravity drop (Fig. 338) comes into view through a small window in a cabinet when the electromagnet is energized and attracts the armature. The current flowing through the drops also flows through the buzzer, or through a relay which operates a buzzer, giving the signal. The drops are reset either by pushing up on a rod projecting through the bottom of the

cabinet, which mechanically raises all the drops, or the rod is forced up by an electromagnet controlled from some location within sight of the cabinet.

The French type of annunciator drop (Fig. 339) is more adaptable to electrical resetting. It may be so connected that all the drops in the cabinet, excepting the one indicating the calling station, are reset automatically, or so that they



(a)



(b)

(c)

FIG 339.—French type, electrical reset communicator drop.

may be reset either individually or collectively from push buttons. A small permanent magnet riveted to the moving arm (Fig. 339 (*a*)) is attracted (Fig. 339 (*b*)) or repelled by the magnetic poles set up at the ends of the coils when current is flowing through the windings. By a study of the wiring diagram (Fig. 339 (*a*)) it is seen that when a push button at a calling station closes the circuit, the right-hand coils which reset the drops, being in series, are all energized. Current from any one of the left-hand coils which cause the drops to indicate, must divide between the right-hand

coils and the buzzer so that the effect of any energized left-hand coil exceeds that of the corresponding resetting coil, and hence causes the drop to indicate. The polarities of the actuating coils must be correct, and should be checked if it is necessary to replace one (see Part I, page 20).

243. Insulated Wires.—The following types of insulated wires are in common use: rubber-covered (R.), weatherproof (WP.), slow-burning (SB.), and, for wiring fixtures, rubber-covered and heat resisting. Varnished cloth insulated (VC.) and slow-burning weatherproof (SBW.) wire are little used at the present time.

Rubber-covered wire for voltages up to 600 volts has an unbroken rubber covering of at least $\frac{3}{64}$ in. (1.2 mm.) in thickness, which is protected by a cotton braid impregnated with a moistureproof compound. Before leaving the factory, all rubber-insulated wire is immersed in water for at least 12 hr. and then subjected to high-voltage and insulation-resistance tests. Conductors from No. 14 A.W.G. to No. 8, inclusive, have but one cotton braid, but sizes larger than No. 8 have two protecting braids.

Since rubber is an unusually good insulator and is also waterproof, it is the best type of insulation for all wiring, except where it is subjected to high temperatures. Although it is more expensive than the other types, its use is required: (*a*) in all concealed wiring; and (*b*) for exposed interior wiring in all damp places.

The current-carrying capacity of rubber-covered wire is not as large as that of the other types (see Appendix I, page 418), since the rubber-covered wire must be used in places where the rubber is the only insulating medium, such as in raceways or conduit. A smaller factor of safety is permissible with other types of insulation, since such wires are usually installed in such a manner that the wires would be insulated by their supports, even if all their insulation were removed.

244. Slow-burning Wire.—Slow-burning wire has three braids of cotton, impregnated with a white compound having fire resisting properties. Slow-burning insulation is not fireproof and is not able to withstand the high temperatures that an asbestos covering will, but it will not burn readily nor conduct

fire along itself. The smooth, hard surface does not hold lint or dust as will the braids of rubber-covered and weatherproof wire when softened under heat. This type of insulation is especially useful in hot, dry places where ordinary insulations would rapidly disintegrate, and in places where wires are massed together as on the back of switchboards or in a wire tower, where the large amount of rubber insulation, because of its highly inflammable character, is objectionable. It can also be used for exposed wiring in dry places, since under these conditions, it does not come in contact with conducting materials. As impregnated cotton braid is not impervious to water, the slow-burning wire must be installed as if there were no insulation at all on the conductor.

245. Weatherproof Wire.—Weatherproof wire is cheaper than wire with other insulations, since it contains no rubber and consists merely of three braids of cotton impregnated with a moisture-proof compound.

This type of insulation is *limited to use out of doors*, as on pole lines where the insulating properties of the circuit are dependent on the supporting glass insulators and not on the insulation on the wires. As soon as weathering removes the impregnating compound and when the wire is wet, the conductors are no better insulated than bare conductors. A good rule to remember, therefore, is that weatherproof wire should not be installed unless bare wire could be used equally well.

246. Fixture Wire.—Fixture wire is made in No. 16 or 18 A.W.G. with rubber-covered or heat-resisting insulation over either solid or stranded conductors. The outer braid may be either cotton or silk. The No. 18 rubber-covered wire has an insulation $\frac{1}{64}$ in. (0.4 mm.) in thickness, and the No. 16 has $\frac{1}{32}$ in. (0.8 mm.). The rubber-covered wire is used in outdoor fixtures, and in places where it is not subject to extreme heat. Slow-burning or other heat-resisting, insulated fixture wire must be used in wiring fixtures for indoor use (see page 402) where the insulation will be subjected to temperatures greater than 120°F., or where the fixtures carry gas-filled incandescent lamps. Since the circulating gas in the gas-filled lamps conducts heat from the filament to the metal base and hence to the socket

and other parts of the fixture, rubber insulation would soon disintegrate, were it used.

247. Splicing Wires.—When wires are spliced or joined they must be made mechanically and electrically secure and have good electrical conduction without being soldered. They must then be soldered, unless the joint is made with a splicing device. The joint must then be covered with an insulation as good at least as that on the wires themselves.

Figure 340 illustrates the method of joining wires under various conditions. Care should be taken in removing the insulation not to cut or nick the copper. If the copper is cut at all, the wire

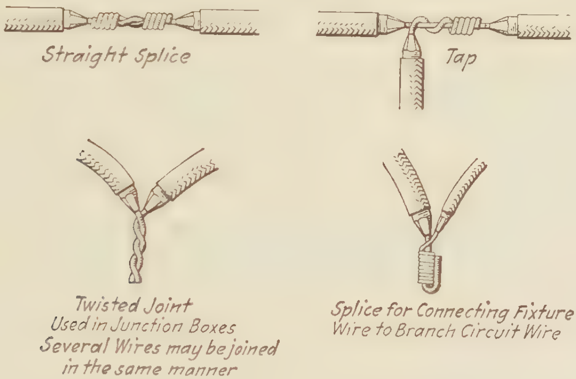


Fig. 340.—Methods of splicing wires.

will break at that point when it has been bent a few times, since most of the bending will be concentrated at that point. When the wires have been thoroughly scraped and cleaned, they should be twisted together or one wrapped around the other (Fig. 340). A small quantity of non-corrosive soldering paste or soldering fluid should then be applied and after the joint is heated with a flame or hot soldering iron, the solder can be applied. The joint should be sufficiently hot to cause the solder to flow freely and the entire surface of the copper should be tinned. Large conductors are usually tinned by pouring hot solder over the joint from one ladle above the wire to another held under the splice. If rubber tape is used and should come in contact with the copper when the copper is not covered with a coating of solder, the sulphur in the rubber compound will cause the copper to corrode.

That is, the sulphur which is necessarily in the rubber in order to vulcanize it will unite with the copper and form a green powder or copper sulphate. This green powder may be seen in old joints, improperly made. If rubber-covered wire is being spliced, a rubber tape or splicing compound should be wrapped around the joint until the rubber is at least equal in thickness to that on the rest of the wire. The rubber must then be covered with friction tape to a thickness equal to the braid. Naturally, rubber splicing compound need not be used on slow-burning wire or on wires which are similarly insulated.

Stranded wires, other than those used in flexible cords, must be soldered together before being fastened under clamps or binding screws. This is to allow the current to flow in all the strands and at the same time to prevent the squeezing out of the wires when flattened under the pressure of the binding screws, thereby allowing only a few of the strands to conduct the current. Usually flexible cords, such as lamp cord (Fig. 341), do not

Fig. 341.—Twisted lamp cord.

carry much current and can be clamped under binding screws after all the strands have been tightly twisted together to prevent any one strand projecting and causing a short-circuit.

Wires larger than No. 8, whether stranded or solid, must be soldered into lugs for all terminal connections, unless a solderless type of connector is used. If No. 6 wire carrying 50 amp., instead of being soldered into a lug, were looped around a terminal stud and the nut tightened, the contact area of the wire would be too small to insure the conducting of the current without heating.

248. Switching Arrangements.—All types of small quick-break switches may be grouped in two classes, surface and flush types. The surface switch is round and protrudes from the surface with either a porcelain or a nickel-plated cover. This type is generally used on exposed or open wiring, and must be mounted on a porcelain sub-base or some metallic conduit or

raceway fitting. Although the greater majority of the surface switches are of the snap type, many surface tumbler switches have been installed.

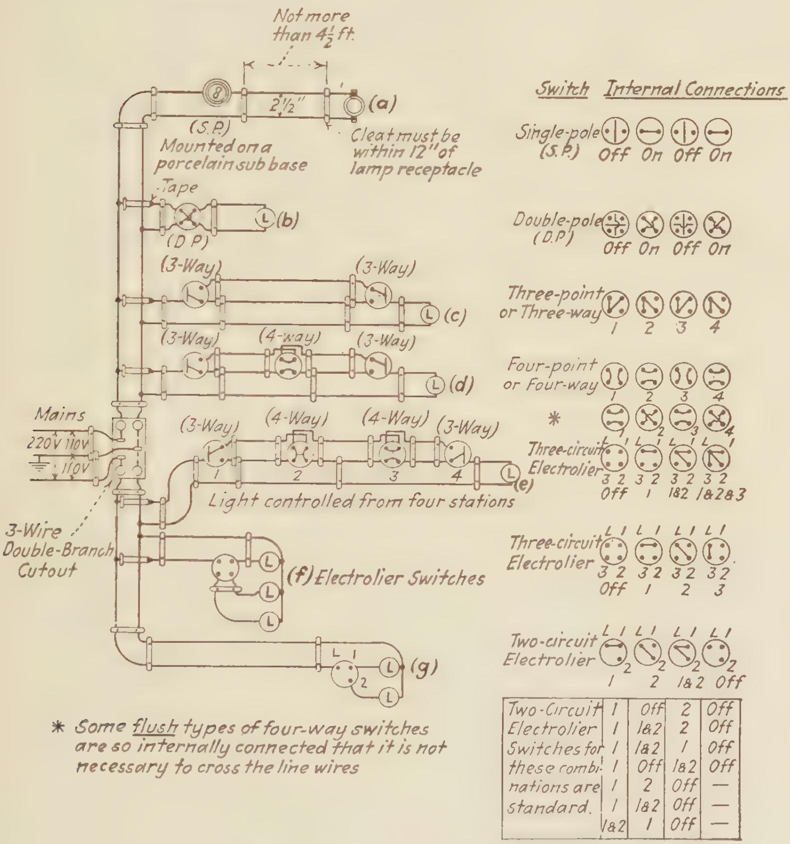


FIG. 342.—Typical switching connections.

The flush switch is set into the wall and inclosed in a metal switchbox. This type of switch is used chiefly with concealed wiring, and is of the push, rotary, or tumbler type.

Single-pole switches (Fig. 342 (*a*)) are extensively employed on branch lighting circuits. Since this type disconnects but one of the line wires, it must be placed in the ungrounded conductor when installed on a circuit which has one of its wires grounded

(Fig. 343). The reason for this is that if the switch is in the grounded side of the line, and an accidental ground occurs between the switch and lamps, the lamps can not be turned off.

Double-pole switches (Fig. 342 (*b*)), which open both line wires, are required only on ungrounded branch circuits to disconnect heating appliances consuming more than 1,320 watts, such as electric ranges. They are also used when a snap switch is desired on an ungrounded branch circuit to a motor of over ¼ hp.

Three-point or *three-way switches* (Fig. 342 (*c*)) are used to control lamps or other devices from two stations. For example, they are used to control a hall lamp located on the first floor from that floor as well as from the second, or to control shed or hall lights from two entrances. Three-point switches are classed as

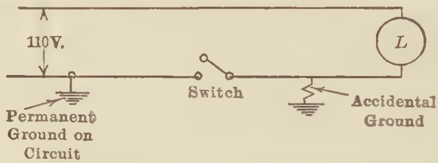


Fig. 343.—Switch in grounded side of line not operative with an accidental ground.

single-pole by code, since they open but one side of the circuit, and hence must be placed only in the ungrounded line. Two adjacent terminal posts are connected together on this type of switch, which may be identified by the word "connected" stamped between the posts. It may be necessary, however, to "ring" between adjacent posts in some cases to make sure that the switch will be installed correctly.

Four-point or *four-way switches* (Fig. 342 (*d*)) are used between three-point switches when it is desirable to control lamps from more than two locations. As many 4-point switches as are needed can be added between the 3-point switches (Fig. 342 (*e*)). Although, in general, all types of switches are connected in the same manner, it is not always necessary to cross two of the wires to a 4-point switch, as is done in Fig. 342. The wires are not crossed with the tumbler and shallow flush types of switches made by some manufacturers. Care should be taken, therefore, either to "ring out" the particular switch to be installed or to obtain the correct connections from the manufacturer's catalogue.

Electrolier switches, as the name implies, are used chiefly to control lamps or groups of lamps on fixtures. A wide choice of switching combinations and sequences is possible with standard switches (Fig. 342 (*f*) and (*g*)). These switches, as with single pole and three way, must be in the ungrounded circuit wire only.

249. Methods of Interior Wiring.—Methods of interior wiring can be grouped as follows: (1) open or exposed wiring; (2) concealed knob and tube wiring; (3) wiring with armored cable; (4) wiring in conduit; and (5) wiring in other ridged raceways, such as wooden or metallic molding.

1. *Open wiring* in dry places for potentials up to 300 volts can be used when permitted by the local inspection departments. The smaller sizes of conductors, such as No. 14 or No. 12 A.W.G., can be supported with 2-wire porcelain cleats (Fig. 342). These cleats space the wires 2½ in. apart and ½ in. from the surface over which the wires are run. The distance between the cleats should not exceed 4½ ft. and should frequently be much less, if the wires are likely to be disturbed. The cleats may be secured with two nails or roundhead screws. When nails are used, a leather washer or "leather head" must be used under the head of each nail to cushion the blow of the hammer when driving the nail and prevent the breaking of the cleat. The nails must penetrate the wood a distance equal to the height of the cleat to insure that the cleat will hold the wires taut. Taut wires run in a straight line denote good workmanship. For voltages from 301 to 600 volts, knobs must be used which will separate the wire 1 in. from the surface wired over, and the wires must be at least 4 in. apart.

Rubber-covered (R.), slow-burning (SB.), or slow-burning, weatherproof (SBW.) can be used in dry places for voltages up to 300, but only rubber-covered wire is permitted in damp places or in buildings subject to moisture.

In damp locations, wires must be run on knobs.

For open wiring in buildings of mill construction, No. 8 wires and larger may be run direct from timber to timber if kept 6 in. apart, being supported by single-wire knobs or cleats on each timber or girder. Although the feeders or mains are frequently run openly in this type of building, the wiring on the side walls is often inclosed in conduit.

Open wiring, where exposed to mechanical injury such as on the lower edge of the floor joists in the cellars of dwelling houses, must be protected by running boards not less than ½ in. in thickness and 3 in. in width, or by guard strips not less than ⅞ in. in thickness and at least as high as the insulating support, placed on each side of the wiring (Fig. 344).

Vertical wires on side walls where exposed to mechanical injury or when passing through floors must be protected by one of the methods shown in Fig. 345. The conduit method (*b*) is preferable to the wooden boxing (*a*), since it is quicker to install and occupies less space. If slow-burning wire is being installed, however, it is necessary to change to rubber-covered wire or encase each of the slow-burning wires in flexible tubing before

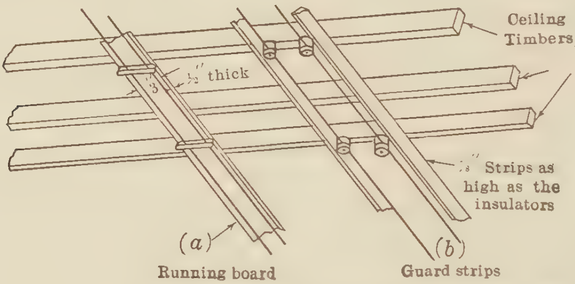


Fig. 344.—Open wiring protected by running boards and guard strips.

drawing them into the conduit. The short piece of protecting conduit need not be grounded (see page 393), if it is out of reach of any grounded material and is not more than 25 ft. in length.

Where wires pass through floors, timbers or walls (Fig. 345), or come within 2 in. of other wires or piping, porcelain tubes must be used. The tubes must always be secured in place by tape (Fig. 342), unless fastened by some other means.

2. *Concealed knob and tube work* (Fig. 346) is employed in wiring wooden buildings in process of construction, and consists in supporting the conductors on knobs between the floors and partitions and in using porcelain tubes whenever the wires pass through timbers or other barriers. While some of the larger cities do not permit this method of wiring, it is still being used in many sections of the country, since it is the cheapest

method, where the cost of material is important. Where the cost of labor is the largest factor, however, the time saved in wiring with armored cable makes its use preferable.

Rubber-covered wire must be used for this class of wiring, as in all places where the conductors are concealed from view and

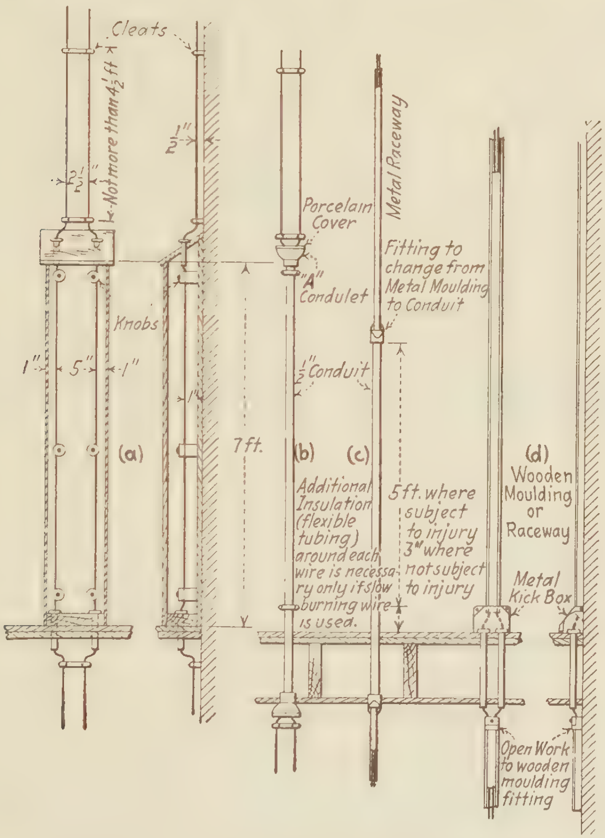


FIG. 345.—Protection of wiring going through floors.

must be supported on knobs which separate the conductors 1 in. from the surface. The wires are usually run on the sides of separate floor timbers or wall joists, as they must be kept at least 5 in. apart, except where the wires must be closer in order to enter outlets or switchboxes. Here each conductor must be

incased in a non-metallic, flexible tubing (Fig. 346), which must extend back to the last support and be secured to the outlet or switchbox by some type of clamping device.

Outlet plates or boxes must be installed at all fixture outlets to provide support for the fixtures. The boxes employed on the ceilings of dwelling houses are usually about ¾ in. deep and are screwed to a board nailed on the under edge of the floor timbers

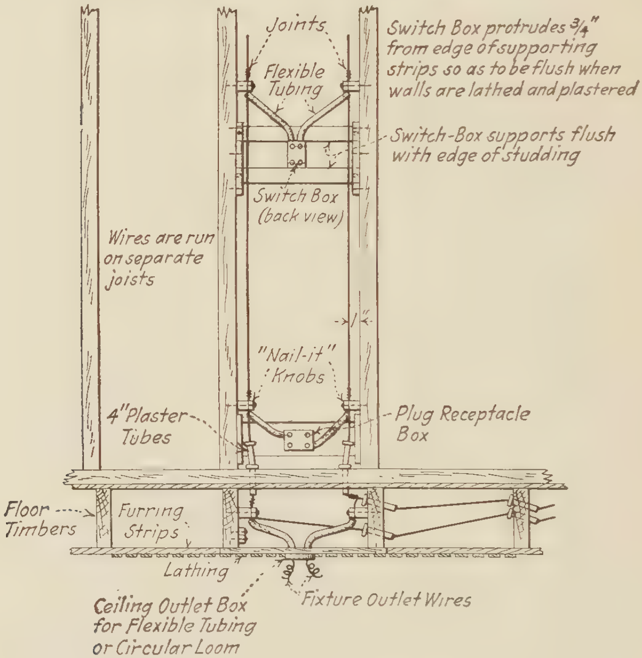


Fig. 346.—Concealed knob and tube wiring.

(Fig. 346). When the lathing and plastering are completed, the edge of the box is flush or protrudes slightly from the finished surface.

When it is necessary to pass through floors or braces on the inside of partitions which will be plastered later, an additional 3- or 4-in. porcelain tube must be placed on top of the tube which insulates the conductor from the timbers (Fig. 346). This additional tube prevents the wires from becoming embedded in the plaster which drops when the partitions are being plastered.

In wiring finished buildings for voltages up to 300 volts, where it is impossible to support the wires between the floors and walls, each wire can be drawn into non-metallic flexible tubing (Fig. 347) and then pulled or fished from one outlet to the next. There must be no breaks in the tubing between outlets or knobs. Flexible tubing cannot be used even for short distances in places subject to moisture, since it contains no rubber, but affords mechanical protection only. When subjected to moisture continuously, armored cable with a lead sheath between the outer braid and the armor or conduit should be installed. These need not be grounded (see page 390), if insulated and out of reach of grounded material and the section does not exceed 25 ft. in length.

3. *Armored cable* wiring in general is limited to Nos. 12 and 14 A.W.G. conductors, but it is preferable to exposed cleat or concealed knob and tube wiring from the viewpoint of fire hazard,

Fig. 347.—Non-metallic flexible tubing.

Fig. 348.—Armored cable or armored conductors.

since the conductors are protected from mechanical injury at all points by the strong steel armor. While in many cases of open wiring the conductors at the time of installation do not require special protection, they may later be subject to injury due to unforeseen conditions. Armored cable may be easily located so as not to be conspicuous and can be secured in place with metal straps held by screws or nails.

The conductors in the armored cable (Fig. 348) are rubber covered and the armor is wound around the wires so that the conductors can be pulled out only in short lengths.

Where armored cable enters junction or outlet boxes, it must be secured to the box by a connector or clamp (Fig. 349) so as to be firm mechanically and to make a good electrical connection. The armor must be at ground potential and the armor connections throughout should have a current-carrying capacity equal to No. 10 wire in order to give the proper protection in case of a "ground" in the system. In order to remove the armor a suffi-

cient distance to make connections inside of a box or fitting, it is only necessary to cut a groove across the metal strips with a fine-toothed hacksaw, and then break the armor by bending it. If the metal is cut entirely through, there is a possibility of cutting the insulation and even nicking the wire. If the copper conductor is nicked, it may not break off until the building is nearly completed and the switches and fixtures are being installed, thereby necessitating considerable extra expense to repair the damage. When the cable is clamped in switchboxes, some cities require a brass ferrule with rounded edges to be screwed back over the armor to prevent the sharp edges of the armor from cutting the insulation on the conductors (Fig. 349)

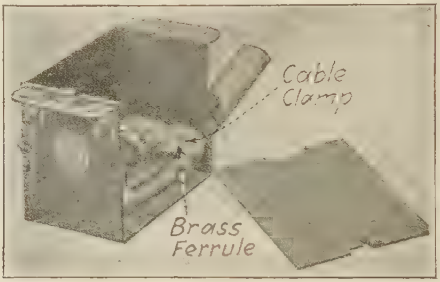


Fig. 349.—Switch box.

One of the conductors in the cable has a white braid (Fig. 348) and, so far as is possible, must be the neutral or grounded conductor throughout the system. Although it may be necessary to use short lengths of the white wire for the ungrounded side, the wiring should be arranged so that the white wire will be the grounded side at all fixture outlets (Fig. 354, page 401).

4. *Conduit* is usually employed to protect wires when the conductors are larger than No. 12, and throughout buildings of concrete or other fireproof construction.

In general, ½-in. conduit is the smallest size which is permitted and care should be taken not to bend it so that the radius of the inner edge is less than 3½ in. A smaller radius flattens or buckles the pipe, which, in addition to being unsightly, may prevent the wires from being drawn in. Since the conductors must be pulled in from one outlet or fitting to another, long runs or many bends in the conduit should be avoided. Not more

than four quarter-bends should be made, not counting those at the fitting. The entire conduit system must be installed and securely fastened in position, and in buildings under construction all the mechanical work on the building should be completed, as far as possible, before the wires are drawn in. Where the conduit enters knock-out holes in junction boxes or cabinets, the conduit must be clamped to the box by a bushing on the end of the pipe inside the cabinet and a locknut on the outside. These bushings with rounded edges to prevent damaging the insulation at the point where the conductors leave the conduit should be used only inside cabinets or boxes, although often their use on the end of the pipe protecting a ground wire (Fig. 353) is allowed. Under all other conditions where conductors leave the conduit or other

FIG. 350.—Flexible conduit.

metal protecting casings, each wire must pass through a separate hole in a piece of porcelain attached to a proper terminal fitting (Fig. 345).

Conduit and the fittings available for use with it are manufactured with both a black enamel and sherardized or galvanized outer coating. When the enameled equipment is installed, the insulating coating must be removed from the threads and other points of connection (see page 400). A few cities even prohibit the use of enameled conduit and fittings because of the possibility of enamel at the points of connection insulating one section from another.

Flexible conduit (Fig. 350) is often installed where the bending of rigid conduit to fit the surfaces to which it must be secured is difficult and also in short lengths to motors where some change in position is necessary in order to tighten belts, chains, etc. (Fig. 351).

5. *Wire raceways,* which include both wooden and metal types, are installed chiefly as branch-circuit extensions to the existing wiring in completed buildings. Hence, their use is limited

to systems of not over 300 volts and in the case of metal moldings to circuits protected by fuses not larger than 20 amp. at 125 volts or 10 amp. at 250 volts. Although moldings cannot be concealed, metallic molding may pass through dry partitions or walls if there are no breaks in the "backing" or "capping"

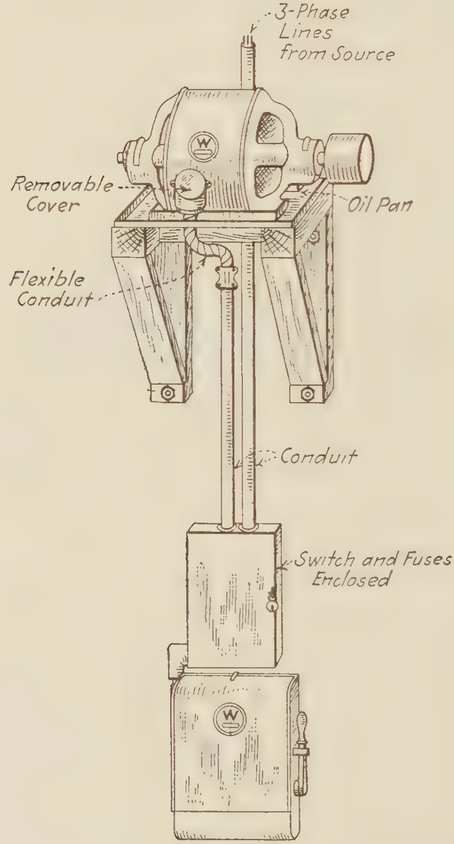


Fig. 351.—Conduit wiring to a motor.

inside the partition. When passing through floors or damp walls, it is better to change from metallic molding to conduit, using the proper connecting fittings (Fig. 345). The conduit need not extend more than 3 in. above the floor where the molding would not be subject to mechanical injury.

Each type of metallic molding, some limited to two conductors and others permitting the use of four No. 14 conductors, has a complete line of fittings and it is possible to install the raceway correctly only when the proper fittings are used.

In making a surface extension with metal raceway from an existing concealed conduit outlet, a box extension ring must be installed over the original outlet box and be both mechanically and electrically secured to it (Fig. 352). The electrical connection is necessary in order to ground the molding (see Par. 252).

In all metallic protecting systems, such as conduit, armored cable, or metal moldings, joints and splices in conductors must be made only in junction boxes or other proper fittings and, hence, these fittings can be located only in accessible places and never concealed in partitions, etc. Splices or joints in the

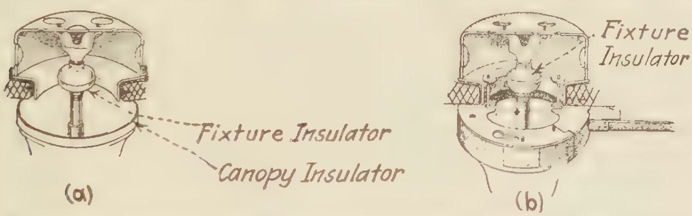


FIG. 352.—Method of making a moulding extension from an existing outlet.

wires must never be in the conduit piping or raceway itself, since the splices are a possible source of trouble from corroding or grounding in case water enters the conduit. Again, if the proper amount of insulation is placed around the splice (see page 386), the diameter of the wire must be larger at that point, which would greatly increase the possibility of injuring it if it were drawn into conduit piping.

250. Alternating-current Systems.—All the conductors of an alternating-current system must be placed in the same conduit, armored cable, metal raceway, or other conducting casing. When, in an alternating-current circuit, the current is increasing and decreasing in value, the magnetic field around any one conductor is also varying. If this single wire with its varying magnetic field is surrounded with any conducting material, short-circuit or eddy currents will be induced in that material. This is similar to short-circuiting the secondary of a transformer.

Instances have occurred where, with large currents, the conduit has become excessively hot. Again this varying magnetic field around one wire induces a reactive voltage in that conductor, which increases the voltage-drop (see pages 41 and 46). If all the conductors of one circuit are placed inside one casing, the net magnetic field around all the conductors is practically zero, since the current flowing in one direction is equal to the return current flowing in the opposite direction. Since many direct-current systems are later changed to alternating-current, it is advisable to place all the conductors of a direct-current circuit in one armor, in order to avoid these effects when the change is made.

251. Service Wires.—Service wires (Fig. 353), the conductors which bring the electric power into a building, should enter the building as near as possible to the service switch. That is, for overhead service the conductors must be run on the outside of the building to a point near the service switch, so that when the switch is open all the electrical conductors and equipment inside the building will be "dead." The service wires must be rubber covered from the point of support on the outside of the building to the service switch or cut-out, and must be at least as large as No. 10 wire, although usually are of No. 8 or larger.

Generally, when the conductors from overhead lines enter a building, the wires are incased in rigid conduit equipped with a weather cap (Fig. 353), the inner end of the service conduit entering a metal service cabinet in which the service fuses and switches are located. The service switch must be operable without opening the cabinet, unless the current of the various separate circuits is metered, as in apartment house installations. The switch and cut-out of each of these circuits must be inclosed and the switches operated externally. This is necessary where it is desirable to be able to open all the circuits in case of emergency by means of one easily accessible switch, but still undesirable to have that switch opened when it is necessary to disconnect the power from one apartment only.

When conduit is not used, drip loops should be formed on the wires which must pass individually upward and inward through slanting porcelain tubes in the side of the building.

All *ungrounded* service wires must be connected to the interior wiring through a blade of the service switch and be fused in the service cabinet. That is, a service switch controlling a 3-wire direct-current or a single-phase system having a grounded wire does not need to open the grounded conductor.

252. Grounding.—Grounding may be classified as: (1) system or service grounding; (2) service-conduit grounding; (3) the grounding of interior conduit, armored cable, metallic raceways and electric equipment (motor, generators, ranges, etc.).

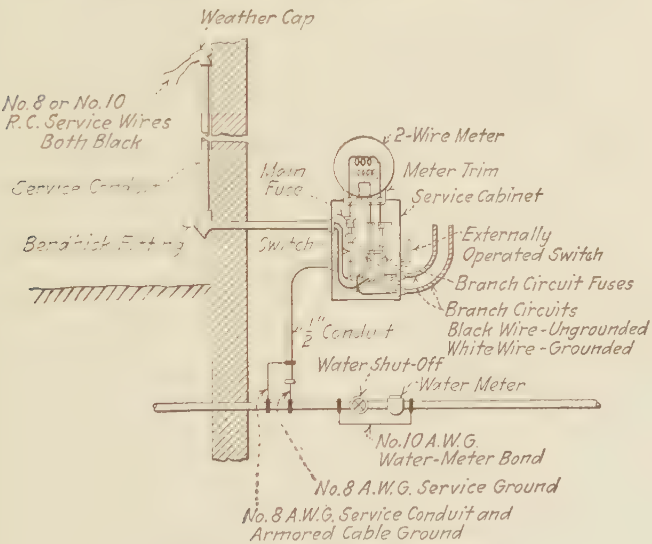


Fig. 353.—Typical service installation.

1. *Direct-current and alternating-current distribution systems* of 300 volts or under are usually grounded. Direct-current systems need only be grounded at the generating stations because the grounded wire is electrically connected to one of the conductors in all the circuits throughout the system. But in alternating-current systems, since one section can be insulated from the other by a transformer, each section of 300 volts or under is grounded at each individual service. The conductor grounding the alternating current system should not be smaller than No. 8 wire and must be run from the supply conductor at the

entrance to the water-piping system on the street side of the water meter (Fig. 353).

2. *The service conduit* (Fig. 353) which protects the service wires on the outside of the building must be grounded by a wire at least as large as No. 8, run direct to ground. This ground wire may also serve to ground the interior conduit, if the interior conduit is connected with the service conduit by means of the service cabinet, but any interior armored cable or conduit cannot serve as a part of the service-conduit ground.

3. *Grounding conduit, armored cable, metal raceways, motors,* and *other electrical equipment* to water-piping systems or to each other, if one is grounded, is necessary since the entire metallic system surrounding the conductors must be at ground potential, or, under certain conditions, current would flow between it and any other grounded metal work with which it makes contact. This current might form an electric arc across some poor connection and, thus, constitute a fire hazard. It is only necessary to ground the metallic system, including motors and other equipment, at one point, provided that each section makes good electrical connection with the next.

Although under normal conditions the metallic casing or the wire grounding the casing does not carry current, it would do so if an inclosed conductor or other live part came in contact with it, and, hence should have a sufficient carrying capacity to blow the fuse protecting the circuit if accidentally grounded. Hence, the wire grounding the conduit or other electrical equipment which incases wires protected by fuses up to 100-amp. capacity, must be at least No. 10 A. W. G. copper wire. This grounding wire must be rubber-covered and be protected from mechanical injury just as if it were above ground potential. Where conduit or equipment grounds are made to the water-piping systems at other places than on the street side of the water meter, the water piping must be bonded around the meter; that is a "jumper" of No. 10 wire must be connected by means of proper ground clamps to the piping on each side of the meter so that the equipment will be properly protected when the meter is removed. (See Fig. 353.)

253. Fixtures and Sockets.—Many electricians assemble the parts and wire the fixtures themselves. All the wiring in the

fixture must be done with at least No. 18 fixture wire which should be tested for "grounds" and correct polarities before the fixture is hung in place. Much time will be saved if the burrs in the tubings, due to the cutting, drilling and threading, are removed, since these burrs will probably cause short-circuits or "grounds"

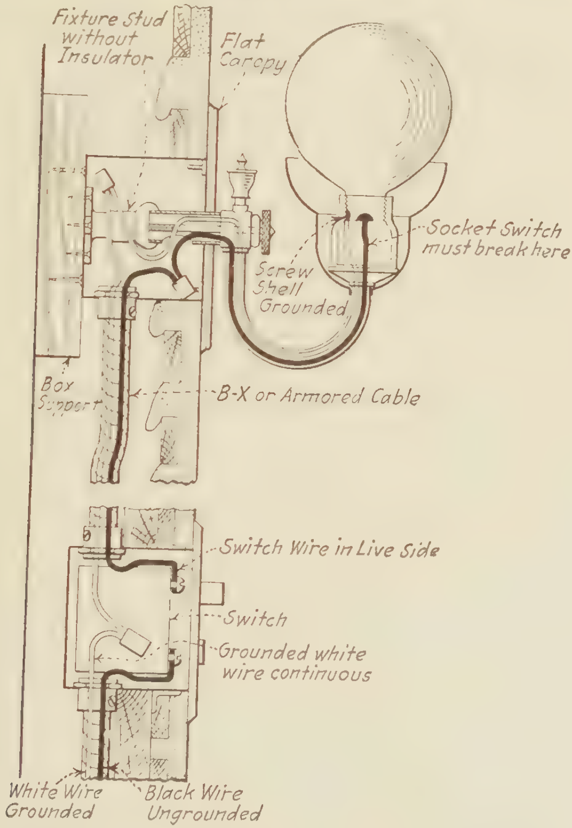


FIG. 354.—Switch and fixture wiring showing correct polarity.

when the fixture wire is drawn in. Often these troubles do not appear when the fixture is tested but come later, due to the handling during installation. They sometimes appear even after the installation is completed and accepted, as the sharp edges cut through the insulation.

Whether the fixture is to be grounded or insulated, the screw shells of the sockets (Fig. 354) should all be connected to the same wire in the fixture stem and this wire marked in some manner so that it may be readily identified. When the fixture is hung in place this marked wire must be connected to the white or ground wire of the conductors supplying power to the fixture outlet. Indoor fixtures should be wired with fixture wire having a heat-resisting insulation since it is likely that gas-filled incandescent lamps will be used in the sockets.

Straight electric fixtures are ordinarily grounded by being mechanically connected to the metal casing which protects the supply conductors. Where the grounding may be doubtful, as when the fixture makes contact with a metal ceiling or where the plaster is supported by metal lathing, the fixture must be either *grounded* or thoroughly *insulated*. It can be considered grounded if it is mechanically connected to conduit, armored cable, metal raceway systems, or to grounded gas piping. Where it is desired to insulate a fixture from metal work, a fixture insulator (Fig. 352 (*b*)) and, if necessary, a canopy insulator (Fig. 352 (*a*)) must be used.

254. Pendants.—The ordinary twisted lamp cord should be used only where pendant lamps hang freely, while reinforced cord should be used if it is likely to come in contact with any object. The weight of the pendant lamps and shades must not be placed on the binding screws in the sockets, ceiling rosettes, or in the joints in ceiling fittings but should be on "Underwriters Knots" (Fig. 355) tied in the cords unless other means are provided to relieve the stress. The socket must have either a pendant cap which has an insulating bushing or else a $\frac{3}{8}$-in. cap with a threaded composition bushing to protect the cord.

255. Outlets.—In frame buildings under construction, outlet boxes from which fixtures are to be hung should be screwed to a $\frac{7}{8}$-in. board fastened to the floor timbers and flush with the back of the lathing. This method should also be used in wiring completed buildings if the fixtures are heavy and the space above the outlet is accessible from roof spaces or by removing flooring. However, when this is difficult, as for example when it would be necessary to lift a polished hardwood floor, a special fixture-

support fitting can be used which distributes the weight of the fixture over a large area on top of the lathing.

Although shallow, cast-iron outlet boxes can be used on side walls, the fixtures installed with this type of box must have canopies in which the splices can be made. In the last few years, however, there has been a greater preference for wall brackets with flat canopies, and since there is no space in the canopy for the connections, outlet boxes having a depth of at least 1½ in. must be installed (Fig. 354).

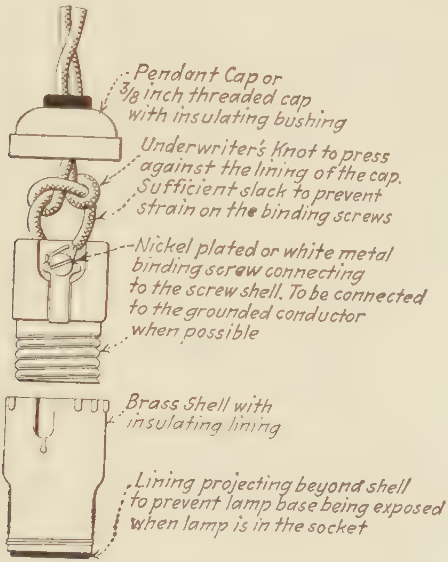


FIG. 355.—Underwriter's knot in a pendant socket.

Without permission of the inspection department, the number of outlets on a 2-wire branch circuit and on either side of a 3-wire branch circuit should not exceed 12, and under ordinary conditions should not reach that number.

256. Overload Protection.—*A fuse* or circuit breaker must be provided in all ungrounded conductors to protect the wires and the connected equipment from injury due to excessive current (see page 299, Part I). Fuses are more commonly employed to protect circuits and equipment taking a current of 100 amp. or less, with the possible exception of induction motors which

more often are protected by overload relays operating the same as automatic circuit breakers (see page 244).

In order to protect wiring properly, one of these automatic cut-outs must be installed at every point where a change is made in the size of the wire. This does not apply to No. 16 or 18 fixture wire at the outlets, since it would be impracticable to put fuses in fixture canopies. Here the fixture wire is considered properly protected by 15-amp. fuses which are the largest that should be used on any 125-volt branch lighting circuit except when supplying current to mogul sockets only.

Except in branch circuits in which the grounded side is not properly connected throughout, fuses must not be placed either in a grounded line (one of the system conductors that is connected at some point to the ground) or in a ground wire (that wire which connects a system wire, conduit or other equipment with the ground). These rulings are necessary, as the grounded line or ground wire maintains many parts of the system at ground potential. For example, if a grounded main line should be opened by the blowing of a fuse placed in it, the screw shells on all the grounded fixtures (insulated only by a thin paper lining) in the various branch circuits would be at 110 volts from the grounded cases. Again, if the grounded neutral on a 3-wire system were opened, the potential between either outer and neutral on some of the branch circuits might approach 220 volts instead of remaining at 110 volts. This voltage would burn out the lamps or other devices on that side of the system.

Fuses, cut-out bases, and *switches* change sizes, as shown in the following table:

Amperes	Fuses and Cut-out Bases 250 and 600 Volt
0–30	Made in both Edison plug (125 volts only) and spring-clip cartridge type (Ferrule contact).
31–60	Spring-clip cartridge (Ferrule contact).
61–100	Knife-blade cartridge type.
101–200	Knife-blade cartridge type.
201–400	Knife-blade cartridge type.
401–600	Knife-blade cartridge type. Maximum size enclosed fuse.
601 and larger.	Minimum number of equal size fuses connected in parallel may be installed on other than motor circuits. Circuit breakers should be used on motor circuits.

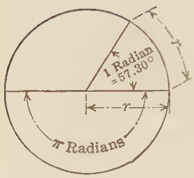
Switches are made in many sizes, from 30 amp. up to many thousands of amperes.

Care must be exercised in selecting the correct size of switch and fuse block. For example, a 65-amp. fuse cannot be inserted in 60-amp. fuse clips, nor a 60-amp. fuse in 100-amp. fuse clips, since the type of fuse and the distance between clips are different. The current rating of both switches and cut-outs is plainly indicated on them and it is the maximum value which they should be permitted to carry.

Since the rating of a fuse is only about 90 per cent. of the current that it will carry indefinitely, and since it may require a few minutes before the heat due to slightly excessive current is sufficient to melt the fuse wire and hence open the circuit, rubber insulation may be permanently damaged if the fuses are larger than the current carrying capacities given by Appendix I, page 418. If fuses selected to conform with the maximum current-carrying capacity of the wire are not large enough to carry the load, either the load must be reduced or the size of the wire increased.

APPENDIX A

CIRCULAR MEASURE—THE RADIAN



The *radian* is a circular angle subtended by an arc equal in length to the radius of its circle as shown in the figure. The circle has a radius of r units and the radian is subtended by an arc whose length is r units.

As the circumference of a circle is $2\pi r$ units, there must be 2π, or 6.283 radians in 360°. Therefore, 1 radian equals $360°/2\pi = 57.30°$. It follows that $180° = \pi$ radians.

Angular velocity is often expressed in radians per second, and the accepted symbol is ω (omega). In every revolution a rotating quantity completes 2π radians. If the rotating quantity makes n revolutions per second, its angular velocity $\omega = 2\pi n$ radians per second.

APPENDIX B

TRIGONOMETRY—SIMPLE FUNCTIONS

1. The sine (sin) of an angle $= \dfrac{\text{opposite side}}{\text{hypotenuse}}$

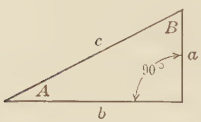
2. The cosine (cos) of an angle $= \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

3. The tangent (tan) of an angle $= \dfrac{\text{opposite side}}{\text{adjacent side}}$

4. The cotangent (cot) $= \dfrac{1}{\tan} = \dfrac{\text{adjacent side}}{\text{opposite side}}$

5. The secant (sec) $= \dfrac{1}{\cos} = \dfrac{\text{hypotenuse}}{\text{adjacent side}}$

6. The cosecant (cosec) $= \dfrac{1}{\sin} = \dfrac{\text{hypotenuse}}{\text{opposite side}}$



7. $\sin A = \dfrac{a}{c}$

8. $\cos A = \dfrac{b}{c}$

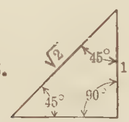
9. $\tan A = \dfrac{a}{b}$

10. $\cot A = \dfrac{b}{a} = \dfrac{1}{\tan A}$

11. $\sec A = \dfrac{c}{b} = \dfrac{1}{\cos A}$

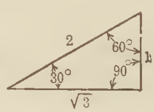
12. $\operatorname{cosec} A = \dfrac{c}{a} = \dfrac{1}{\sin A}$

13.



Ratio of sides in a right isosceles triangle

14.



Ratio of sides in a 30–60° right triangle

15. $\sin B = \dfrac{b}{c} = \cos A = \cos(90° - B)$, since $A = 90° - B$

16. $\cos B = \dfrac{a}{c} = \sin A = \sin(90° - B)$

17. $\dfrac{\sin A}{\cos A} = \dfrac{a/c}{b/c} = \dfrac{a}{b} = \tan A$

18. $\sin\ 30° = 0.5$

19. $\cos\ 30° = \sqrt{3}/2 = 0.866$

20. $\sin\ 60° = \sqrt{3}/2 = 0.866$

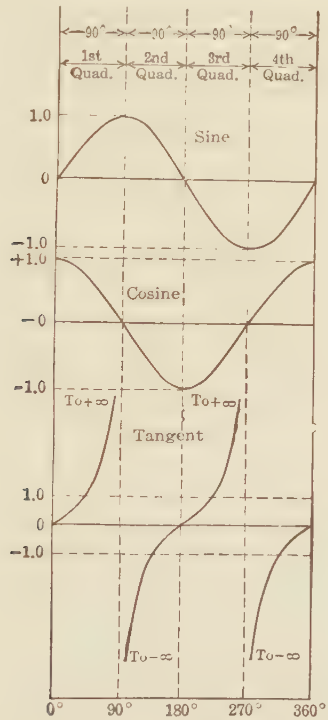
21. $\cos\ 60° = 0.5$

22. $\tan\ 30° = 1/\sqrt{3} = 0.577$

23. $\tan\ 60° = \sqrt{3} = 1.732$

24. $\sin\ 45° = \cos\ 45° = 1/\sqrt{2} = 0.707$

25. $\tan\ 45° = 1.0$



Graphic Representation of Trigonometric Functions.

26. $\sin\ (90° + x) = \cos x$

27. $\cos\ (90° + x) = -\sin x$

28. $\tan\ (90° + x) = -\cot x$

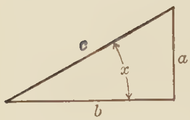
29. $\sin\ (180° - x) = \sin x$

30. $\cos\ (180° - x) = -\cos x$

31. $\tan\ (180° - x) = -\tan x$

APPENDIX C

SIMPLE TRIGONOMETRIC FORMULAS



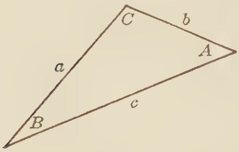
$$a^2 + b^2 = c^2$$

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2} = 1$$

since $\sin x = \frac{a}{c}$ $\qquad \cos x = \frac{b}{c}$

32. $\therefore \sin^2 x + \cos^2 x = 1$

Law of Sines.
In *any* triangle

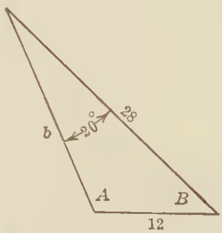
33. 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example.—Given $a = 28$, $c = 12$, $C = 20°$
Find A, B, and b.

$$\frac{28}{\sin A} = \frac{12}{\sin 20°} \quad \sin A = \sin 20° \frac{28}{12} = 0.342 \frac{28}{12} = 0.798$$

A is obviously greater than 90°.
$\sin A = \sin (180° - A)$ (See **29**)
$\sin 52.9° = \sin 127.1°$
Hence, $A = 127.1°$. *Ans.*



$$B = 180° - 20° - 127.1° = 32.9°. \quad \textit{Ans.}$$

$$\frac{b}{\sin 32.9°} = \frac{12}{\sin 20°} \quad b = 12 \frac{0.543}{0.342} = 19.05. \quad \textit{Ans.}$$

Law of Cosines.—In any triangle the square of any side is equal to the sum of the squares of the other two sides minus twice the product of these two sides into the cosine of their included angle.

That is:

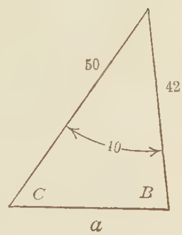
34. $a^2 = b^2 + c^2 - 2bc \cos A$ (See triangle in **33**)

35. $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

36. $\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$

37. $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Example.—Given two sides of 50 and 42 and included angle 40°. Find a, B, and C.



$$a^2 = 50^2 + 42^2 - 2 \times 50 \times 42 \cos 40°$$
$$= 2{,}500 + 1{,}764 - 4{,}200 \times 0.766$$
$$= 1{,}047$$
$$a = 32.36. \quad \textit{Ans.}$$
$$\frac{32.36}{\sin 40°} = \frac{50}{\sin B}$$
$$\sin B = \sin 40 \frac{50}{32.36} = 0.6428 \times 1.545 = 0.9931$$
$$B = 83.3°. \quad \textit{Ans.}$$
$$C = 180° - 83.3° - 40° = 56.7°. \quad \textit{Ans.}$$

(See page 469 for problems in trigonometry.)

APPENDIX D

MATHEMATICAL TABLES

Natural Sines and Cosines

NOTE.—For cosines use right-hand column of degrees and lower line of tenths.

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
0°	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	89
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	88
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506	87
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680	86
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854	85
5	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028	84
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201	83
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374	82
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547	81
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719	80°
10°	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891	79
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062	78
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232	77
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402	76
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571	75
15	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740	74
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907	73
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074	72
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239	71
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404	70°
20°	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567	69
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730	68
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891	67
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051	66
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210	65
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368	64
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524	63
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679	62
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833	61
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985	60°
30°	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135	59
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284	58
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432	57
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577	56
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721	55
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864	54
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004	53
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143	52
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280	51
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414	50°
40°	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547	49
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678	48
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807	47
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934	46
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059	45
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

Natural Sines and Cosines (*Concluded*)

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181	44
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302	43
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420	42
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536	41
49	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649	40°
50°	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760	39
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869	38
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976	37
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080	36
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181	35
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281	34
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377	33
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471	32
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563	31
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652	30°
60°	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738	29
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821	28
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902	27
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980	26
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056	25
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128	24
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198	23
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265	22
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330	21
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391	20°
70°	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449	19
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505	18
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558	17
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608	16
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655	15
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699	14
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740	13
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778	12
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813	11
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845	10°
80°	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874	9
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900	8
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923	7
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943	6
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960	5
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	4
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985	3
87	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993	2
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998	1
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000	0°
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

APPENDIX E

Natural Tangents and Cotangents

NOTE.—For cotangents use right-hand column of degrees and lower line of tenths.

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
0°	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	89
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	88
2	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507	87
3	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682	86
4	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857	85
5	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0 0998	0.1016	0.1033	84
6	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210	83
7	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388	82
8	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566	81
9	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745	80°
10°	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926	79
11	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107	78
12	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290	77
13	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475	76
14	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661	75
15	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849	74
16	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038	73
17	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230	72
18	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424	71
19	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620	70°
20°	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819	69
21	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020	68
22	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224	67
23	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431	66
24	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642	65
25	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856	64
26	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073	63
27	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295	62
28	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520	61
29	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750	60°
30°	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985	59
31	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224	58
32	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469	57
33	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720	56
34	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976	55
35	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239	54
36	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508	53
37	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785	52
38	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069	51
39	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361	50°
40°	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662	49
41	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972	48
42	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293	47
43	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623	46
44	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965	45
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

Natural Tangents and Cotangents (*Concluded*)

Deg.	°0.0	°0.1	°0.2	°0.3	°0.4	°0.5	°0.6	°0.7	°0.8	°0.9	
45	1 .0000	1 .0035	1 .0070	1 .0105	1 .0141	1 .0176	1 .0212	1 .0247	1 .0283	1 .0319	44
46	1 .0355	1 .0392	1 .0428	1 .0464	1 .0501	1 .0538	1 .0575	1 .0612	1 .0649	1 .0686	43
47	1 .0724	1 .0761	1 .0799	1 .0837	1 .0875	1 .0913	1 .0951	1 .0990	1 .1028	1 .1067	42
48	1 .1106	1 .1145	1 .1184	1 .1224	1 .1263	1 .1303	1 .1343	1 .1383	1 .1423	1 .1463	41
49	1 .1504	1 .1544	1 .1585	1 .1626	1 .1667	1 .1708	1 .1750	1 .1792	1 .1833	1 .1875	40°
50°	1 .1918	1 .1960	1 .2002	1 .2045	1 .2088	1 .2131	1 .2174	1 .2218	1 .2261	1 .2305	39
51	1 .2349	1 .2393	1 .2437	1 .2482	1 .2527	1 .2572	1 .2617	1 .2662	1 .2708	1 .2753	38
52	1 .2799	1 .2846	1 .2892	1 .2938	1 .3985	1 .3032	1 .3079	1 .3127	1 .3175	1 .3222	37
53	1 .3270	1 .3319	1 .3367	1 .3416	1 .3465	1 .3514	1 .3564	1 .3613	1 .3663	1 .3713	36
54	1 .3764	1 .3814	1 .3865	1 .3916	1 .3968	1 .4019	1 .4071	1 .4124	1 .4176	1 .4229	35
55	1 .4281	1 .4335	1 .4388	1 .4442	1 .4496	1 .4550	1 .4605	1 .4659	1 .4715	1 .4770	34
56	1 .4826	1 .4882	1 .4938	1 .4994	1 .5051	1 .5108	1 .5166	1 .5224	1 .5282	1 .5340	33
57	1 .5399	1 .5458	1 .5517	1 .5577	1 .5637	1 .5697	1 .5757	1 .5818	1 .5880	1 .5941	32
58	1 .6003	1 .6066	1 .6128	1 .6191	1 .6255	1 .6319	1 .6383	1 .6447	1 .6512	1 .6577	31
59	1 .6643	1 .6709	1 .6775	1 .6842	1 .6909	1 .6977	1 .7045	1 .7113	1 .7182	1 .7251	30°
60°	1 .7321	1 .7391	1 .7461	1 .7532	1 .7603	1 .7675	1 .7747	1 .7820	1 .7893	1 .7966	29
61	1 .8040	1 .8115	1 .8190	1 .8265	1 .8341	1 .8418	1 .8495	1 .8572	1 .8650	1 .8728	28
62	1 .8807	1 .8887	1 .8967	1 .9047	1 .9128	1 .9210	1 .9292	1 .9375	1 .9458	1 .9542	27
63	1 .9626	1 .9711	1 .9797	1 .9883	1 .9970	2 .0057	2 .0145	2 .0233	2 .0323	2 .0413	26
64	2 .0503	2 .0594	2 .0686	2 .0778	2 .0872	2 .0965	2 .1060	2 .1155	2 .1251	2 .1348	25
65	2 .1445	2 .1543	2 .1642	2 .1742	2 .1842	2 .1943	2 .2045	2 .2148	2 .2251	2 .2355	24
66	2 .2460	2 .2566	2 .2673	2 .2781	3 .2889	2 .2998	2 .3109	2 .3220	2 .3332	2 .3445	23
67	2 .3559	2 .3673	2 .3789	2 .3906	2 .4023	2 .4142	2 .4262	2 .4383	2 .4504	2 .4627	22
68	2 .4751	2 .4876	2 .5002	2 .5129	2 .5257	2 .5386	2 .5517	2 .5649	2 .5782	2 .5916	21
69	2 .6051	2 .6187	2 .6325	2 .6464	2 .6605	2 .6746	2 .6889	2 .7031	2 .7179	2 .7326	20°
70°	2 .7475	2 .7625	2 .7776	2 .7929	2 .8083	2 .8239	2 .8397	2 .8556	2 .8716	2 .8878	19
71	2 .9042	2 .9208	2 .9375	2 .9544	2 .9714	2 .9887	3 .0061	3 .0237	2 .0415	3 .0595	18
72	3 .0777	3 .0961	3 .1146	3 .1334	3 .1524	2 .1716	3 .1910	3 .2106	3 .2305	3 .2506	17
73	3 .2709	3 .2914	3 .3122	3 .3332	3 .3544	3 .3759	3 .3977	3 .4197	3 .4420	3 .4646	16
74	3 .4874	3 .5105	3 .5339	3 .5576	3 .5816	3 .6059	3 .6305	3 .6554	3 .6806	3 .7062	15
75	3 .7321	3 .7583	3 .7848	3 .8118	3 .8391	3 .8667	3 .8947	3 .9232	3 .9520	3 .9812	14
76	4 .0108	4 .0408	4 .0713	4 .1022	4 .1335	4 .1653	4 .1976	4 .2303	4 .2635	4 .2972	13
77	4 .3315	4 .3662	4 .4015	4 .4374	4 .4737	4 .5107	4 .5483	4 .5864	4 .6252	4 .6646	12
78	4 .7046	4 .7453	4 .7867	4 .8288	4 .8716	4 .9152	4 .9594	5 .0045	5 .0504	5 .0970	11
79	5 .1446	5 .1929	5 .2422	5 .2924	5 .3435	5 .3955	5 .4486	5 .5026	5 .5578	5 .6140	10°
80°	5 .6713	5 .7297	5 .7894	5 .8502	5 .9124	5 .9758	6 .0405	6 .1066	6 .1742	6 .2432	9
81	6 .3138	6 .3859	6 .4596	6 .5350	6 .6122	6 .6912	6 .7720	6 .8548	6 .9395	7 .0264	8
82	7 .1154	7 .2066	7 .3002	7 .3962	7 .4947	7 .5958	7 .6996	7 .8062	7 .9158	8 .0285	7
83	8 .1443	8 .2636	8 .3863	8 .5126	8 .6427	8 .7769	8 .9152	9 .0579	9 .2052	9 .3572	6
84	9 .5144	9 .677	9 .845	10 .02	10 .20	10 .39	10 .58	10 .78	10 .99	11 .20	5
85	11 .43	11 .66	11 .91	12 .16	12 .43	12 .71	13 .00	13 .30	13 .62	13 .95	4
86	14 .30	14 .67	15 .06	15 .46	15 .89	16 .35	16 .83	17 .34	17 .89	18 .46	3
87	19 .08	19 .74	20 .45	21 .20	22 .02	22 .90	23 .86	24 .90	26 .03	27 .27	2
88	28 .64	30 .14	31 .82	33 .69	35 .80	38 .19	40 .92	44 .07	47 .74	52 .08	1
89	57 .29	63 .66	71 .62	81 .85	95 .49	114 .6	143 .2	191 .0	286 .5	573 .0	0°
	°1.0	°0.9	°0.8	°0.7	°0.6	°0.5	°0.4	°0.3	°0.2	°0.1	Deg.

APPENDIX F

Logarithms of Numbers

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

Logarithms of Numbers (*Concluded*)

N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8456	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

APPENDIX G

Resistance of Copper Wire, Ohms per Mile 25°C. (77°F.)

Size, cir. mils, A.W.G.	Number of wires	Outside diam., mils	Ohms per mile
		STRANDED	
500,000	37	814	0.1130
450,000	37	772	0.1267
400,000	37	728	0.1426
350,000	37	681	0.1626
300,000	37	630	0.1900
250,000	37	575	0.2278
0000	19	528	0.2690
000	19	470	0.339
00	19	418	0.428
0	19	373	0.538
1	19	332	0.681
2	7	292	0.856
3	7	260	1.083
4	7	232	1.367
		SOLID	
0000		460	0.264
000		410	0.333
00		365	0.420
0		325	0.528
1		289	0.665
2		258	0.839
3		229	1.061
4		204	1.335

For more detailed tables, see Part I, pages 47 and 48.

APPENDIX H

Inductive Reactance per Single Conductor, Ohms per Mile[1]

Stranded

Size cir. mils A.W.G.	60 cycles per sec. Spacing, in.												
	12	24	36	48	60	72	84	96	108	120	132	144	156
500,000	0.451	0.535	0.584	0.619	0.647	0.669	0.688	0.703	0.718	0.730	0.742	0.752	0.762
450,000	0.458	0.541	0.591	0.625	0.653	0.675	0.693	0.709	0.724	0.736	0.748	0.758	0.767
400,000	0.464	0.548	0.598	0.632	0.660	0.682	0.700	0.716	0.731	0.743	0.755	0.765	0.775
350,000	0.472	0.556	0.606	0.640	0.668	0.690	0.708	0.724	0.739	0.751	0.763	0.774	0.783
300,000	0.482	0.566	0.615	0.650	0.677	0.699	0.718	0.734	0.748	0.760	0.772	0.783	0.792
250,000	0.493	0.577	0.626	0.661	0.688	0.711	0.729	0.745	0.759	0.772	0.783	0.794	0.804
0000	0.503	0.587	0.636	0.672	0.698	0.722	0.739	0.755	0.770	0.782	0.793	0.804	0.814
000	0.517	0.601	0.650	0.685	0.713	0.735	0.754	0.769	0.784	0.796	0.808	0.818	0.828
00	0.531	0.615	0.664	0.699	0.726	0.748	0.767	0.782	0.798	0.810	0.822	0.832	0.842
0	0.546	0.629	0.678	0.714	0.740	0.762	0.781	0.797	0.812	0.824	0.836	0.846	0.856

Solid

Size cir. mils A.W.G.	12	24	36	48	60	72	84	96	108	120	132	144	156
0000	0.510	0.594	0.642	0.677	0.704	0.726	0.746	0.762	0.776	0.788	0.800	0.810	0.820
000	0.524	0.608	0.656	0.692	0.718	0.740	0.760	0.776	0.790	0.802	0.814	0.824	0.834
00	0.538	0.622	0.670	0.706	0.732	0.754	0.774	0.790	0.804	0.816	0.828	0.838	0.848
0	0.552	0.636	0.684	0.720	0.746	0.768	0.788	0.804	0.818	0.830	0.842	0.852	0.862
1	0.566	0.649	0.698	0.734	0.760	0.782	0.802	0.818	0.832	0.844	0.856	0.866	0.876
2	0.580	0.664	0.712	0.748	0.774	0.796	0.816	0.832	0.846	0.858	0.870	0.880	0.890
3	0.594	0.678	0.726	0.762	0.788	0.810	0.829	0.846	0.860	0.872	0.884	0.894	0.904
4	0.608	0.692	0.740	0.776	0.803	0.824	0.843	0.860	0.874	0.886	0.898	0.908	0.918
5	0.622	0.706	0.754	0.790	0.817	0.838	0.858	0.874	0.888	0.900	0.912	0.922	0.932
6	0.636	0.720	0.768	0.804	0.831	0.853	0.872	0.888	0.902	0.915	0.926	0.936	0.946

[1] From formula $x = 2\pi f\left(80 + 741.1 \log \frac{D}{r}\right)10^{-6}$.

APPENDIX I

Allowable Carrying Capacities of Wires

A. W. G.	Diameter of solid wires in mils	Area in circular mils	Table A, rubber insulation, amperes	Table B, varnished cloth insulation, amperes	Table C, other insulation, amperes
18	40.3	1,624	3		5
16	50.8	2,583	6		10
14	64.1	4,107	15	18	20
12	80.8	6,530	20	25	25
10	101.9	10,380	25	30	30
8	128.5	16,510	35	40	50
6	162.0	26,250	50	60	70
5	181.9	33,100	55	65	80
4	204.3	41,740	70	85	90
3	229.4	52,630	80	95	100
2	257.6	66,370	90	110	125
1	289.3	83,690	100	120	150
0	325.0	105,500	125	150	200
00	364.8	133,100	150	180	225
000	409.6	167,800	175	210	275
		200,000	200	240	300
0000	460	211,600	225	270	325
		250,000	250	300	350
		300,000	275	330	400
		350,000	300	360	450
		400,000	325	390	500
		500,000	400	480	600
		600,000	450	540	680
		700,000	500	600	760
		800,000	550	660	840
		900,000	600	720	920
		1,000,000	650	780	1,000
		1,100,000	690	830	1,080
		1,200,000	730	880	1,150
		1,300,000	770	920	1,220
		1,400,000	810	970	1,290
		1,500,000	850	1,020	1,360
		1,600,000	890	1,070	1,430
		1,700,000	930	1,120	1,490
		1,800,000	970	1,160	1,550
		1,900,000	1,010	1,210	1,610
		2,000,000	1,050	1,260	1,670

1 mil = 0.001 in.

QUESTIONS ON CHAPTER I

1. Give five reasons for generating electrical energy as alternating current, even although some of this energy must ultimately be converted to direct current. Explain carefully the influence of the commutator and the transformer on the choice of direct or alternating current.

2. What is meant by a sine curve? Show how a sine curve may be plotted with the aid of a table of sines. Show how a sine curve may be plotted without tables.

3. What is meant by a cosine curve? How is such a curve plotted without tables; by means of cosine tables?

4. Define a radian. How is the value of the radian determined in degrees?

5. What result is obtained if sine or cosine curves, having the same scale of abscissas, are combined by adding their ordinates?

6. Why does a simple plane-coil rotating in a uniform magnetic field generate an e.m.f. that varies as the sine of its angular displacement from the neutral plane? Show that such an e.m.f. must also vary sinusoidally with time. What relation exists between degrees and time?

7. Define cycle; alternation. How many cycles does a single coil generate per revolution in a 2-pole field; in a 4-pole field? Derive the equation giving the relation among poles, speed, and frequency.

8. What three frequencies are in common use in this country? State the objections to higher frequencies; to lower frequencies. For what class of service are the higher frequencies used; the lower?

9. Describe briefly the construction of commercial alternators. What is meant by sinusoidal flux distribution? Show with a simple fundamental relation that with sinusoidal flux distribution, a sine e.m.f. is generated in the conductors of an alternator.

10. What is meant by angular velocity? What is its relation to the frequency?

11. Define an alternating-current ampere. What is meant by the r.m.s., or the effective value of the current? What is the relation of the effective to the maximum value if the current is sinusoidal?

12. Show how to determine graphically the effective value of any current curve?

13. How is the average current over a half-cycle determined? What is the ratio of the effective to the average value?

14. Distinguish vector quantities from scalar quantities. Give examples of each. How are vector quantities added; subtracted? Explain the parallelogram of forces and the polygon of forces.

15. Show that values of an alternating current, varying sinusoidally with time, may be determined at any instant by the projection of a rotating vector on a vertical axis. What determines the length of the rotating vector; its speed in r.p.s.?

16. Show the relation between the two rotating vectors which represent two currents differing in phase. How is the angle between the vectors determined?

17. In what fundamental manner may two currents in time-phase with each other be added to find their sum? What is the relation of the resultant current to the component currents?

18. In what fundamental manner may two currents differing in phase be added to find their sum?

19. Show that if the two rotating vectors which determine the individual current curves are added vectorially, the resultant rotating vector determines in phase and in magnitude the resultant curve found by adding the component current curves.

20. Why may the effective value of the resultant current be similarly determined, representing the effective values of the component currents by vectors?

21. Why is this method of adding currents also applicable to voltages?

PROBLEMS ON CHAPTER I

NOTE: Problems marked with asterisk * are more difficult than the average and are intended for special assignment.

1. Plot the values of sin x from sine tables (page 410, also see page 407) with values of x in degrees as abscissas. Take values of x at every 30°. That is, sin 30° = ? sin 60° = ? sin 210° = ?

2. A sine function, $y = 12 \sin x$, has a maximum value of 12. Using the values of sines from question 1, plot values of y as ordinates with values of x in degrees as abscissas.

3. Determine graphically the sine curve corresponding to $y = 12 \sin x$ (see Fig. 5, page 7).

4. Plot the values of cos x from cosine tables (page 410, also see page 407) with values of x in degrees as abscissas. Take values of x at every 30°.

5. Plot values of y as ordinates for various values of x in the equation $y = 20 \cos x$.

6. Determine graphically the cosine curve corresponding to $y = 20 \cos x$.

7. To how many radians are the following angles equivalent? (*a*) 30°; (*b*) 77°; (*c*) 139°; (*d*) 342°; (*e*) 476°.

8. A rotating vector completes 15 revolutions in a second. (*a*) Through how many radians does it go in one revolution? (*b*) What is its angular velocity in radians per second?

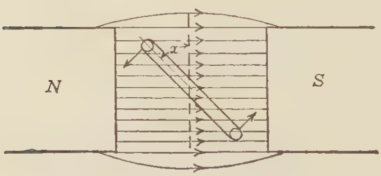


FIG. 9 (*A*).

***9.** Figure 9 (*A*) shows a single-turn coil, having an axial length of 40 cm. and a breadth of 25 cm., and rotating at a speed of 15 r.p.s. in a uniform magnetic field having a density of 2,000 gausses. (*a*) What is its peripheral speed in centimeters per second? (*b*) Using the equation $e = Blv$

10^{-8} volts (page 18), what is the induced e.m.f. per coil side when the plane of the coil is perpendicular to the magnetic field? (*c*) When the plane of the coil is parallel to the magnetic field? (*d*) When the angle x is 45°? (*e*) What is the frequency of the e.m.f.?

*10. Plot values of this e.m.f. as ordinates and values of the angle x as abscissas (problem 9). Also mark a time-scale every 90° along the axis of abscissas.

11. Through how many radians does this e.m.f. go in a second? Through how many radians will a 25-cycle e.m.f. go in a second; a 60-cycle e.m.f.?

12. A 4-pole alternator has a speed of 1,800 r.p.m. (*a*) What is the frequency? (*b*) What is the frequency of a 4-pole alternator whose speed is 1,500 r.p.m.?

13. What speed in r.p.m. must a 6-pole, 50-cycle alternator have?

14. A 500-kw. geared turbo-alternator has 12 poles and rotates at 600 r.p.m. What is the frequency in cycles per second?

15. At what speed does a 10-pole, 60-cycle, vertical turbo-alternator rotate?

16. A 25-cycle Corliss-engine-driven generator has a speed of 75 r.p.m. How many poles has it?

17. How many poles must a 60-cycle, 90-r.p.m. Corliss-engine-driven alternator have?

18. A 60-cycle e.m.f. has an instantaneous maximum value of 150 volts. Through how many radians does it go in one second? What is its equation?

19. Repeat problem 18 for a 25-cycle e.m.f. of the same value.

20. An alternating current has an equation, $i = 14.14 \sin 314t$. What is the frequency?

21. Through how many radians has the current in problem 20 gone when the time t is equal to 0.004 sec.? Through how many degrees has it gone? What is the value of the current at this instant?

22. Find the value of the current (problem 20) when the time t is equal to 0.014 sec.

23. An alternating current has an instantaneous maximum value of 59.39 amp. What is its effective, or r.m.s. value?

24. What is the maximum value of an alternating voltage which has an effective value of 110 volts?

25. What is the effective value of the current in problem 20?

26. What is the average current (problem 23), over a half-cycle?

27. An alternating current has an instantaneous maximum value of 21.12 amp. What average power will it dissipate when it flows through a resistance of 2.0 ohms?

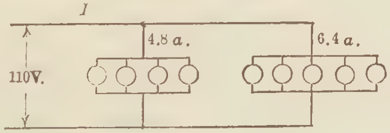


Fig. 28 (*A*).

28. Two lamp loads (Fig. 28 (*A*)) are connected in parallel across 110-volt, 60-cycle mains. One lamp bank takes 4.8 amp. (r.m.s.) and the other takes 6.4. amp. (r.m.s.). Since incandescent lamps are practically pure resistance, the current in each lamp-bank is in phase with the voltage. (*a*) What

is the value of the total current I, supplied by the mains? (b) What is the maximum value of this current?

29. A resistance, when connected across 100-volt, 60-cycle mains takes a current I_1 of 2.4 amp. (r.m.s.) (Fig. 29 (A)). A condenser connected in parallel with this resistance takes a current I_2 of 1.6 amp. (r.m.s.). The current I_2 leads the current I_1 by 90°. (a) Plot the two currents (see Fig. 29, page 33). (b) Find the resultant current curve by adding ordinates. (c) Find the maximum value of the resultant current. (d) Find the resultant r.m.s. current by adding vectors each equal in length to the r.m.s. value of the corresponding current. (e) Compare (d) with the effective value obtained from (c).

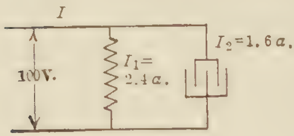


FIG. 29 (A).

30. Each of the two alternators, 1 and 2 (Fig. 30 (A)) delivers 40 amp. (r.m.s.) to the load. Find the load current I if these two currents differ in phase by 45°, I_1 leading I_2.

31. Find the resultant current I when 1 (Fig. 30 (A)) delivers 40 amp. (r.m.s.) and 2 delivers 50 amp. (r.m.s.) the 40 amp. leading the 50 amp. by 45°.

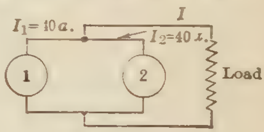


FIG. 30 (A).

32. Find the resultant current (problem 30) assuming that the two currents differ in phase by 30°.

33. The two phases A and B of a 2-phase alternator each generate an e.m.f. of 600 volts (r.m.s.) (Fig. 33 (A)). The voltage in phase A leads the voltage in phase B by 90°. These two phases are connected at one end. Find the voltage across the other two ends CD.

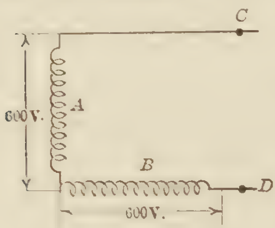


FIG. 33 (A).

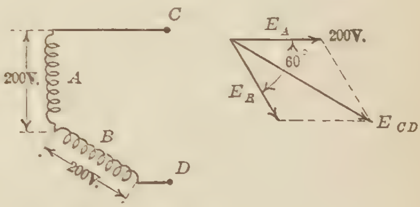


FIG. 34 (A).

34. The e.m.fs. generated in two phases A and B (Fig. 34 (A)) of a 3-phase alternator are each equal to 200 volts (r.m.s.). These e.m.fs. differ in phase by 60°, the e.m.f. E_A in phase-A leading the e.m.f. E_B in phase-B, as shown by the vector diagram. Find the voltage E_{CD} across the open ends CD of these phases.

35. Phase-B, is reversed (problem 34). This makes a phase difference of 120 degrees between the two phases, the e.m.f. of phase-B now leading that of A. Find the resultant voltage across the open ends of the phases.

Hint: Reverse the vector E_B (Fig. 34 (A)).

QUESTIONS ON CHAPTER II

1. With resistance only in the circuit, why must the impressed voltage be equal to the resistance-drop? What phase relation exists between the current and the voltage with resistance only in circuit? What relation exists among the current, voltage, and resistance under these conditions? How are the foregoing relations shown vectorially?

2. Why does inductance have an effect on the flow of alternating current, whereas it has no effect on the flow of a steady direct current? How may this be demonstrated experimentally?

3. Show that inductance in opposing change in current prevents an alternating current reaching the value which it would attain, were inductance not present.

4. At what instants during the cycle is an alternating current not changing with time? At what instants is it changing at the maximum rate? At what instants is the e.m.f., of self-induction zero? A maximum?

5. What phase relation exists between the e.m.f. of self-induction and the current? Between the e.m.f. of self-induction and the impressed voltage with inductance only in circuit? Between the impressed voltage and the current?

6. How may the phase relation between current and voltage be shown vectorially?

7. How does the current in an inductance vary with the voltage; the frequency; the inductance? Give the equation which shows the relation among voltage, current, inductance, and frequency. Define inductive reactance.

8. Describe a mechanical system which is analogous in mechanics to an alternating-current circuit containing inductance only. What factor in the mechanical system corresponds to current; to inductance?

9. What hydraulic system is analogous to a condenser connected across an alternating voltage? What factor corresponds to voltage; to current? Show that the flow of water *leads* the pressure.

10. Demonstrate that the current flowing into a condenser leads the voltage by 90° when the impressed voltage varies sinusoidally with time. Compare this current flow with the flow of water (question 9).

11. To what three factors is the flow of current in a condenser proportional? Why?

12. Define condensive reactance.

13. Fundamentally, to what two factors is electrical power proportional? Show how this principle may be utilized in determining instantaneous values of power in alternating-current circuits.

14. Sketch a current, a voltage, and a power curve when the current and the voltage are in phase with each other. How many times each current cycle is the power zero? What is the frequency of the power curve in terms of that of current and voltage?

15. To what is the average power over a cycle equal when current and voltage are in phase with each other?

16. A battery is connected across a certain circuit and is acting as a source of energy. Under what conditions is the battery power positive; negative?

17. If the battery is considered as being a translating device, that is a device which receives energy, under what conditions is the power positive; negative?

18. Sketch voltage, current, and power curves with inductance only in the circuit. How many times each current cycle is the power zero? What is the frequency of the power curve in terms of the frequency of the voltage and current?

19. Indicate the periods during which the circuit receives energy from the source (question 18). Indicate the periods during which the circuit delivers energy to the source. What is the average power over a cycle?

20. Sketch voltage, current, and power curves with capacitance only in the circuit. How many times each current cycle is the power zero? What is the frequency of the power curve in terms of the frequency of the voltage and current?

21. Indicate the periods during which the circuit receives energy from the source (question 20). Indicate the periods during which the circuit delivers energy to the source. What is the average power over a cycle?

22. Sketch voltage, current, and power curves when the current lags the voltage by an angle θ, where θ is greater than zero and less than 90°. What is the frequency of the power curve in terms of the frequency of the voltage and current? How many times each cycle is the power zero?

23. Indicate the periods during which the power is positive and during which it is negative (question 22). Which periods are of longer duration? Why cannot the average power over the cycle be equal to the product of volts and amperes? Why must it be greater than zero?

24. Define power-factor. Show its use in determining the power.

25. To what is the impedance of a circuit containing resistance and inductance in series equal? In what units is impedance expressed? Knowing the voltage and impedance, how is the current found?

26. To what ratio is the tangent of the lag angle equal; the cosine?

27. With resistance and inductance in series, account for the power dissipated in the circuit. To what is this power equal in terms of current and resistance; voltage, current, and power-factor?

28. Write the expression which gives the impedance of a circuit containing resistance and capacitance in series. To the ratio of what factors is the tangent of the angle of current lead equal? To what is the cosine of this angle equal?

29. With resistance and capacitance in series, account for the power dissipated in the circuit. To what is this power equal in terms of current and resistance; voltage, current, and power-factor?

30. Derive the general expression for the impedance of a circuit containing resistance, inductance, and capacitance in series, giving this expression in terms of resistance and simple *reactances*. Also give this expression in terms of resistance, frequency, inductance, and capacitance.

31. To what ratio is the tangent of the phase angle equal? How is it determined whether the current lags or leads?

32. To what expression is the cosine of the phase angle equal?

33. Account for the power dissipated in a series circuit containing resistance, inductance, and capacitance in series. To what is this power equal in terms of current and resistance; voltage, current, and power-factor?

34. With a fixed resistance, under what conditions is the impedance of a series circuit containing resistance, inductance, and capacitance in series a minimum? To what is the impedance equal when it is a minimum? How does the value of current when the circuit is in resonance compare with its value when the circuit is non-resonant?

35. Compare the voltage across the inductance with the voltage across the capacitance when the circuit is in resonance. Compare the voltage across the resistance with the voltage impressed across the circuit. What is the power-factor of the circuit under these conditions?

36. What is the "natural frequency" of a circuit? To what is it equal? How is it utilized in tuning radio circuits?

37. Explain why it is impossible practically to have pure inductance. Within how many degrees do practical inductance coils approach pure inductances? How may a practical inductance coil be considered so far as its effect in alternating-current circuits is concerned?

38. Why do practical condensers usually come within a fraction of a degree to being pure capacitances?

39. Why is the parallel circuit met more commonly in practice than the series circuit? What factor is common to all branches of the parallel circuit and what position is it convenient to give this factor in vector diagrams of the circuit?

40. What is the natural method of solving parallel-circuit problems?

41. With resistance and inductance in parallel, to what is the tangent of the power-factor angle equal? The cosine? Repeat for resistance and capacitance in parallel.

42. With resistance, inductance, and capacitance in parallel, what quantities in the vector diagram tend to cancel each other?

43. Under what conditions does resonance occur in the parallel circuit? Compare the value of current with the value occurring in the circuit when it is not in resonance.

44. Show that with pure inductance and pure capacitance in parallel current may flow in the inductance and current may flow in the capacitance and yet in the line supplying this parallel circuit, the current may be zero.

PROBLEMS ON CHAPTER II

36. A 100-watt Mazda C lamp having a hot resistance of 125 ohms is connected across 110-volt (r.m.s.),[1] 60-cycle mains. (*a*) What r.m.s. current flows? (*b*) What is the maximum instantaneous value of current? (*c*) What is the equation of this current curve?

37. An 8- and a 13-ohm non-inductive resistance are connected in series across 220-volt, 50-cycle mains. Find: (*a*) the r.m.s. current; (*b*) the

[1] Unless otherwise specified, all values of current and voltage are *effective* or r.m.s. values

voltage across the 8-ohm resistance; (*c*) the voltage across the 13-ohm resistance.

38. What is the reactance of a 0.1-henry inductance at 60 cycles; at 50 cycles?

39. Find the current in the 0.1-henry inductance (problem 38) when it is connected across 100-volt, 60-cycle mains. What is the maximum instantaneous value of current?

40. Find the current in the 0.1-henry inductance (problem 38) when it is connected across 100-volt, 50-cycle mains. What is the maximum instantaneous value of current?

41. Find the reactance of a 0.2-henry inductance at 25 cycles per second. Find its reactance at the telephonic frequency of 1,000 cycles per second. Find the voltage across the inductance at each frequency, when the current is 0.2 amp.

42. What is the reactance of a 0.000040-farad condenser at a frequency of 25 cycles; at a frequency of 60 cycles?

43. (*a*) What current will the condenser (problem 42) take when connected across 200-volt, 25-cycle mains? (*b*) 200-volt, 60-cycle mains? (*c*) Find the maximum instantaneous current in (*a*); in (*b*).

44. Determine the reactance of a 2.0-μf. condenser at a frequency of 50 cycles per second; at the telephonic frequency of 1,000 cycles per second.

45. (*a*) Determine the voltage across the condenser (problem 44), when a current of 0.2 amp. at 50 cycles flows in the condenser. (*b*) When a current of 0.2 amp. at 1,000 cycles flows in the condenser.

46. A 10- and a 5-μf. condenser are connected in series across a 200-volt, 500-cycle circuit. (*a*) What is the equivalent capacitance of the combination? (*b*) What current flows through the combination? (*c*) What is the voltage across each condenser?

47. (*a*) Find the average power over a cycle (problem (36). (*b*) What is the maximum instantaneous value of the power.

48. (*a*) Find the average power over a cycle (problem 37). (*b*) What is the maximum instantaneous value of the power.

49. A "Sunbowl" (a parabolic-reflector heater whose heating element consists of a non-inductive resistance) takes 620 watts from 115-volt, 50-cycle mains. What current does it take?

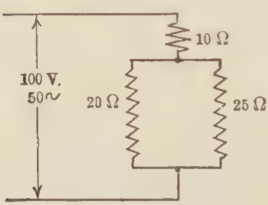


FIG. 51 (*A*).

50. What current will the "Sunbowl" (problem 49) take from 115-volt, 25-cycle mains?

51. A resistance of 10 ohms is connected in series with a combination of 20 and 25 ohms in parallel (Fig. 51 (*A*)). The entire combination is connected across 100-volt, 50-cycle mains. Find: (*a*) the total current; (*b*) the total power; (*c*) the current in each resistance; (*d*) the power dissipated by each resistance

52. A single-phase, 60-cycle, alternating-current repulsion-induction motor takes 400 watts and 5.4 amp. at 110 volts. At what power-factor is it operating?

53. How many watts is a small single-phase induction motor taking when the voltage is 110 volts, 60 cycles, the current is 2.2 amp. and the power-factor is 0.65.

54. A certain single-phase load takes 4.3 amp. at 220 volts, 25 cycles and the current lags the voltage by an angle of 36.9°. (*a*) What is the cosine of the angle of lag? (*b*) What is the load power-factor? (*c*) How much power is the load taking?

55. A series circuit consists of a 30-ohm non-inductive resistance and an inductive reactance of 40 ohms at 60 cycles (Fig. 55 (*A*)). (*a*) What is the impedance of the circuit? (*b*) What current flows when this circuit is connected across 200-volt, 60-cycle mains? (*c*) Determine the voltage across the resistance. (*d*) Determine the voltage across the reactance. (*e*) Find the vector sum of these voltages. (*f*) How much power is dissipated in the resistance? (*g*) How much power is dissipated in the reactance? (*h*) What is the total power taken by this circuit? (*i*) What are the volt-amperes taken by this circuit? (*j*) What is the power-factor of this circuit? (*k*) What is the angle of phase difference between voltage and current? (Use both the cosine and tangent formula.) (*l*) Draw a vector diagram.

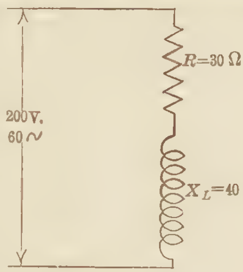


Fig. 55 (*A*).

56. A resistance of 60 ohms and an inductance of 0.51 henry are connected across 100-volt, 60-cycle mains. Find: (*a*) the inductive reactance of the circuit; (*b*) the impedance of the circuit; (*c*) the current; (*d*) the power; (*e*) the power-factor; (*f*) the lag angle; (*g*) draw a vector diagram.

57. A telephone relay having a resistance of 200 ohms and an inductance of 0.4 henry is connected in series with an 1,800-ohm resistance unit. (*a*) What is the reactance of the circuit at 796 cycles per second? (*b*) What is the total series resistance of the circuit? (*c*) What is the impedance of the circuit at 796 cycles? (*d*) If it requires 5 milliamp. at 796 cycles to operate the relay, what voltage must be impressed across the circuit?

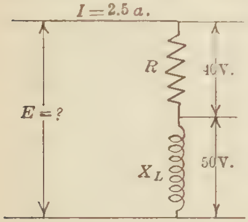


Fig. 58 (*A*).

58. The voltage across a non-inductive resistance is 40 volts, that across an inductive reactance is 50 volts, and the current is 2.5 amp when the resistance and reactance are connected in series across a 50-cycle voltage (Fig. 58 (*A*)). Find: (*a*) the resistance of the circuit; (*b*) the reactance of the circuit; (*c*) the impedance of the circuit; (*d*) the power taken by the circuit; (*e*) the power-factor of the circuit; (*f*) the power-factor angle; (*g*) the circuit voltage.

59. A 400-ohm resistance and a condensive reactance of 300 ohms are connected in series across 100-volt, 60-cycle mains. (*a*) What is the impedance of the circuit? (*b*) What current flows? (*c*) What is the voltage across the resistance? (*d*) What is the voltage across the reactance? (*e*) Draw a vector diagram.

60. Find the capacitance of the condenser, problem 59.

61. Repeat problem 59, (*a*) to (*d*) inclusive, when this circuit is connected across 100-volt, 25-cycle mains, using the same *condenser*.

62. A 10-ohm resistance is connected in series with a 2.0-μf. condenser across 20-volt, 60-cycle mains. (*a*) What is the condensive reactance of this circuit? (*b*) What current does it take?

63. Repeat problem 62, substituting a 20-volt, 1,000-cycle circuit for the 60-cycle circuit.

***64.** An alternating-current circuit consisting of a condenser and a resistance in series takes 1.5 amp. and 120 watts when connected across 100-volt, 60-cycle mains. (*a*) What is the resistance of the circuit? (*b*) What is the impedance of the circuit? (*c*) What is the reactance of the circuit? (*d*) What is the capacitance of the condenser? (*e*) Find the power-factor angle.

***65.** In problem 59, find: (*a*) the power taken by the circuit; (*b*) the volt-amperes of the circuit; (*c*) the power-factor of the circuit; (*d*) the power-factor angle.

66. It is desired to obtain 50 amp., leading 90°, by connecting a condenser across 1,100-volt, 60-cycle mains. (*a*) What should be the capacitance of the condenser? (*b*) How many volt-amperes does it take? (*c*) How many kilovolt-amperes does it take?

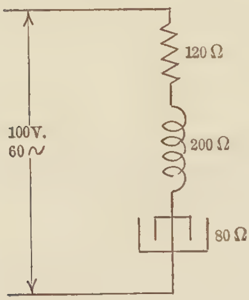


Fig. 68 (*A*).

67. An 80-ohm resistance is connected in series with a 70μf. condenser across 200-volt, 25-cycle mains. Find: (*a*) the reactance of the circuit; (*b*) the impedance of the circuit; (*c*) the current; (*d*) the power; (*e*) the volt-amperes; (*f*) the power-factor; (*g*) the power-factor angle; (*h*) the voltage across the resistance and across the condenser; (*i*) draw a vector diagram.

68. A resistance of 120 ohms, an inductive reactance of 200 ohms, and a condensive reactance of 80 ohms are all connected in series across 100-volt, 60-cycle mains (Fig. 68 (*A*)). Find: (*a*) the impedance of the circuit; (*b*) the current; (*c*) the voltage across the resistance; (*d*) the voltage across the inductive reactance; (*e*) the voltage across the condensive reactance; (*f*) the power; (*g*) the volt-amperes; (*h*) the power-factor; (*i*) the power-factor angle; (*j*) draw a vector diagram.

69. Find the inductance and the capacitance, problem 68.

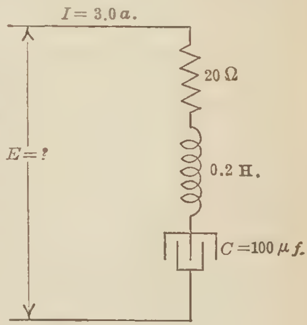


Fig. 70 (*A*).

70. When a resistance of 20 ohms, an inductance of 0.2 henry, and a capacitance of 100μf. are connected in series across a 25-cycle circuit, a current of 3.0 amp. flows (Fig. 70 (*A*)).

Find: (*a*) the inductive reactance of the circuit; (*b*) the condensive reactance of the circuit; (*c*) the impedance of the circuit; (*d*) the circuit voltage; (*e*) the power; (*f*) the power-factor; (*g*) the power-factor angle; (*h*) the voltage across the resistance, the inductance, and the capacitance; (*i*) draw a vector diagram.

71. Repeat problem 70 for a 60-cycle circuit.

72. A series circuit consists of 40-ohms resistance and 50-ohms nductive reactance. (*a*) What condensive reactance should be connected in series with these two in order that the circuit may be in resonance? (*b*) If this circuit is connected across 120-volt, 60-cycle mains, what current flows? (*c*) What power does the circuit take? (*d*) What is the voltage across the inductive reactance? (*e*) What is the voltage across the condensive reactance? (*f*) Draw a vector diagram.

73. Find the value of the inductance and the value of the capacitance (problem 72). Check these values by equation (35) (page 74).

74. A certain loud speaker (Fig. 74 (*A*)) has a resistance of 6,000 ohms and an inductance of 2.0 henrys at a frequency of 796 cycles per second. ($\omega = 5{,}000$.) What capacitance should be connected in series with it in order that the current taken by the loud speaker at this frequency may be in phase with the voltage?

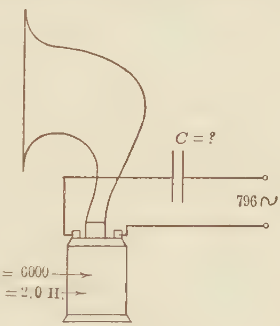


FIG. 74 (*A*).

75. A series circuit consists of a resistance of 120 ohms, an inductance of 0.3 henry, and a capacitance of 5.87μf. At what frequency will this circuit be in resonance?

76. If the circuit (problem 75) is connected across a 240-volt, 120-cycle source of power: (*a*) What current flows? (*b*) What power does it take? (*c*) What is the voltage across the inductance? (*d*) What is the voltage across the capacitance? (*e*) Draw a vector diagram.

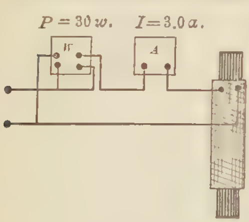


FIG. 77 (*A*).

77. An impedance coil (Fig. 77 (*A*)), when connected across 110-volt, 60-cycle mains takes 3.0 amp. and 30 watts. (*a*) What is its impedance? (*b*) What is its power-factor? (*c*) What is its apparent resistance? (*d*) What is its reactance? (*e*) What is its inductance? (*f*) By what angle does the current lag the voltage?

Hint: In (*c*) the fact that watts = I^2R may be used to find the resistance. The apparent resistance includes iron losses. The iron losses may, however, be considered as increasing the resistance of the coil.

78. When an impedance coil is connected across 100-volt, 25-cycle mains the angle between the voltage and the current is 75° and the current is 2.8 amp. (*a*) What volt-amperes does it take? (*b*) What is its power-

factor? (*c*) What power does it take? (*d*) What is its apparent resistance? (*e*) What is its reactance? (*f*) What is the inductance of the coil?

79. A 50-ohm resistance and a 50-ohm inductive reactance are connected in parallel across 100-volt, 60-cycle mains. (*a*) What current flows in the resistance? (*b*) What current flows in the reactance? (*c*) What is the resultant current? (*d*) What power does the circuit take? (*e*) What are the volt-amperes? (*f*) What is the power-factor of the circuit? (*g*) What is the power-factor angle? (*h*) Draw a vector diagram.

80. A resistance of 20 ohms and an inductance of 0.08 henry are connected in parallel across 100-volt, 60-cycle mains. Repeat (*a*) to (*h*) inclusive, problem 79.

81. Repeat problem 80 with the resistance and inductance connected across 100-volt, 25-cycle mains.

82. A 60-ohm condensive reactance and a 60-ohm resistance are connected in parallel across 120-volt, 50-cycle mains. Find: (*a*) the current in the condensive reactance; (*b*) the current in the resistance; (*c*) the resultant current; (*d*) the power taken by the circuit; (*e*) the volt-amperes of the circuit; (*f*) the power-factor of the circuit; (*g*) the power-factor angle; (*h*) draw a vector diagram.

83. A 40-ohm resistance and an 80μf. condenser are connected in parallel across 120-volt, 60-cycle mains. Repeat (*a*) to (*h*) inclusive, problem 82.

84. Repeat problem 83 with the resistance and condenser connected across 120-volt, 25-cycle mains.

85. A 100-ohm resistance, a 40-ohm inductive reactance, and a 60-ohm condensive reactance are all connected in parallel across 120-volt, 60-cycle mains (Fig. 85 (*A*)) Find: (*a*) the current in the resistance, in the inductive reactance, and in the condensive reactance; (*b*) the resultant current, using a vector diagram; (*c*) the power taken by the circuit; (*d*) the volt-amperes of the circuit; (*e*) the power-factor of the circuit; (*f*) the power-factor angle of the circuit.

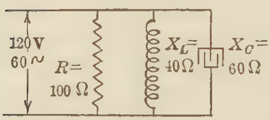


Fig. 85 (*A*).

86. An 80-ohm resistance, an inductance of 0.3 henry, and a capacitance of 40-μf. are connected in parallel across 120-volt, 60-cycle mains (Fig. 86 (*A*)). Find: (*a*) the reactance of the inductive branch and the current in this branch; (*b*) the reactance of the condensive branch and the current in this branch; (*c*) the total circuit current using a vector diagram; (*d*) the power taken by the circuit; (*e*) the volt-amperes of the circuit; (*f*) the power-factor of the circuit; (*g*) the power-factor angle of the circuit.

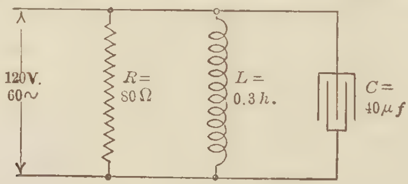


Fig. 86 (*A*).

87. Repeat problem 86 when the circuit is connected across 120-volt, 25-cycle mains.

88. A 30-ohm resistance and a 60-ohm inductive reactance are connected in parallel across 120-volt, 60-cycle mains (Fig. 88 (*A*)). It is desired that the power-factor of the entire circuit be made unity by connecting a condenser in parallel with the resistance and the inductive reactance. (*a*) What current does the inductive reactance take? (*b*) What current should the condenser take? (*c*) What should be the reactance of the condenser? (*d*) What should be the capacitance of the condenser? (*e*) What power does the circuit take? (*f*) What are the volt-amperes of the circuit? (*g*) Draw a vector diagram of the circuit.

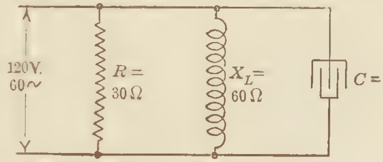


Fig. 88 (*A*).

89. A resistance of 50 ohms and an inductance of 0.2 henry are connected in parallel across 110-volt, 25-cycle mains. What capacitance, connected in parallel, will make the power-factor of the circuit unity?

90. Find the current in the resistance, the inductance, and the capacitance, and also the total current (problem 89). What power does the circuit take, what is its power-factor, and what is its power-factor angle? Draw a vector diagram.

91. An inductance of 0.5 henry and a capacitance of 0.0506μf. in parallel are bridged across the two wires of a telephone circuit. (*a*) What is the impedance of each at 1,000 cycles? (*b*) Neglecting losses, what current does each take with 10 volts across its terminals? (*c*) What current does the combination take? (*d*) What is the impedance of the combination?

QUESTIONS ON CHAPTER III

1. Describe the Siemens Dynamometer. Why does the moving coil tend to turn when current flows through the two coils in series? Explain the method of measuring the turning moment of the coil. Why is this type of instrument used only for very special measurements?

2. Describe the indicating electrodynamometer. What electromagnetic principle does its operation illustrate? What makes it more portable than the Siemens dynamometer?

3. How should the coils of an electrodynamometer be wound in order that the instrument may operate as a voltmeter?

4. How should the coils and the instrument be connected in circuit? Explain the use of the instrument for direct current.

5. Why is the voltmeter-ammeter method of measuring power not ordinarily practicable with alternating current? Upon what principle does the wattmeter operate? Explain briefly why the torque of the instrument is nearly proportional to the power.

6. Show how a wattmeter is connected to measure the power consumed in its own current-coil in addition to the power being measured. Show how it is connected to measure the power consumed in its own potential-circuit, in addition to the power being measured. What is the order of magnitude of the power consumed by each coil of the instrument and how is correction made?

7. Explain how it is possible to burn out either the current-coil or the potential-circuit of a wattmeter, and yet the instrument indication be well on the scale. What means may be used to determine whether or not the current- or the potential-coil is overloaded?

8. Why is a polyphase wattmeter often desirable? Describe its construction.

9. Describe the construction and operation of the Weston iron-vane type of instrument. How is it adapted to operate as a voltmeter; as an ammeter?

10. Describe the construction and operation of the General Electric inclined-coil, iron-vane type of instrument. How does the voltmeter differ from the ammeter?

11. Name one important use of the hot-wire instrument.

12. Why is the direct-current watthour meter operative with alternating currents? What error is introduced in its use at low power-factors?

13. Describe the construction and the connections of the induction watthour meter. Why is a power-factor compensating coil necessary? What adjustment is made if the meter is slow with lagging current; fast? Repeat for a leading current.

14. In what manner is the error due to the friction torque compensated? Explain carefully the operation of the "shading coil."

15. Explain briefly the method by which the driving torque is obtained; the retarding torque.

16. Make a diagram of the connections which would be used in calibrating an alternating-current watthour meter. State the procedure to follow at unity power-factor; at low power-factor.

17. Give the advantages of the induction watthour meter over the direct-current type of meter.

18. Upon what mechanical principle does the vibrating-reed type of frequency meter operate? Why are the steel reeds polarized?

19. For what purpose is a synchroscope used? How does the synchroscope indicate that the incoming alternator has the same frequency as the bus-bars? How does it indicate that the proper phase relation between the incoming alternator and the bus-bar exists for connecting in parallel?

20. Upon what simple principle does the oscillograph operate? What feature of its design enables it to follow rapid changes in the electric circuit? How is the deflection of the vibrator determined? In what manner is the element of time introduced?

21. Describe the general construction and operation of a laboratory type of oscillograph. How is a photographic record obtained? Show the method of connecting the two vibrators into an electric circuit.

QUESTIONS ON CHAPTER IV

1. Give three reasons for the use of polyphase rather than single-phase power. Describe an elementary generator capable of generating 3-phase voltage. Why do these three voltages differ in phase by 120 time-degrees? Show that at every instant the algebraic sum of the ordinates of the three e.m.f. curves is zero.

2. How may each of the 3-phase voltages be connected to the external circuit?

3. Describe briefly symbolic notation, giving the relation of the order of subscripts to the direction of flow of energy. What is the order of the subscripts when two voltages are added?

4. Two coils of a 3-phase generator are connected in series, the outer end of one coil being connected to the inner end of the other. What is the relation of the resultant voltage to the voltage of the individual coils?

5. Repeat question 4 with the two inner ends of the coils connected.

6. Show by means of vector addition (or subtraction) that with a Y-system the line voltage is $\sqrt{3}$ times the coil voltage and also that there is a phase difference of 30° between line and coil voltage.

7. In the Y-system, what relation exists between the line currents and the individual coil currents? Why must the instantaneous algebraic sum and the vector sum of the 3-coil currents equal zero?

8. At unity power-factor, what is the phase relation between coil voltage and coil current? If the power-factor is other than unity, what phase relation exists between coil voltage and coil current?

9. At unity power-factor, what phase relation exists between coil current and line voltage; between line voltage and line current? If the power-factor is other than unity, what phase relation exists between line voltage and line current?

10. At unity power-factor, what power does the Y-system develop in terms of coil voltage and coil current? In terms of line voltage and line current?

11. Repeat question 10 for power-factors other than unity.

12. To what are the volt-amperes of a 3-phase system equal? The kilovolt-amperes? Give the expression for the power-factor in terms of volts, amperes, and watts. What is the relation of the power-factor of the system to the power-factor of the individual coils?

13. Develop the delta-connection from three coils connected in Y. In the delta-system, what relation exists between the line voltage and the coil terminal voltage?

14. In the delta-system, what is the numerical relation between the coil currents and the line currents? What phase relation exists between any one coil current and a corresponding line current?

15. In the delta-system, what phase relation exists between line currents and line voltages at unity power-factor?

16. Derive the equation giving the power in watts of a delta-system in terms of line voltage, line current, and coil phase angle. To what are the kilovolt-amperes equal; the power-factor?

17. Why is it ordinarily better to connect alternators of large capacity in Y rather than in delta? Why is this not ordinarily true with small alternators? Give other uses of the Y- and delta-connections.

18. Make a wiring diagram, showing the use of the 3-wattmeter method of measuring 3-phase power when a neutral is and is not accessible. Under what conditions do all three wattmeters read the same?

19. Make a wiring diagram showing the use of the 2-wattmeter method for measuring 3-phase power. To what is the total power equal? Under what conditions are the readings of the two wattmeters equal? Under what conditions does one wattmeter read negatively?

20. How may the power-factor of a balanced 3-phase system be determined from the wattmeter readings alone?

21. In what type of 3-phase system can the 2-wattmeter method *not* be used?

22. How are the e.m.fs. of a 2-phase system generated? What is their phase relation?

23. What is meant by a 2-phase insulated system? How are the loads applied? How many different values of voltages are available?

24. How may an interconnected, 5-wire, quarter-phase system be obtained from a 2-phase system? How many different values of voltage are available?

25. Show the connections of the 2-phase, 3-wire system. What relation exists among the three line voltages; the three line currents? Why is this system little used?

26. Show a mesh-connected, 2-phase system.

27. Make a diagram showing the manner of measuring power in an insulated 2-phase system; in an interconnected, 2-phase system; and in a 3-wire, 2-phase system.

28. Show diagrammatically a star-connected, 6-phase system and a mesh-connected, 6-phase system. Give the relations among voltages in the two systems.

PROBLEMS ON CHAPTER IV

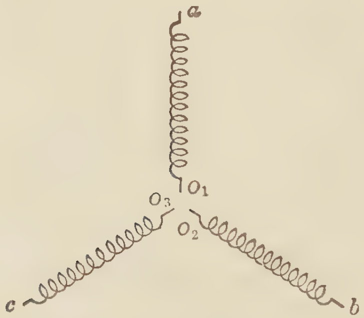


FIG. 92 (*A*).

* **92.** Figure 92 (*A*) shows the three 3-phase coil-circuits o_1a, o_2b, and o_3c of an alternator, each of which generates 250 volts. The sequence of phase rotation is E_{o_1a}, E_{o_2b}, and E_{o_3c}. (*a*) Find the voltage across the ends ab when o_1 is connected to o_2. (*b*) What is the phase angle between the voltage E_{ab} and E_{o_2b}? (*c*) Between E_{ab} and E_{ao_1}?

* **93.** The end o_1 is connected to end b (Fig. 92 (*A*)). Find the voltage across ao_2. (*b*) What is the phase angle between the voltage E_{o_2a} and E_{o_1a}? (*c*) Between E_{o_2a} and E_{o_2b}?

*94. Find the voltage across ends ac (Fig. 92 (A)) when o_1 is connected to o_3.

*95. Find the voltage across ends o_1c (Fig. 92 (A)) when a is connected to o_3.

96. It is desired that a 10,000 kv-a., 3-phase, 60-cycle alternator have a rated voltage of 13,800 volts across its terminals. This alternator is to be Y-connected. What is the voltage rating of each of its coil-circuits?

97. (*a*) What is the kilovolt-ampere rating per coil-circuit of the alternator, problem 96? (*b*) What is the rated coil-circuit current? (*c*) What is the rated line current?

98. Each of three 3-phase, Y-connected alternator coils (Fig. 98 (*A*)) generates 254 volts. (*a*) What is the no-load terminal voltage of the alternator? (*b*) If the current per coil is 433 amp., what is the kilovolt-ampere rating per coil? (*c*) What is the kilovolt-ampere rating of the alternator? (Check by means of equation (42) (page 112), using line values.) (*d*) If the generator operates at unity power-factor with the foregoing values of voltage and current, what is its kilowatt output? (Neglect the voltage-drop in the armature coils in (*c*) and (*d*).)

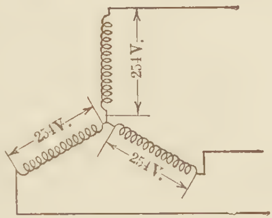


FIG. 98 (*A*).

99. (*a*) If the current in each coil lags its voltage by 45° (problem 98), what is the coil power factor? (*b*) What is the system power factor? (*c*) What is the system output in kilowatts? (*d*) What is the system kilovolt-ampere output?

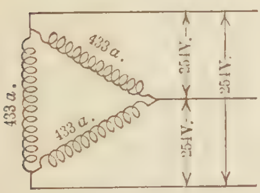


FIG. 100 (*A*).

100. The three coils (problem 98), are connected in delta (Fig. 100 (*A*)) and each coil still delivers 433 amp. at 254 volts. (*a*) What is the line voltage? (*b*) What is the line current? (*c*) What is the kilovolt-ampere output of the system? (*d*) If the coil current lags its terminal voltage by 45°, what is the kilowatt output of the system?

101. A 3-phase, 5,000 kv-a., 11,000-volt, 25-cycle alternator is Y-connected. (*a*) What is the rated voltage per coil-circuit? (*b*) What is the rated current per coil-circuit? (*c*) If the coil power-factor at rated kilovolt-ampere load is 0.8, what is the kilowatt output under these conditions?

102. A 3-phase 100 kv-a., 600-volt, 25-cycle alternator is delta-connected. (*a*) What is the rated current per coil-circuit? (*b*) If the coil current lags the coil voltage by 30° at rated kilovolt-ampere load, what is the power factor of the alternator? (*c*) What is the kilowatt output of the alternator under these conditions?

103. Three 20-ohm resistances are connected in delta across 110-volt, 3-phase, 60-cycle mains (Fig. 103 (*A*)). (*a*) What current does each resistance take? (*b*) What is the line current? (*c*) What is the total power delivered to the three resistances?

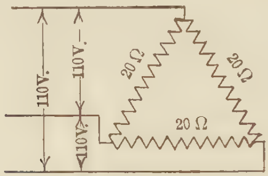


FIG. 103 (*A*).

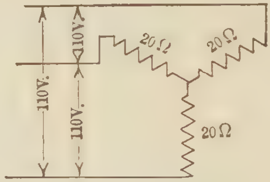


FIG. 104 (*A*).

104. The three 20-ohm resistances of problem 103 are connected in Y across the same 110-volt, 3-phase mains (Fig. 104 (*A*)). (*a*) What is the voltage across each resistance? (*b*) What current does each resistance take? (*c*) What power is the line supplying to this load?

105. It is desired that the three 20-ohm resistances connected in Y (problem 104), take the same power as when connected in delta (problem 103). What should be the line voltage under these conditions?

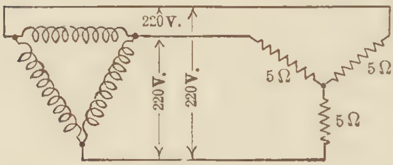


FIG. 106 (*A*).

106. A 60-cycle, 220-volt, delta-connected alternator (Fig. 106 (*A*)) supplies power to three 5-ohm resistances connected in Y. (*a*) What is the voltage across each resistance? (*b*) What current does each resistance take? (*c*) What current flows in each coil-circuit of the alternator? (*d*) What power does the load take?

107. Repeat problem 106, (*a*) to (*d*), inclusive, with the three alternator coils connected in Y, each coil generating 220-volts as before.

108. Each coil of a Y-connected generator (Fig. 108 (*A*)) delivers 63.5 volts, 25 cycles at its terminals when a load consisting of three delta-con-

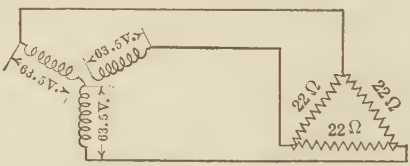


FIG. 108 (*A*).

nected impedances, each having 22 ohms, is connected across the generator terminals. The ratio of resistance to reactance in each of the impedances is such that each current lags its voltage by 35.9°. (*a*) What is the line voltage of the system? (*b*) What current does each impedance take? (*c*)

What is the value of the line current? (*d*) What is the power-factor of each impedance? (*e*) What power does each impedance take? (*f*) What is the system power? (*g*) What are the kilovolt-amperes of the system?

109. Figure 109 (*A*) shows an unbalanced 3-phase, 4-wire, 60-cycle system operating with 220 volts between line wires and neutral. Three resistances of 20, 25, and 11 ohms are connected between line wires and neutral. (*a*) What does each of the wattmeters W_1, W_2, and W_3 read? (*b*) What is the total system power?

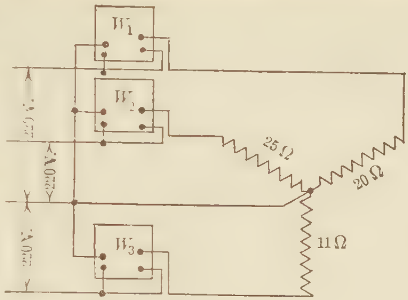


FIG. 109 (*A*).

110. Repeat problem 109 with the resistances connected to the neutral of a balanced, symmetrical 3-phase system in which there is 300 volts between *line* wires.

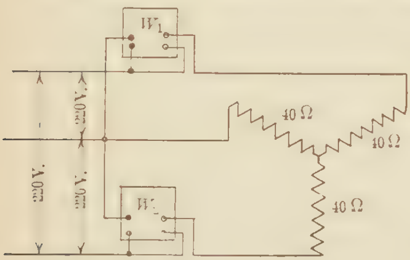


FIG. 111 (*A*).

111. The 2-wattmeter method is used to measure the power taken by a balanced Y-connected load connected across 220-volt, 60-cycle mains (Fig. 111 (*A*)). Each leg of the load consists of a 40-ohm non-inductive resistance. (*a*) How much power is taken by each of the Y-loads? (*b*) What is the total power taken by the load? (*c*) What does each wattmeter read?

112. Repeat problem 110 with the three resistances connected in delta across the same 3-phase mains.

113. The input to a 220-volt, three-phase induction motor is measured by the 2-wattmeter method (Fig. 113 (*A*)). The load is balanced. Near

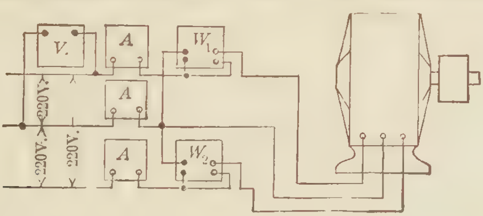


FIG. 113 (*A*).

full load on the motor, wattmeter W_1 reads 3,300 watts and wattmeter W_2 reads 1,400 watts. Both instruments are known to be reading up scale. (*a*) What is the total power taken by the motor? (*b*) Using the curve of Fig. 105 (page 122) determine the power-factor of the motor. (*c*) What

are the volt-amperes taken by the motor? (*d*) What do the three ammeters read?

114. Repeat problem 113 when wattmeter W_1 reads 1,400 watts and wattmeter W_2 reads 560 watts. In order that W_2 may read up scale, it is found necessary to reverse its current-coil connection.

115. (*a*) What is the motor power-factor (problem 113) when W_1 reads 1,600 watts and W_2 reads zero? (*b*) What are the kilovolt-amperes of the system? (*c*) What current does the motor take?

116. A certain balanced 3-phase load consists of three similar Y-connected circuits, each consisting of a resistance and a condenser in parallel (Fig. 116 (*A*)). The power to this load is measured by means of the 2-wattmeter

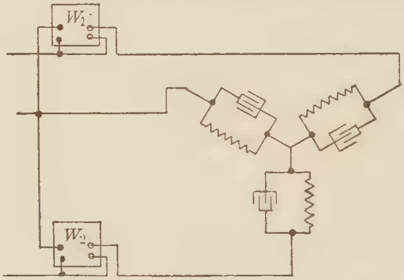


Fig. 116 (*A*).

method. W_1 reads -300 watts and W_2 reads $+1{,}200$ watts. The line voltage is 200 volts. Find: (*a*) the power-factor of the load (use Fig. 105, page 122); (*b*) the kilovolt-amperes of the load; (*c*) the line current.

117. A 2-phase, 3-wire system is shown in Fig. 117 (*A*). Phase-*AB* and phase-*BC* are connected at *B*. The voltage of each phase is 220 volts and the current in each phase is 25 amp. The voltage and current in phase-*AB* are in quadrature with the voltage and current in phase-*BC* respectively. (*a*) What is the voltage across the outer wires *A*-*C*? (*b*) What current flows in the common wire *BB′*?

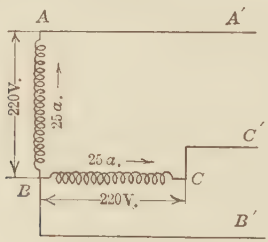


Fig. 117 (*A*).

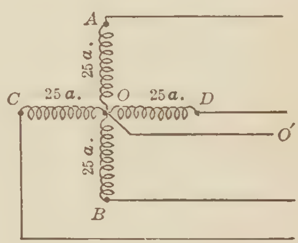


Fig. 118 (*A*).

118. Phase-*AB* and phase-*BC* (problem 117) are connected together at their neutral points *O* (Fig. 118 (*A*)) and a neutral wire *OO′* is also connected. This constitutes a quarter-phase, 5-wire system. The voltages across *AB*

and CD are each equal to 220 volts. (*a*) Find the voltages from each outer wire to neutral. (*b*) What is the voltage across adjacent outer wires such as A-D, D-B etc.? (*c*) If the current per phase is 25 amp., what is the kilovolt-ampere output of the system?

119. A 2-phase, mesh-connected system is shown in Fig. 119 (A). The voltage per coil is 110 volts and the current per coil is 25 amp. (*a*) What is the voltage across the diametrical connections A-C and B-D? (*b*) What current flows in each line wire?

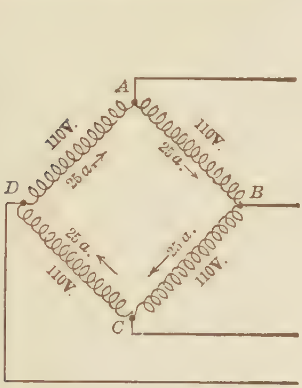


FIG. 119 (A).

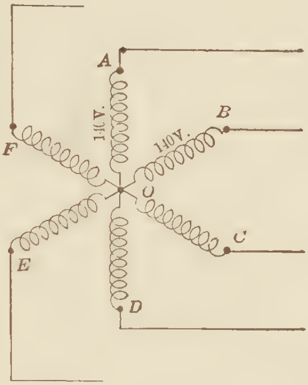


FIG. 120 (A).

120. Figure 120 (A) shows a star-connected, 6-phase system. There is 140 volts from each coil terminal A, B, C, etc. to the neutral O. (*a*) What is the voltage between adjacent line terminals A-B, B-C, etc.? (*b*) What is the voltage across the diametrically opposite terminals A-D, B-E, etc.? (*c*) What is the voltage between the alternate terminals A-C, C-E, etc.?

QUESTIONS ON CHAPTER V

1. In what way does the relation of armature to field in an alternator differ from their relation in a direct-current generator? Why is this relation possible with the alternator?

2. Give three advantages of a stationary over a rotating armature.

3. Describe the method by which the field induces alternating e.m.fs. in the stator or armature coils. To what fundamental laws are these e.m.fs. due? How is Fleming's right-hand rule applied to the stationary-armature type of alternator?

4. What two fundamental principles govern the design of alternator windings?

5. Why are simple single-phase windings little used in alternators?

6. Sketch a 4-pole, single-phase, lap-winding in which there are four slots per pole. Repeat for a wave-winding. Compare the e.m.fs. of the two windings, the number of conductors, etc. in each being the same.

7. Show the difference between half-coil and whole-coil windings. Why are armatures having but one slot per pole little used?

8. In what way does the spiral winding differ from the lap- and wave-windings? What is meant by "single-range?"

9. Show how a simple 2-phase winding may be evolved from two single-phase windings. Sketch such a winding.

10. Sketch a 4-pole, 2-phase, spiral winding having four slots per pole. How many ranges must such a winding have?

11. Sketch a 4-pole, 2-phase, lap-winding having eight slots per pole or four slots per pole per phase.

12. In what manner does the 3-phase winding differ from the single-phase winding? What relation exists between the coil-sides of adjacent phases?

13. In what way does a fractional-pitch winding differ from a full-pitch winding? How do the coil-sides in the top layer of a fractional-pitch winding compare with those of a similar full-pitch winding. How do the coil-sides in the bottom layers of the two windings compare?

14. What are the advantages of fractional-pitch windings?

15. Why is it necessary that the stators of alternators be made of laminations? Describe the construction of a slow-speed alternator. How are the stator stampings held to the frame?

16. In what way does the construction of turbo-alternators differ from that of low-speed alternators? Why are ventilating ducts through the iron stator necessary? How are the coil-ends braced and why is this bracing necessary?

17. Sketch two types of alternator slot. How are the conductors held in the slot? What is the advantage of each type and under what conditions is each used?

18. How are the field-poles of low-speed alternators constructed? What two methods are used to hold them to the spider?

19. Describe the construction of moderate-speed rotors. How are the poles held?

20. Why is it not possible to use salient poles with high-speed turbo-alternators? Of what material are the rotors of high-speed turbo-alternators made?

21. Compare the parallel-slot with the radial-slot rotor. Where is each used?

22. What is meant by a distributed-field winding? How does the flux distribution with this type of rotor compare with that of the ordinary salient-pole rotor?

23. Why does the construction of turbo-alternators make it particularly difficult to secure adequate ventilation? How is the air forced through the machine and what paths does it take? Why is an air-washer used?

24. To what three factors is the induced e.m.f. in an alternator proportional? What is meant by "pitch-factor;" "belt-factor?"

25. Describe in detail the manner of "phasing" alternator coils in Y; in delta.

26. What factor ordinarily determines the rating of electric machinery? Why is the output of an alternator determined by its *current* output rather

than by its *kilowatt* output. What determines the rating of the prime mover?

PROBLEMS ON CHAPTER V

121. Figure 121 (*A*) shows the stator and field structure of a 4-pole alternator. There are eight stator slots 1–8, inclusive. Sketch a half-coil, single-phase winding in which there are four coils and two slots per pole.

122. Sketch a two-layer, whole-coil winding having eight coils for the stator of Fig. 121 (*A*).

123. Make a sketch of a half-coil, single-layer, wave-winding for a stator similar to that of Fig. 121 (*A*), except that there are four slots per pole.

124. Repeat problem 123 for a lap-winding.

125. Sketch a stator and show a half-coil, 2-phase winding having two slots per pole per phase placed on a stator similar to that shown in Fig. 121 (*A*), with the exception of the number of slots.

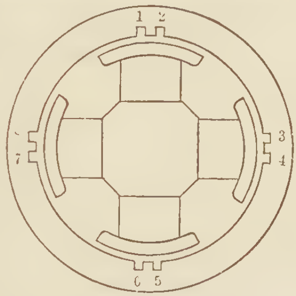


FIG. 121 (*A*).

126. Figure 126 (*A*) shows 20 slots, each containing two coil-sides of an alternator winding having eight slots per pole. Connect these coil-sides

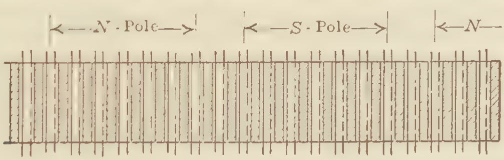


FIG. 126 (*A*).

so as to give a portion of a 2-phase, half-coil, full-pitch, lap-winding. The conductors shown solid lie in the top of the slots and the conductors shown dotted lie in the bottom of the slots.

127. Repeat problem 126 with connections to give a wave-winding.

128. Sketch a 2-phase, 2-layer, ¾-pitch, lap-winding for a machine having eight slots per pole (see Figs. 126 and 130, pages 140 and 143).

129. Show a portion of a 3-phase, 2-layer, full-pitch, lap-winding for a machine having nine slots per pole (see Fig. 128, page 142).

130. Repeat problem 129 making the winding 7/9 pitch.

Note: In the following problems the flux may be considered as being distributed sinusoidally along the gap.

***131.** An alternator coil having 12 turns is linked by a total flux of 2,500,000 lines leaving one north pole (Fig. 131 (*A*)). In a half-cycle a south pole will be directly under this same coil, and the flux has been completely reversed in direction. Hence, in a quarter cycle *no* flux links this coil (also see Part I, page 206, Fig. 181). This is a 60-cycle generator so that in 1/240 sec.

after the maximum flux links the coil, no flux links the coil. (*a*) What is the average induced e.m.f. in the coil during a quarter-cycle; (*b*) during a half-cycle? (*c*) The ratio of effective to average e.m.f. is 1 : 11 (see page 25). What is the effective e.m.f. of this coil during a half cycle? (*d*) Will the e.m.f. over a complete-cycle differ from that over a quarter-cycle; a half-cycle? Explain.

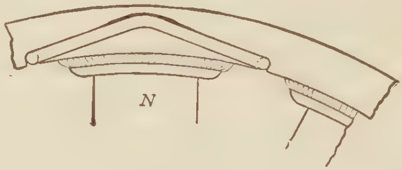


FIG. 131 (*A*).

***132.** Repeat problem 131 for a 25-cycle machine.

133. A 4-pole, single-phase, 50-cycle alternator has one slot per pole and there are 20 conductors per slot. A flux of 2,500,000 lines leaves each north pole and enters the armature. What voltage is induced in this alternator?

***134.** A 4-pole, 2-phase, 50-cycle alternator has one slot per pole per phase. There are 12 conductors per slot and a flux of 2,000,000 lines enters the armature from each north pole. (*a*) What is the induced voltage per phase? (*b*) If the two phases are connected to give a 2-phase, 3-wire system, what is the voltage across the open ends (see Fig. 109, page 125)?

***135.** A 6-pole, 2-phase 60-cycle alternator has three slots per pole per phase and there are eight conductors per slot. The three slots per pole give a belt factor of 0.91. A flux of 1,500,000 lines leaves each north pole and enters the armature. (*a*) Find the induced voltage per phase. (*b*) What are the voltages across the machine terminals, if the phases are star-connected.

***136.** In an 8-pole, 25-cycle, 2-phase alternator there are four slots per pole per phase, and 18 conductors per slot. A flux of 4,000,000 lines enters the armature from each north pole. The belt factor is 0.91. Find the induced e.m.f. per phase.

***137.** Find the induced e.m.f. (problem 136) for a 6/8-pitch winding, the pitch factor being 0.92.

***138.** A 3-phase, 6-pole, 60-cycle, delta-connected alternator has 12 slots per pole, and there are 12 conductors per slot. A 5/6-pitch winding is used. A flux of 3,200,000 lines enters the armature from each north pole. The belt factor is 0.958 and the pitch factor is 0.966. Find the induced e.m.f. across each pair of terminals.

139. A 25-cycle, 3-phase, 2,300-volt alternator has a rating of 2,000 kilowatts at 0.8 power-factor. (*a*) Find its kilovolt-ampere rating. (*b*) What is the rated current per terminal? (*c*) If the alternator is Y-connected, what is its rated coil voltage?

140. A 6,600-volt, 3-phase turbo-alternator is rated at 438 amp. per terminal. (*a*) What is its kilovolt-ampere rating? (*b*) What is its kilowatt

rating at unity power-factor. (c) What is its kilowatt rating at 0.75 power-factor?

141. If the alternator (problem 140) has an efficiency of 0.94, excluding the field loss, what is the horsepower of the turbine required to drive it at the ratings given in (b) and in (c)?

142. A 3-phase, 60-cycle turbo-alternator is required to deliver 12,000 kw. at 6,600 volts and at 0.8 power-factor. (a) What is its kilovolt-ampere rating? (b) What is its current rating? (c) If its efficiency, neglecting field loss, is 0.95, what is the horsepower which the turbine delivers when the alternator delivers its rated load?

QUESTIONS ON CHAPTER VI

1. Why is the variation of voltage with load in alternators ordinarily much greater, proportionately, than the variation of voltage with load in direct-current generators? Why does the selection of system apparatus make it desirable to know something of the characteristics of the alternators connected to the system? What three factors tend to cause the terminal voltage of alternators to drop with increase of load?

2. Make a sketch demonstrating that considerable flux links the conductors embedded in the slots of an alternator armature. Show that this flux also links the armature coils. What flux other than that which crosses the slot, links the armature coils?

3. How is the armature inductance affected by the slot dimensions? How is the armature reactance affected by the slot dimensions? How is the armature reactance affected by frequency?

4. Give three reasons why the resistance of the alternator armature to alternating current is greater than it is to direct current. What is meant by 'effective resistance?" Approximately what is the ratio of effective to ohmic resistance in commercial alternators?

5 Show with a diagram that when the armature current is in phase with the induced e.m.f., its m.m.f. distorts the field of the alternator but alters the magnitude of the field but slightly. In what direction, relative to the rotation of the armature coil, is the field crowded?

6. Show with a diagram that when the armature current lags the induced e.m.f. by 90°, its m.m.f. weakens the alternator field.

7. Show with a diagram that when the armature current leads the induced e.m.f. by 90° its m.m.f. strengthens the alternator field.

8. What voltage-drop, other than the resistance-drop, exists in the alternator armature? How is the armature impedance-drop found?

9. Knowing the armature terminal voltage, the value of the current, the phase difference between the current and the terminal voltage, and the resistance and reactance of the armature how may the *induced* e.m.f. be found?

10. Compare the relative magnitudes of the terminal voltage and the induced e.m.f. when the current is in phase with the terminal voltage; when the current lags the terminal voltage; when the current leads the terminal voltage.

11. Why is the induced e.m.f. in an alternator not equal to the no-load e.m.f., even with constant field excitation? Define alternator regulation. What two effects combine to give poor regulation with lagging current?

12. Give two reasons why the rated-load terminal voltage may exceed in magnitude the no-load voltage when the current leads.

13. Show with a sketch the approximate voltage—load characteristics of an alternator when the current is in phase with the terminal voltage; when the current lags the terminal voltage; when the current leads the terminal voltage.

14. Make a diagram of the connections which would be used to determine the saturation curve of a 3-phase, delta-connected alternator; a 3-phase, Y-connected alternator.

15. Make a diagram of the connections which would be used for determining the voltage—load characteristics of a 3-phase alternator. Discuss the methods which can be used in connecting the load. What difficulties are encountered in making such load tests?

16. Discuss the use of the Tirrill regulator in maintaining constant the terminal voltage of alternators.

17. Show that a change in field excitation cannot change appreciably the division of power load between alternators operating in parallel. What factors do determine the division of power load between alternators in parallel?

18. How may the division of load between alternators operating in parallel be changed? Why are alternators operating in parallel in stable equilibrium? What occurs when the driving torque of one alternator is entirely removed?

19. Describe in general the effects which occur with two alternators operating in parallel if the field of one is strengthened and the field of the other is weakened. In which machine does the current lead more; lag more?

20. What effects occur in each of the two machines (*a*) which tend to equalize their *induced* e.m.fs.; (*b*) which cause their terminal voltages to be equal?

21. What two conditions must be fulfilled before alternators can be connected in parallel? How may the equality of terminal voltages be determined? How may the correct polarity be determined? Make a diagram of connections.

22. What does the flickering of the synchronizing lamps show? What are the disadvantages of the "three-dark" method of synchronizing? Explain how these difficulties are eliminated in the "two-light-and-one-dark" method. What is a synchroscope and what does it show?

23. What is meant by the "hunting" of alternators? What causes hunting? What methods are employed to minimize and even to eliminate hunting?

PROBLEMS ON CHAPTER VI

143. The ohmic resistance between each pair of terminals of a 3-phase alternator is measured and found to average 0.12 ohm. The ratio of effective to ohmic resistance is known to be 1.4 to 1.0 (*a*) What is the

ohmic resistance per coil-circuit? (*b*) What is the effective resistance per coil circuit?

144. Each coil or coil-circuit, *ab*, *bc*, and *ca*, of a delta-connected alternator Fig. 144 (*A*) is known to have an ohmic resistance of 0.18 ohm. If the ohmic resistance is measured between terminals *a* and *b*, what will be its value? If the ratio of effective to ohmic resistance is known to be 1.5 to 1.0, what is the effective resistance of each coil?

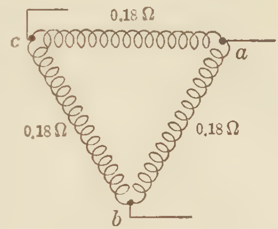


Fig. 144 (*A*).

145. The ohmic resistance between each pair of terminals, such as *ab*, *bc*, and *ca*, Fig. 144 (*A*), of a delta-connected alternator is measured with direct current and found to be 0.06 ohm. What is the ohmic resistance per coil? If the ratio of effective to ohmic resistance is 1.5, to 1.0 what is the effective resistance per coil?

146. A 50-kv-a., single-phase, 550-volt, 60-cycle alternator has an effective resistance of 0.25 ohm and an armature reactance of 1.5 ohms. (*a*) What is the rated current of this alternator? (*b*) What is the resistance-drop at rated-load current? (*c*) What is the reactance-drop at rated-load current?

147. Determine the e.m f. induced in the armature of this alternator (problem 146) when it delivers its rated current and the power-factor of the load is unity (see Fig. 153, page 167).

***148.** Determine the e.m.f. induced in the armature of the alternator (problem 146) when the load power-factor is 0.707, lagging current (see Fig. 154, page 168).

***149.** Repeat problem 148 for a load power-factor of 0.707, leading current.

***150.** A 25-kv-a., single-phase, 220-volt, 60-cycle alternator has an effective resistance of 0.1 ohm and an armature reactance of 0.5 ohm. Find: (*a*) the rated-load current of this alternator; (*b*) the armature resistance-drop; (*c*) the armature reactance-drop; (*d*) the induced e.m.f. at rated-load when the load power-factor is unity; (*e*) when the load power-factor is 0.80, lagging current; (*f*) when the load power-factor is 0.80, leading current.

***151.** The terminal voltage of a 15 kv-a., 3-phase, 220-volt, 60-cycle alternator is adjusted to its rated value at rated current under each of the following conditions, and the load is then removed: (*a*) unity power-factor; (*b*) 0.8 power-factor, lagging current; (*c*) 0.8 power-factor, leading current. In (*a*) the no-load voltage is found to be 242 volts; in (*b*) it is found to be 285 volts; in (*c*) it is found to be 210 volts. Find the regulation under each condition.

***152.** Find the power delivered by the alternator under each condition of load (problem 151).

153. A 500 kv-a., 3-phase, 2,300-volt, 25-cycle alternator has its terminal voltage adjusted to rated value at rated-load and regulates as follows: unity power-factor load, 8 per cent; 0.8 power-factor load, current lagging,

25 per cent.; 0.8 power-factor load, current leading −5 per cent. What is the value of the no-load voltage in each case?

154. Find the power delivered by the alternator under each condition of load (problem 153).

155. The terminal voltage of a 400-kv-a., 600-volt, 3-phase, 60-cycle alternator is adjusted to its rated value when the load power-factor is unity, when the load power-factor is 0.85, lagging current, and when the load power-factor is 0.85, leading current. When the load is removed under each of these conditions, the no-load voltages are 645, 720, and 590 volts, respectively. What is the regulation in each case?

156. Find the power delivered by the alternator under each condition of load (problem 155). What are the kilovolt-amperes in each case?

157. Two 3-phase, 25-cycle alternators, having ratings of 100 and 50 kv-a., operate in parallel. The governor characteristics of the engines driving these alternators are such that the speed of the 100-kv-a. unit drops at a uniform rate from 25 to 23 cycles when the power changes from zero to 100-kw. and the speed of the 50-kv-a. unit drops at a uniform rate from 25 to 22.5 cycles when it changes from zero to 50-kw. (see Fig. 157 (*A*)). Find the power delivered by the 50-kv-a. unit when the 100 kv-a. unit delivers 100 kw.

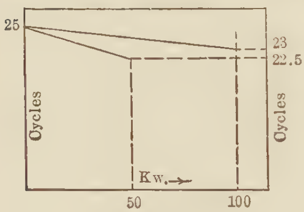


Fig. 157 (*A*).

158. Find the power delivered by the 100-kv-a. unit when the 50-kv-a. unit delivers 25 kw. What is the total power delivered to the load?

159. If the speed-load characteristic of the 100-kv-a. unit (problem 158) is a straight line to 25 per cent. overload, find the power which it delivers when the 50-kv-a. unit delivers its rated kilowatts at unity power-factor.

***160.** Two 500-kv-a., 2,300-volt, 3-phase, 60-cycle turbo-alternator units 1 and 2, operating in parallel, have such speed-load characteristics that the speed of 1 increases from 60 to 64 cycles when a 500-kw. load is removed, and the speed of 2 increases from 60 to 63 cycles under the same conditions. The speed-load characteristic of each unit is a straight line. What load does 1 deliver when 2 delivers 250 kw.?

***161.** In problem 160, what load does 2 deliver when 1 delivers 400 kw.?

***162.** In problem 160, what load does 1 deliver when the load on 2 is zero?

***163.** The governor of unit 1 (problem 160), is so adjusted that its no-load speed is 63 cycles and its speed at 500 kw. output is 59 cycles. What load does 2 deliver when 1 delivers 250 kw.?

***164.** Two 500-kv-a., 2,300-volt, 3-phase, 60-cycle alternators operate in parallel and each delivers 60 amp. at unity power-factor to a non-inductive load taking 478 kw. The field of one alternator is strengthened until it delivers a current which lags 30° and the field of the other is simultaneously weakened so that the bus-bar voltage does not change. The load remains constant. What current does each alternator deliver under these conditions and at what power-factor does each operate?

***165.** Repeat problem 164 when the field adjustments cause the current of the first alternator to lag 45°, other conditions remaining unchanged.

QUESTIONS ON CHAPTER VII

1. Define a static transformer. Through what medium is the energy transferred? Why is the transformer a very useful piece of power apparatus?

2. Upon what fundamental principle does the induced e.m.f. in the secondary depend? In what manner does the production of e.m.f. in the transformer differ from that in the generator?

3. Define the *primary;* the *secondary.*

4. To what three factors is the induced e.m.f. proportional? Show that the induced e.m.f. in the primary and the induced e.m.f. in the secondary are proportional to their respective number of turns. Why are the terminal voltages practically proportional to their respective number of turns?

5. What two functions does the no-load current perform? What is the order of magnitude of the no-load current; the no-load power-factor?

6. Show that the flux in the core of the ordinary transformer remains practically constant over the operating range of the transformer.

7. Why do the secondary ampere-turns *oppose* the flux? Analyze the sequence of reactions which follow the application of load to the secondary and which cause more energy to enter the primary from the line.

8. Why must the secondary ampere-turns be practically equal to the increase of primary ampere-turns over the no-load ampere-turns? To what is the ratio of primary to secondary current equal?

9. What m.m.f. produces the mutual flux; the primary leakage flux? Discuss the effect of the primary leakage flux on the operation of the transformer. Define primary leakage reactance.

10. To what is the secondary leakage flux proportional? Discuss its effect on the operation of the transformer. Define secondary leakage reactance.

11. What is the general effect of leakage flux on the operation of the transformer as a whole. When are small and when are large leakage reactances desirable?

12. Why is it difficult to determine transformer efficiency and regulation accurately by measurements of output and input?

13. What measurements are necessary for determining the copper losses and what precautions are necessary in making these measurements?

14. To what are the core losses due and how are they minimized? Sketch a diagram of connections which would be used in measuring such losses. Why do these losses remain practically constant at all ordinary loads?

15. Describe core- and shell-type transformers. How are the cores built up? Where is each used? In the core type, why are a portion of the primary and a portion of the secondary placed on each of the two legs?

16. Describe the Type H transformer of the General Electric Company. Discuss the disposition of the two windings, and of the insulation. What are oil channels?

17. Why does the problem of cooling transformers become more important with increase of rating? How does oil cool the transformer? How is the heat in the smaller units dissipated?

18. When the corrugated surface becomes inadequate to dissipate heat, what means are taken to increase the heat-radiating surface?

19. What limits the size to which tubular construction can be built? What method of cooling is not subject to these limitations?

20. How are the core and windings of the larger transformers supported and why?

21. How is air cooling obtained? Water cooling?

22. Sketch the core and windings of a 3-phase, core-type transformer. Compare a 3-phase transformer with three single-phase transformers of the same total rating with reference to weight, space, reliability, etc.

23. In what manner do auto-transformers differ from the ordinary transformer? Where would auto-transformers be used? Why is it not feasible to use them for large ratios of transformation? Sketch the connections of a two-to-one, 3-phase, step-down auto-transformer. Show the connections which would be used to obtain a 220-110, 3-wire system from a 220-volt, 2-wire system. What is a "balance coil?"

24. Why is it highly undesirable to connect at random two primary coils in series directly across a power source? Describe two methods by which the correct phase relation of such coils for both series and parallel connection may be determined without danger of short-circuit. Describe methods which may be used to phase secondary coils.

25. Sketch typical series and parallel connections of both the primary and secondary coils of a distribution transformer. How is a 3-wire secondary system obtained?

26. State the advantages of a delta-delta transformer bank. Of a Y-Y bank. Under what conditions is it not practicable to use a Y-Y bank?

27. Under what conditions is the delta-Y bank used; the Y-delta bank? What is the distinct advantage of using Y-connected transformers for high voltage? When must the proper phase relations between primaries be observed? How are proper phase relations between secondaries determined?

28. Show, by means of a delta-connection, that a V-V connection gives 3-phase to 3-phase transformation of power. What is the ratio of the kilovolt-ampere rating of a delta-bank to a V-bank, if units of the same kilovolt-ampere capacity are used in each?

29. In the T-transformer connection, why do the e.m.fs. in the two halves of the main transformer differ in phase by 180°? With equal ratios of transformation, why are the secondary e.m.fs. unequal? How may equality of secondary e.m.fs. be obtained? Where is the Scott or T-connection used?

30. Show that when the load on a constant-current transformer is changed, reactions develop which tend to maintain the current at a constant value. What part does leakage flux play in maintaining constant current and why?

What precautions should be taken in the operation of constant-current transformers?

31. Give three reasons for the necessity of using instrument transformers. Describe a potential transformer. At what standard secondary voltage do such transformers operate?

32. In what manner does the current transformer differ from the constant-potential transformer? At what current are the secondaries ordinarily rated? What precautions should be taken in the operation of such transformers?

PROBLEMS ON CHAPTER VII

166. A 2,000-kv-a. 6,600/70,000-volt, step-up transformer has 80 low-side turns. (*a*) What is the ratio of transformation? (*b*) How many high-side turns has the transformer?

167. A 10-kv-a., 60-cycle transformer operates with 2,200 volts across its primary or high side. There are 1,000 high-side turns. It is desired that either 225 or 235 volts be obtainable from the secondary by having tap *b* in the low-voltage winding (Fig. 167 (*A*)). (*a*) How many turns in the portion *ab* of the secondary winding? (*b*) How many turns in the entire winding *ac*?

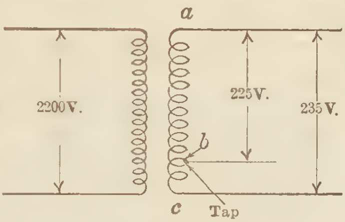


Fig. 167 (*A*).

168. A 400-kv-a., 11,000/2,300-volt, 60-cycle transformer has 1,000 high-side turns and the maximum instantaneous flux is 4,140,000 maxwells. (*a*) Find the low-side turns. (*b*) Determine the volts per turn. (*c*) Find the induced e.m.f. in the high side. (*d*) Determine the induced volts per turn (use equation (54), page 190).

169. A 15-kv-a., 2,300/230-volt, 60-cycle transformer has 800 high-side turns. (*a*) How many low-side turns has it? (*b*) What is the maximum instantaneous flux in the core?

170. A 40-kv-a., 4,400/550-volt, 25-cycle transformer operates with a flux, having a maximum instantaneous value of 3,300,000 maxwells. Find the number of turns required for the high and for the low side.

171. With what maximum instantaneous flux will the transformer (problem 170) operate, if the high side operates at 4,400 volts, 60 cycles?

172. In problem 166, find: (*a*) the rated high-side current; (*b*) the rated low-side current; (*c*) the primary and secondary ampere-turns, neglecting exciting current.

173. (*a*) In problem 167, find the high-side current when a load of 44.5 amp. is applied across the 225-volt portion *ab* of the secondary. (*b*) Find the high-side current when a load of 44.5 amp. is applied across the entire secondary winding *ac*. Neglect the exciting current.

174. A 5,000-kv-a., 25-cycle transformer steps down the voltage from 13,800 to 2,300 volts. There are 640 turns in the high-side winding. (*a*)

Find the rated high-side current. (*b*) Determine the high-side ampere-turns at rated current. (*c*) Determine the low-side ampere-turns, neglecting the exciting current. (*d*) Determine the low-side current. (Links *ab* and *cd* together are connected across *bc* (Fig. 175 (*A*)), connecting coils *a'c* and *d'd* in series.)

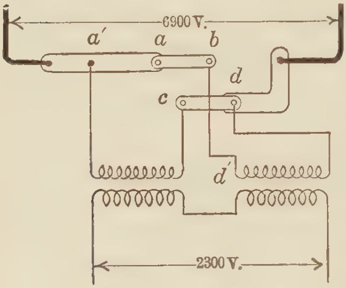


FIG. 175 (*A*).

175. The high-side winding (problem 174) is in two sections rated at 6,900 volts each. These coils are paralleled by means of the links *ab* and *cd* (Fig. 175 (*A*)) so that the transformer high side operates at 6,900 volts and rated current. Under these conditions, determine: (*a*) the total rated high-side current; (*b*) the high-side ampere-turns; (*c*) the low-side ampere-turns. Compare these results with those obtained in problem 174.

176. A 25-kv-a., 2,200/550-volt, 60-cycle transformer has a no-load loss or core loss of 550 watts when operating at its rated voltage. The rated high-side current is 25,000/2,200, or 11.4 amp. The high-side resistance is 1.50 ohms and the low-side resistance is 0.09 ohm. Find: (*a*) the low-side current; (*b*) the high-side copper loss; (*c*) the low-side copper loss; (*d*) the total losses at rated kilovolt-ampere load; (*e*) the efficiency at rated kilovolt-ampere load and unity power-factor.

177. Find the efficiency of the transformer (problem 176) at half its rated kilovolt-ampere load and unity power-factor.

178. Find the efficiency of the transformer (problem 176) at rated kilovolt-ampere load and 0.7 power-factor, lagging current.

179. A 500-kv-a., 22,000/2,200-volt, 25-cycle transformer has a core loss of 6,000 watts at rated voltage. The high-side resistance is 6.0 ohms and the low-side resistance is 0.056 ohm. Find: (*a*) the rated-load high-side and low-side currents; (*b*) the high-side copper loss at rated current; (*c*) the low-side copper loss at rated current; (*d*) the total loss at rated kilovolt-ampere load; (*e*) the efficiency at rated kilovolt-ampere load and unity power-factor.

180. Find the efficiency of the transformer (problem 179) at rated kilovolt-ampere load and 0.8 power-factor, lagging current; at 0.8 power-factor, leading current.

181. Find the efficiency of the transformer (problem 179) at $\frac{3}{4}$ rated kv-a. load and 0.8 power-factor.

182. A 5-kv-a., 13,200/220-volt, 60-cycle transformer has a core loss of 75 watts at no-load and rated voltage. The high-side resistance is 400 ohms and the low-side resistance is 0.107 ohm. Find the transformer efficiency at rated kilovolt-ampere load and unity power-factor.

183. Determine the efficiency of the transformer (problem 182), at rated kv-a., load and at 0.75 power-factor, leading current.

184. An auto-transformer (Fig. 184 (*A*)) is used to transform from 550 to 440 volts. The load at $b'c'$ takes 10 kw. at unity power-factor. Neglecting

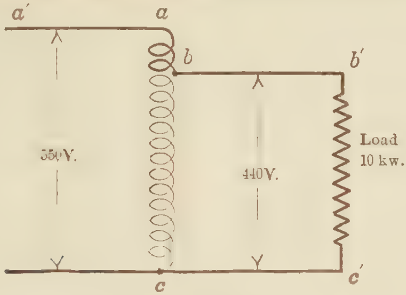


Fig. 184 (*A*).

all losses and the magnetizing current, find: (*a*) the current in bb'; (*b*) the current in *ab*; (*c*) the current in *cb*.

185. An auto-transformer (Fig. 185 (*A*)) is used to boost the voltage on a 2,200-volt feeder to 2,400 volts. The load requires 50 kw. at unity power-

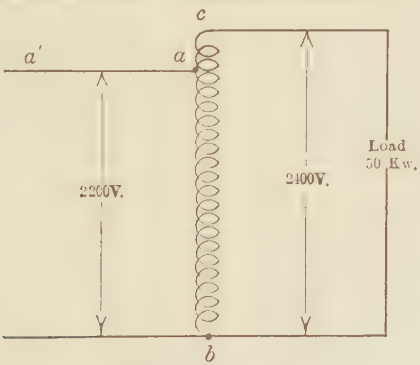


Fig. 185 (*A*).

factor. Neglecting all losses and the magnetizing current, determine: (*a*) the current in *ac*; (*b*) the current in $a'a$; (*c*) the current in *ba*.

186. A 220-volt starting compensator or auto-transformer used for starting alternating-current motors at reduced voltage supplies the motor on starting

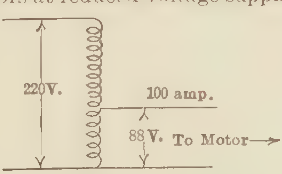


Fig. 186 (*A*).

from 40 per cent. taps (Fig. 186 (*A*)), giving 88 volts across the motor terminals. Determine the line current when the motor takes 100 amp.

187. A Y-Y transformer bank is used to transform from 6,600 to 2,200 volts (Fig. 187 (*A*)). Find: (*a*) the voltage ratings of the transformer

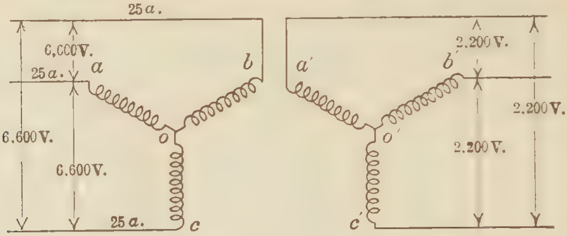


Fig. 187 (*A*).

primaries; (*b*) the voltage ratings of the transformer secondaries; (*c*) the secondary currents when the primary currents are 25 amp.

188. A delta-Y transformer bank (Fig. 188 (*A*)) is used to transform from 13,200 volts to a 4,000-volt, 4-wire distribution system. Find: (*a*) the voltage ratio of the transformer bank; (*b*) the voltage ratios of the individual

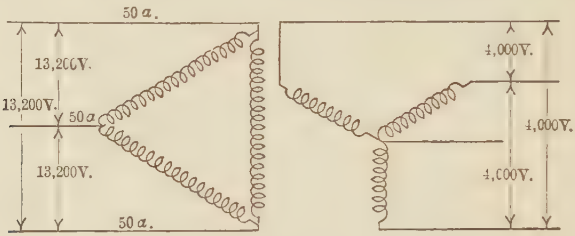


Fig. 188 (*A*).

transformers; (*c*) the primary coil currents, when the primary line current is 50 amp.; (*d*) the secondary coil currents; (*e*) the voltage to neutral on the secondary.

189. What is the kilovolt-ampere rating of each transformer (problem 188)?

190. Determine the voltage and current ratings, and the ratio of transformation of the individual units of a delta-delta transformer bank to transform the same 3-phase power at the same 3-phase ratio, in problem 188.

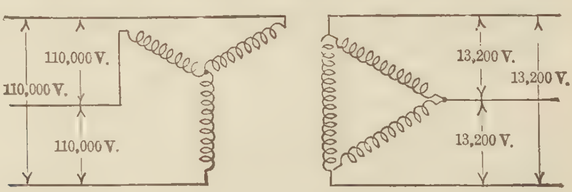


Fig. 191 (*A*).

191. It is desired to transform 6,000-kv-a., 3-phase 60-cycles at unity power-factor from a transmission voltage of 110,000 volts to 13,200 volts by means of a Y-delta transformer bank (Fig. 191 (*A*)). Find: (*a*) the

primary line current; (*b*) the rated voltages of the individual primaries; (*c*) the ratio of 3-phase to 3-phase transformation; (*d*) the ratio of transformation of each unit; (*e*) the secondary line currents; (*f*) the secondary coil currents; (*g*) the required kilovolt-ampere rating of each unit.

192. Figure 192 (*A*) gives the currents and voltages in the primary line of a delta-Y transformer bank used for stepping up the generated voltage of 6,600 to 66,000 volts for transmission. Find: (*a*) the current in each

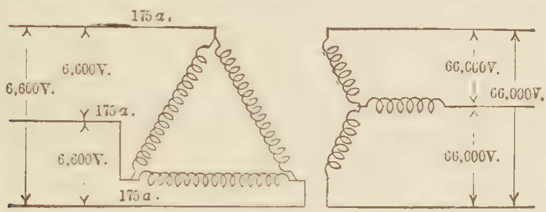


Fig. 192 (*A*).

transformer primary; (*b*) the current in each transformer secondary; (*c*) the voltage rating of each secondary; (*d*) the individual transformer ratios; (*e*) the secondary line current; (*f*) the kilovolt-ampere output of each transformer.

193. It is desired to transform 100 kw. at unity power-factor and 60 cycles from 6,900 to 2,300 volts by means of V-connected transformers (Fig. 193

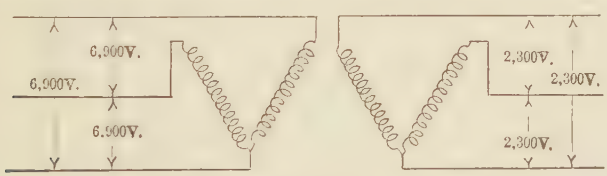


Fig. 193 (*A*).

(*A*)). Find: (*a*) the primary line current; (*b*) the kilovolt-ampere rating of each transformer; (*c*) the kilowatt rating of each transformer.

194. If a third transformer similar to the other two be added to the V-bank (problem 193) to give a delta-connection, what will be the kilovolt-ampere and the kilowatt rating at unity power-factor of the entire transformer bank?

195. Two transformers, each rated at 2,200/550 volts and 4.5 amp., pri. are connected in V to supply power to a machine shop. (*a*) Assuming unity power-factor load, how many kilowatts can these transformers deliver without exceeding their ratings? (*b*) What is the secondary line current?

196. What will be the power rating at unity power-factor of the transformer bank (problem 195) if a third transformer similar to the other two is added to complete a delta-connection?

197. A T-connected transformer bank (Fig. 197 (A)) transforms from 600 volts, 3-phase, to 230 volts 2-phase. Determine: (a) the value of the volt-

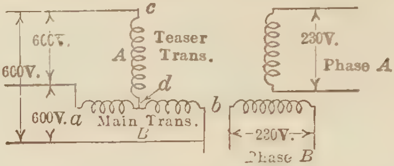


FIG. 197 (A).

ages E_{ad}, E_{bd}, and E_{cd}; (b) the transformation ratio of the main transformer; (c) the transformation ratio of the teaser transformer.

QUESTIONS ON CHAPTER VIII

1. Describe a simple experiment which shows that if a flux is made to cut a conducting body, the currents induced in that body have such a direction that the body tends to follow the direction of motion of the flux.

2. Under these circumstances why must a force exist between the flux and the conducting body? Why can the body never attain the speed of the flux?

3. By means of sketches show that a rotating magnetic field may be produced by 2-phase currents flowing in a 2-phase, 4-pole winding. How many space-degrees does the field advance for each time-degree advance of the current? What is the relation of the speed of the field in r.p.s. to the frequency of the currents?

4. Show by sketches how a 2-pole, rotating magnetic field may be produced by 2-phase currents flowing in a drum winding. What is the relation of the speed of the field in r.p.s. to the frequency of the currents?

5. What is meant by the *synchronous speed* of an induction motor? Derive the equation which gives the synchronous speed in terms of frequency and number of poles.

6. What is meant by the *revolutions slip* of an induction motor? To what is the *slip* equal?

7. Show that the rotor frequency f_2 is equal to the product of the stator frequency and the slip. To what s the rotor frequency equal at starting? Of what order of magnitude is the rotor frequency under ordinary operating conditions?

8 In what important respect does the change of torque with load for the induction motor differ from that of the direct-current shunt motor?

9 Sketch a typical torque-slip curve for a motor having an ordinary rotor. At low values of slip, how does the torque vary with increase of load? What occurs when the break-down torque is reached? What is the order of magnitude of the starting torque?

10. Show that for any given value of slip the torque varies with the square of the impressed voltage? Explain why it is important to keep this fact in mind.

11. Describe the construction of the squirrel-cage rotor. In what different ways are the squirrel-cage bars fastened to the end-rings? What type of slot is used and why?

12. Why must the slip of the motor increase with increase of load? Why is the slip of the squirrel-cage motor comparatively low? Give the order of magnitude of the full-load slip for small and for large motors.

13. Sketch curves of current, efficiency, slip, speed, and power-factor for an induction motor, using horsepower output as abscissas. Discuss each of these curves. Why is the no-load current comparatively large and the no-load power-factor low? Why does the power-factor increase with load?

14. Name one distinct disadvantage of the squirrel-cage induction motor. Why is the no-load torque low? State the advantages of this type of motor. For what classes of work is it best adapted? For what classes of work is it not adapted?

15. Why do squirrel-cage motors take relatively large currents when connected directly across the line? Show the connections which may be used when the motor is started by being connected directly across the line, the connections being such that the motor has the necessary overload protection on starting.

16. Discuss the use of resistance for limiting the starting current.

17. Make a wiring diagram, showing how a triple-pole, double-throw switch may be used for starting, by changing the stator connections from Y to delta.

18. Make a diagram which shows the construction and method of operation of two types of starting compensators, discussing the method of obtaining suitable fuse protection.

19. Analyze the effect on the operation of an induction motor of introducing resistance into the rotor circuit when the motor is carrying a load which requires some definite torque.

20. How does the introduction of resistance in the rotor circuit affect the relation of break-down torque to slip? Show that large starting torque may be obtained by the introduction of resistance in the rotor circuit.

21. Discuss speed control of the induction motor by the introduction of resistance in the rotor circuit. State two distinct disadvantages of this method of speed control.

22. Make a diagram showing the connections which are used when resistance is introduced in the rotor circuit.

23. How is good starting torque sometimes obtained without the use of slip-rings and resistance exterior to the motor?

24. Enumerate some of the industrial applications of the wound-rotor induction motor. How does its cost compare with that of the squirrel-cage motor?

25. What three factors completely determine the speed of an induction motor?

26. Discuss change of slip as a method of speed control; change of frequency; change of the number of poles. Where is each of these methods used?

27. How may an induction motor be made to act as a generator? What determines the frequency of the induction generator? Why is it called *asynchronous?*

28. Why must the induction generator always take a lagging current, and deliver a leading current? Give two reasons which make it necessary for the induction generator always to operate in parallel with a synchronous machine.

29. State the disadvantages of the induction generator. Give its industrial applications.

30. Describe briefly the principle underlying the stroboscopic method for measuring slip. Describe two other methods.

31. Describe the operation of the induction regulator, showing how "boost" and "buck" are obtained. Why is a short-circuited winding necessary? Show with a diagram the method of connecting the regulator to a feeder circuit.

32. Describe the 3-phase regulator.

PROBLEMS ON CHAPTER VIII

198. A 4-pole induction motor is operated from 50-cycle mains. What is the speed of its rotating field in r.p.s.? What is its synchronous speed in r.p.m.?

199. A 6-pole, 60-cycle induction motor operates from a circuit whose frequency is 58 cycles per second. What is its synchronous speed in r.p.m.?

200. Find the synchronous speed of a 4-pole, 25-cycle induction motor which operates on a circuit whose frequency is 24 cycles per second.

201. The synchronous speed of an induction motor is 600 r.p.m. when operating on a 60-cycle circuit. How many poles has it?

202. How many poles has an induction motor whose synchronous speed is 300 r.p.m. when operating on 25-cycle mains?

203. The rotor speed of a 6-pole, 10-hp. induction motor is 1,140 r.p.m. when it operates on a 60-cycle circuit. What is its slip?

204. Determine the slip of the motor (problem 202) if its rotor speed is 291 r.p.m.

205. The rotor speed of a 4-pole, 5-hp. induction motor is 1,710 r.p.m. when it operates on 60-cycle mains. What is its slip?

206. Determine the slip of an 8-pole, 50-cycle induction motor when its rotor speed is 720 r.p.m.

207. What is the rotor frequency in problem 203? What is the rotor frequency in problem 206?

208. A 10-hp., 440-volt, 60-cycle induction motor develops a starting torque of 20 lb.-ft. when connected directly across 440-volt, 3-phase mains. What starting torque does it develop when a starting compensator is used, the motor being connected to the 50 per cent. taps, giving half voltage?

209. Determine the starting torque (problem 208), when the motor is connected to the 40 per cent. starting taps.

210. The motor (problem 208) develops 40 lb.-ft. torque when operating at 440 volts and at 5 per cent slip. What torque does it develop when operating at this same slip, if the line voltage is reduced to 400 volts?

211. This same motor (problem 208), has a break-down torque of 100 lb.-ft. when across a 440-volt circuit. What does the break-down torque become if the voltage drops to 400 volts?

212. The following data were taken during the brake test of a 5-hp., 220-volt, 6-pole, 3-phase, 60-cycle, squirrel-cage induction motor. The power input was measured by means of the 2-wattmeter method. A prony brake having a 2-ft. arm was used to measure the output (see Part I, page 284, Fig. 252). The tare was +2.7 lb.; line voltage, 220 volts; frequency, 60 cycles; current, 14.9 amp.; watts, $W_1 = +3{,}220$, $W_2 = +1{,}230$; revolutions slip, 65 r.p.m.; balance reading, 14.3 lb. Find the slip, speed, torque output, efficiency, and power-factor at this load.

213. The following data were taken with the motor (problem 212), when the load was reduced to approximately 30 per cent. of the rated output: Line voltage, 220 volts; frequency, 60 cycles; current, 12.7 amp.; watts, $W_1 = +2{,}130$, $W_2 = -160$; revolutions slip, 21 r.p.m.; balance reading = 6.3 lb. Find the slip, speed, etc., as in problem 212.

***214.** The electrical efficiency of the rotor (ratio of mechanical power developed in rotor to power transferred across the gap) of a 6-pole, 60-cycle, wound-rotor induction motor with all external resistance cut out is 0.93 when the slip is 0.07. (*a*) At what speed is it operating? (*b*) What is the electrical efficiency of the rotor when the speed is reduced to 800 r.p.m. by introducing resistance in the rotor circuit? (*c*) What is the corresponding efficiency at 500 r.p.m.?

215. The 7,500-hp. wound-rotor induction motor, such as are used to drive the battleship *Tennessee*, has two windings, one for 36 poles and the other for 24 poles. If the frequency is 30 cycles, what two synchronous speeds are obtainable?

216. A 60-cycle, 15-hp., elevator motor has two windings, one of which has 6 poles and the other 18 poles. What two synchronous speeds has this motor?

QUESTIONS ON CHAPTER IX

1. Why do both the direct-current shunt motor and the direct-current series motor develop torque in one direction when supplied with alternating current? What factor prevents the shunt motor from becoming a commercial alternating-current motor?

2. Why does the series motor have possibilities as an alternating-current motor?

3. Before the motor becomes operative with alternating current, what changes must be made in the following and why: field structure; series-field turns; frequency; air-gap?

4. Why is a compensating winding necessary? Define "conductive compensation" and "inductive compensation."

5. Sketch the characteristic curves of this type of motor including speed, torque, horsepower, efficiency, and power-factor.

6. Give the industrial applications of this type of motor. What are its limitations?

7. What is meant by a "universal motor?" What are the advantages of this type of motor? Why do such motors ordinarily operate at high speed? In how large capacities are they manufactured? Give some typical industrial applications of this type of motor.

8. What type of armature is used with the repulsion motor? Show that the armature of the repulsion motor behaves like a transformer secondary.

9. Between what two points on the armature does zero potential difference exist and why? Between what two points does maximum potential difference exist?

10. Why does the motor develop no torque when the brushes are in the geometrical neutral; in the plane of the pole axis? For what positions of the brush axis does the motor develop torque? Give reasons.

11. How are the stator and rotor ordinarily constructed? Why? Describe briefly the operating characteristics of the repulsion motor. State its industrial applications.

12. Discuss the reactions which occur in the single-phase induction motor when its stator is supplied with single-phase current and the short-circuited rotor is stationary. Show that the motor develops no torque under these conditions.

13. What reaction occurs when the armature is rotating, which causes it to develop a rotating field? In what directions does this field rotate?

14. Compare the characteristics of the single-phase induction motor with those of the polyphase induction motor. Compare its output and size with those of the polyphase motor.

15. Discuss the reactions which follow the opening of one phase of the 3-phase induction motor, when it is rotating. How is the rating of the motor affected by the opening of a phase?

16. Show that a polyphase motor may inadvertently operate single phase. How is such operation detected?

17. Describe two split-phase methods of starting single-phase induction motors. How is the direction of rotation reversed in one of these methods?

18. Describe in detail the "shaded-pole" method of starting a single-phase induction motor. In what types of motor is this method used? Name one other industrial application of the shaded pole.

19. What is meant by "repulsion-motor starting" of single-phase motors?

QUESTIONS ON CHAPTER X

1. Compare the construction of the synchronous motor with that of the alternator.

2. Describe the principle on which the synchronous motor operates. Show that the motor must operate at synchronous speed or not operate at all.

3. Show that the rotor of the synchronous motor may be considered as being a salient-pole magnet dragged around by a synchronously rotating magnetic field. Give a mechanical analogue.

4. When load is applied to the rotor of the synchronous motor, why can it not take additional energy from the line by a decrease in the magnitude of

its induced e.m.f.? What actually does occur when the mechanical load is increased? Give a mechanical analogue. How may the effect of applying load be observed visually?

5. Why can the synchronous motor not act like the shunt motor when its field excitation is increased? What two reactions result when the field of the synchronous motor is overexcited? Show that a leading current in a synchronous motor weakens the field and also gives a counter e.m.f. which may exceed the terminal voltage in magnitude.

6. Why can the synchronous motor not act like the shunt motor when its field excitation is decreased? What two reactious result when the field of the synchronous motor is underexcited? Show that a lagging current in a synchronous motor strengthens the field and at the same time gives a counter e.m.f. which is less than the terminal voltage. Where does the synchronous motor obtain a portion of its excitation when it is operating underexcited?

7. Sketch the connections, with all instruments, which would be used in testing a synchronous motor.

8. Show by a graph the variation of armature current and power-factor as the field current of the synchronous motor is increased from small to large values.

9. What is meant by a "synchronous condenser?" How may the synchronous condenser be used to improve power-factor? What is meant by "energy current;" "quadrature current?" Show that if a synchronous condenser running light takes current equal to the quadrature current of the inductive load, the power-factor of the system is brought practically to unity. Show how a synchronous motor may also be used to improve power-factor.

10. Under what conditions can a synchronous motor or condenser be made to regulate the voltage at the end of a transmission line?

11. What is the purpose of amortisseur windings or dampers? Analyze their operation.

12. Describe the procedure which is followed in starting a synchronous motor when it is connected to a direct-current generator which may be operated as a motor. How is the synchronous motor, operating as a generator when starting, made to operate as a motor after being connected to the bus-bars?

13. Why will the synchronous motor start from rest and come up to speed when polyphase currents are supplied to its stator? Why can the motor pull into synchronism, even without its direct-current excitation? Show that the application of excitation may cause a line disturbance. How may this disturbance be minimized?

14. To what danger is the field winding subjected during starting? How is this danger minimized?

15. State some industrial applications of the synchronous motor.

16. Describe the principle of operation of synchronous motors of very small size. For what purposes are they used?

17. Show that the principle of the synchronous converter is related to the generation of e.m.f. in the direct-current machine. How is the converter ordinarily operated? Give six other methods of operation.

18. How are the armature connections of the slip-rings ordinarily made? What relation exists among the number of phases, number of poles and number of slip-ring taps?

19. In a general way, how does the rating of a converter vary as the number of its phases is increased? How is the rating affected by power-factor?

20. Unless the shape of the flux-distribution curve changes, why is the ratio of the alternating- to the direct-current *induced* e.m.fs. in a converter armature constant under all conditions? Show that the ratio of direct-current e.m.f. to single-phase e.m.f. (r.m.s.) must be $\sqrt{2}$. Give a graphical method for determining the ratio of direct- to alternating-current e.m.fs. for different numbers of phases.

21. How does change of excitation affect the ratio of alternating- to direct-current induced e.m.f.? Why does change of excitation change the ratio of slip-ring to commutator voltage? Why does change of excitation ordinarily change the slip-ring voltage also? State the objections to voltage control by change of excitation.

22. Discuss the use of transformer taps for controlling the direct-current voltage.

23. Describe the series booster. How is it used to control the direct-current voltage?

24. What is meant by an inverted synchronous converter? What precaution is necessary when it is in operation and why?

25. Show that the converter armature starts as an induction motor when polyphase alternating currents are supplied to its armature. Describe the precautions which must be taken as regards the shunt and series fields and the brushes, when starting in this manner. How may the converter be caused to come up with the correct direct-current polarity?

26. What is the procedure followed when the converter is started from the direct-current side? What difficulty appears when attempt is made to synchronize?

27. What are dampers? Analyze their operation.

28. Make a wiring diagram showing the connections which would be used for making a laboratory test of a synchronous converter.

29. Describe the rectifying commutator. What limits its output?

30. Upon what principle does the mercury-arc rectifier operate? Why are two anodes necessary? Why is inductance necessary? Describe the operation of the starting anode.

31. Upon what principle does the tungar operate? What is meant by "ionized gas?" Why does the tungar conduct current only in one direction? Make a diagram of connections. What are the approximate efficiencies and outputs of commercial tungars?

32. Upon what principle do electrolytic rectifiers operate? Make a diagram of connections.

PROBLEMS ON CHAPTER X

217. Determine the no-load and the rated-load speed of a 500-hp., 25-cycle, 3-phase 16-pole synchronous motor which is direct-connected to an ammonia compressor operating in an artificial-ice-making plant.

218. It is desired to drive an ammonia compressor at 225 r.p.m. by means of a 300-hp. synchronous motor operating from 2,300-volt, 3-phase, 60-cycle supply. How many poles must the motor have?

219. How many poles has a 450-hp., 100-r.p.m., 50-cycle, 3-phase, synchronous motor?

220. At what speed does a 250-hp., 22-pole, 25-cycle, 3-phase, synchronous motor operate?

221. A 500-hp., 2,300-volt, 12-pole, 60-cycle, 3-phase synchronous motor takes 96 amp. at 2,300 volts, three-phase and its power-factor is 0.85 leading current when it is driving a direct-current generator which requires 400 hp. Disregard field loss. (*a*) What power is the motor armature taking? (*b*) What is its efficiency? (*c*) What is the torque at the coupling? (*d*) If the direct-current generator delivers 1,210 amp. at 230 volts, what is the generator efficiency?

222. When the motor (problem 221) is operating near full load, it takes 110 amp. at 2,300 volts, three phase, and the power-factor is 0.9 leading current. If its efficiency is 0.935 (excluding field loss) at this load, find: (*a*) its input in kilowatts; (*b*) its output in horsepower; (*c*) its efficiency; (*d*) the torque at the coupling; (*e*) the generator current if the voltage is 230 and the generator efficiency is 0.93; (*f*) the overall efficiency of the set.

223. It is desired to drive a 1,000-kw., 600-volt, 750-r.p.m. railway generator by a synchronous motor which must take its power from a 6,600-volt, 3-phase, 25-cycle line. The generator efficiency at rated load is 0.94. Determine the horsepower rating and the number of poles that the synchronous motor must have.

224. The motor (problem 223), has an efficiency of 0.938 at unity power-factor, disregarding field loss. What is its ampere rating?

225. If the motor (problem 223), when delivering its rated load must operate at 0.8 power-factor, what must be its kilovolt-ampere rating? The efficiency at this load and power-factor is 0.930, disregarding field loss.

***226.** A 10-hp., 220-volt, 60-cycle, single-phase induction motor operates at 0.707 power-factor and delivers 7.4 hp. when it takes 40 amp. at 220 volts. Determine: (*a*) the energy current of the motor; (*b*) the quadrature current. (*c*) How many amperes leading quadrature current will bring the system power-factor to unity? (*d*) How many quadrature volt-amperes will bring the system power-factor to unity?

***227.** Determine the motor input in watts (problem 226). Determine its efficiency.

***228.** A certain mill, whose load consists almost entirely of induction motors, takes 100 kw. at 600 volts, 60 cycles and 0.60 power-factor. Determine: (*a*) the total current; (*b*) the energy current; (*c*) the quadrature current; (*d*) the kilovolt-amperes which a synchronous condenser, located

in the mill, must take in order to bring the power-factor of the mill to unity. (For simplicity, assume that the load is single phase.)

***229.** A certain load takes 300 kw. at 2,300 volts, 25 cycles, single phase, and 0.8 power-factor. Determine: (*a*) the total current; (*b*) the energy current; (*c*) the quadrature current; (*d*) the synchronous condenser kilovolt-ampere necessary to raise the system power-factor to unity.

Note: In the foregoing problems single-phase loads have been assumed for simplicity. The same principles apply to 3-phase loads. A 3-phase load may be assumed to consist of three single-phase loads to neutral.

230. A 6-phase, 25-cycle, 230-volt synchronous converter has a rating of 600 kw. What is its rating if it becomes necessary to operate it three phase (see table, page 293, Par. 166)?

231. If the converter (problem 230) were directly connected to a prime mover and operated as a 230-volt, direct-current generator, what would its rating be?

232. A 4-phase, 60-cycle, 600-volt synchronous converter is rated at 400 kw. What is the maximum load which it can safely carry if one of the two phases, supplying energy to two of the four slip-rings, becomes disabled and the converter operates single phase?

233. What is the rating of the converter (problem 232) if it is operated as a direct-current generator?

234. Determine the rating of the converter (problem 230) when it operates six phase and at 0.9 power-factor. Repeat for 3-phase operation and 0.9 power-factor.

235. Find the rating of the 400-kw. converter (problem 232), when it operates four phase and at 0.9 power-factor.

236. In problem 230 the direct-current voltage of the 600-kw. synchronous converter is 230 volts. When it operates six phase, determine: (*a*) its direct-current rating; (*b*) the 6-phase voltage to neutral or across adjacent line terminals; (*c*) the rated alternating current per slip-ring. Assume unity power-factor and neglect losses.

237. A 1,000-kw., 6-phase, 60-cycle, synchronous converter is used to supply a railway with power at 600 volts. Determine: (*a*) the rated direct current of the converter; (*b*) the 6-phase voltage at the slip-rings; (*c*) the current per slip-ring. Assume unity power-factor and neglect losses.

***238.** A synchronous converter is to be used to supply a laboratory with 10-kw., direct-current power at 115 volts, from 230-volt, 3-phase, 60-cycle laboratory mains. The transformers are connected delta-delta, the converter being operated three phase. Determine: (*a*) the direct-current rating of this converter; (*b*) the rated slip-ring current of the converter; (*c*) the slip-ring voltage of the converter; (*d*) the kilovolt-ampere rating of each transformer; (*e*) the voltage ratings of each transformer; (*f*) the primary current of each transformer; (*g*) the 3-phase line current. Make a complete wiring diagram, showing all voltages and currents. Assume unity power-factor and neglect losses.

QUESTIONS ON CHAPTER XI

1. Name some of the factors which determine the location of power stations. What makes it possible to transmit and utilize energy at places which are situated many miles from the generating plant?

2. In a typical large power system, at what voltages is the energy ordinarily generated? If it is to be transmitted a long distance, what voltage might be used for transmission? Give reasons.

3. Why is the transmission voltage reduced before entering the more thickly populated districts? What is the function of the distributing sub-station?

4. At what voltages, frequencies, etc. are the following services usually rendered: (*a*) trolley service; (*b*) alternating-current lighting service; (*c*) direct-current lighting service; (*d*) street lighting; (*e*) alternating-current power service?

5. What types of apparatus and what connections would be used to give each class of service in 4?

6. How does the weight of copper vary with the transmission voltage, other factors being equal? What factors limit the transmission voltage?

7. Show that a transmission line has self-inductance; capacitance.

8. What three types of structures are most frequently used for transmission-line supports? State the field of use of each.

9. Why is porcelain the most satisfactory material for line insulators? Where is the pin-type insulator used and what are its limitations? State the advantage of the suspension-type insulator.

10. What are the functions of lightning arresters? Describe a low-voltage, low-power type. Describe the oxide-film arrester. How should lightning arresters be connected to the system which they are designed to protect?

11. Where would direct current be used for general lighting and power service? Give reasons.

12. Distinguish between feeders and mains. How is the voltage at the end of a feeder measured from the station? How is it maintained constant at the desired value?

13. How are the trolley and the rail, in street railway systems, connected with the generator? What is meant by the "ladder system" of feeding? Make a diagram showing the sectionalized trolley method of feeding. What are the advantages and the disadvantages of this last method of feeding?

14. State the underlying cause of electrolysis of water mains, gas mains, etc. What measures are taken to minimize electrolysis?

15. What factors determine the best value of voltage for lighting and household service? State the advantages and the disadvantages of the series-parallel system.

16. Describe the Edison 3-wire system, giving its advantages. Why should every precaution be taken to prevent the neutral becoming opened?

17. Describe the 2-generator and the balancer-set methods of maintaining the neutral in a 3-wire system. What connections of the machines of the balancer set give better voltage balancing?

18. Why are storage batteries seldom used in large power systems to give a more uniform load curve? For what purposes are storage batteries used? How are their charge and discharge controlled?

19. State the advantages of series distribution. Where are such systems used? How are the loads connected in and removed from service? What is a "film cut-out?" Make a diagram showing the parallel loop system of series distribution.

PROBLEMS ON CHAPTER XI

239. Figure 239 (*A*) shows a 5-kw. direct-current load taking 50 amp. over a 250-ft. (76.2 m.) of No. 6 A.W.G. feeder, having a resistance of 0.10 ohm per conductor. The load voltage is 100 volts. (*a*) What is the bus-bar voltage? (*b*) How much power is lost in the feeder? (*c*) What is the efficiency of transmission? (*d*) What weight of copper is involved in the feeder?

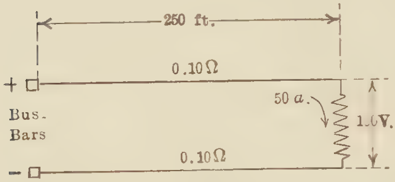


FIG. 239 (*A*).

240. A 5-kw. direct-current load taking power at 200 volts is supplied over the system shown in Fig. 239 (*A*). Repeat (*a*), (*b*), and (*c*) of problem 239 and compare.

241. (*a*) What size wire should be used in problem 240, in order that the power loss in the two systems should be the same? (*b*) How does the weight of this wire compare with that of the wire used for the feeder (problem 239)? (*c*) Has this wire, if weatherproof, sufficient carrying capacity (see page 418)?

242. A 2,300-volt, single-phase, 60-cycle distribution line has a resistance of 3.23 ohms per conductor. (*a*) Determine the current when a 100-kw. load whose power-factor is unity is connected at the far end, the voltage at the load being 2,300 volts. (*b*) Determine the total copper loss. (*c*) Determine the power input to the line. (*d*) Determine the efficiency of transmission.

243. The line (problem 242) is 12 miles (19.3 km.) long and the conductors are stranded copper. Determine: (*a*) the resistance per mile; (*b*) the size wire (see Appendix G, page 416); (*c*) the wire cross-section; (*d*) the weight of copper.

244. It is desired to transmit 100 kw. unity power-factor, single phase, 60 cycles, 12 miles (19.3 km.) at 4,600 volts. Each conductor has a resistance of 12.92 ohms. Determine: (*a*) the current; (*b*) the total copper loss; (*c*) the input to the line; (*d*) the efficiency of transmission.

245. Find the circular mils in problem 244 from data in (*b*), problem 243. How does the weight of copper in the two cases compare?

246. Assume the power-factor to be 0.8, lagging current (problem 244). Determine: (*a*) the current; (*b*) the copper loss; (*c*) the efficiency of transmission. What is the effect of power-factor on efficiency?

247. It is desired to transmit 200 kw., three phase, 60 cycles, a distance of 15 miles (24.2 km.) at 6,600 volts and at unity power-factor. The resistance per conductor is 10 ohms. Determine: (*a*) the current per conductor; (*b*) the power loss per conductor; (*c*) the total transmission loss; (*d*) the efficiency of transmission.

248. Repeat problem 247 with the power-factor lowered to 0.75, the other factors remaining unchanged.

249. A single-phase transmission line (Fig. 249 (*A*)) has a resistance of 2.8 ohms and a reactance of 4.6 ohms per conductor at 60 cycles. A load of

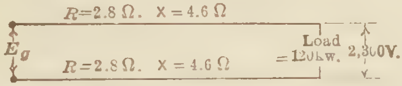


FIG. 249 (*A*).

120 kw. at 2,300 volts unity power-factor and 60 cycles is applied to its far end. (*a*) What is the current? (*b*) Determine the resistance- and the reactance-drop. (*c*) Find the sending-end voltage, *Eg*.

***250.** Repeat problem 249 for an equal kilowatt load whose power-factor is 0.8, lagging current.

***251.** Repeat problem 249 for an equal kilowatt load whose power-factor is 0.8, leading current.

***252.** A single-phase transmission line 34 miles long consists of two 000 stranded copper conductors spaced 5 ft. (1.525 m.) on centers. Determine: (*a*) the total resistance of the line; (*b*) the total reactance of the line. (*c*) When a load of 3,000 kw. at 27,000 volts, 60 cycles and unity power-factor is connected to the far end, determine the voltage at the sending end (consult Appendix G and H).

***253.** Repeat problem 252 for an equal kilowatt load whose power-factor is 0.707, lagging current.

***254.** Repeat problem 252 for an equal kilowatt load whose power-factor is 0.707, leading current.

255. Six lamps, all having the following ratings at 115 volts, are connected across 115-volt, direct-current mains (Fig. 255 (*A*)): two 60-watt lamps;

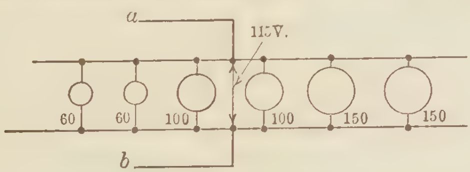


FIG. 255 (*A*).

two 100-watt lamps, and two 150-watt lamps. Find the current supplied to this main by the feeder *ab*.

256. At 8½ cts. per kilowatt-hour, what is the monthly lighting bill (problem 255), if these lamps are operated on the average of 5½ hr. a day on 30 days each month?

257. The voltage at the end of a d.-c. feeder supplying a 150-kw. load is 600 volts and this load is fed from bus-bars 6,000 ft. (1.83 km.) away. The feeder consists of two 350,000-C.M. cables, having a resistance of 0.03 ohm per 1,000 ft. (Fig. 257 (*A*)). (*a*) What current does the load take? (*b*) What is the total feeder resistance? (*c*) What is the bus-bar voltage? (*d*) What power is lost in the feeder? (*e*) What is the efficiency of transmission?

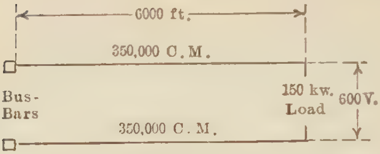


FIG. 257 (*A*).

258. With the same power if the load voltage (problem 257) were 300 volts, find: (*a*) the current; (*b*) the feeder resistance which would produce the loss found in (*d*), problem 257; (*c*) the resistance per 1,000 ft. of such a feeder; (*d*) the size in circular mils of such a feeder; (*e*) compare the weight of copper in the two cases.

259. A 1,000-ft. (305-m.) length of 10,000-C.M. copper conductor weighs 31.4 lb. (see pt. 1, p. 45). It is desired to transmit 100 kw. a distance of 2,000 ft. (610 m.). With 110 volts at the load and with a line drop equal to 10 per cent of the load voltage, find: (*a*) the size of the necessary copper conductor; (*b*) the weight of the copper; (*c*) with copper costing 20 cts. per pound ($0.441 per kilogram), find the cost of the copper in the feeder.

260. A trolley system extends 4.5 miles from the station (Fig. 260 (*A*)), a 0000 trolley having a resistance of 0.27 ohm per mile being used. The rail and ground return have a combined resistance of 0.10 ohm per mile. Find the voltage at a car 3.5 miles from the station and taking 65 amp. What is the voltage at the end of the line under these conditions?

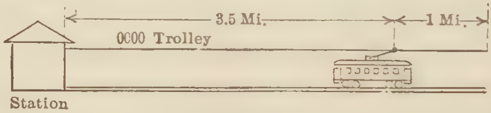


FIG. 260 (*A*).

261. Find the voltage at the car (problem 260) when the car is at the far end of the line and is taking 90 amp. at starting.

262. An Edison 3-wire system has two loads of 2.0 and 6.0 amp. on its positive side, and two loads of 3.0 and 8.0 amp. on its negative side. What current flows in the neutral and what is its direction? (Currents flowing outwards are considered as positive.)

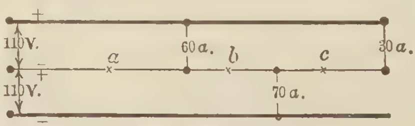


FIG 263 (*A*).

263. An Edison 3-wire system is shown diagrammatically (Fig. 263 (*A*)) with loads of 60 and 30 amp. on the positive side and a single load of 70

amp. on the negative side. Determine the current and its direction at points *a*, *b*, and *c* in the neutral.

264. Figure 264 (*A*) shows an Edison 3-wire system and its accompanying loads. Find the currents at points *a*, *b*, *c*, *d*, *e*, *f*, and *g*, and show their directions.

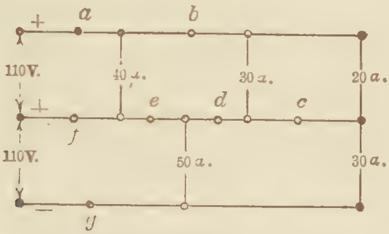


FIG. 264 (*A*).

265. A 6.6-amp. series arc system has a loop length of 8.6 miles (13.85 km.) and consists of No. 6 A.W.G. underground cable, having a resistance of 0.442 ohm per 1,000 ft. (305 m.) at 50°C. the operating temperature in the ducts. On this circuit are 45 500-watt magnetite lamps. (*a*) What is the voltage-drop in the conductor? (*b*) What is the entire circuit voltage? (*c*) What is the transmission efficiency of the system?

266. Repeat problem 265 for a similar system having a loop length of 10.8 miles and 42 magnetite lamps.

QUESTIONS ON CHAPTER XII

1. What is the unit of luminous intensity? Why is the candle not used as a working standard of luminous intensity?

2. Define mean spherical candlepower; spherical reduction factor.

3. Show that the intensity of illumination on a surface varies inversely as the square of its distance from the light source, if the light source is considered as a point source. In what unit is intensity of illumination expressed?

4. Under what conditions do bodies ordinarily emit light? Give examples of luminescent bodies.

5. Why could the carbon-filament lamp not become a highly efficient illuminant? Give the efficiency of lamps having untreated carbon filaments and of "G.E.M." lamps in which the carbon filament is given a metallized appearance and characteristic.

6. What three properties of tungsten make it very desirable for lamp filaments? Describe the Mazda B lamp. Give the approximate efficiency of this type of lamp. What factor limits its efficiency?

7. What is the underlying principle of the Mazda C lamp? What factor tends to reduce its efficiency materially? What factor increases its luminous efficiency? Compare the disposition of the filament with that of the Mazda B lamp, giving the reason for any differences. Why is the shape of the bulb

different from that of the Mazda B lamp? Give the approximate efficiencies of the type C lamp.

8. Describe the carbon arc lamp. From where does the light come? Why is ballast necessary when the lamp is operated on multiple circuits?

9. What is the fundamental difference between the metallic-electrode arc lamp and the carbon arc lamp? From where does the light come? Why must the copper always be positive?

10. State two reasons why the metallic-electrode arc lamp cannot operate with alternating current. How does this lamp "feed?"

11. Describe the Bunsen photometer. Why does the grease spot on the screen appear darker than the rest of the screen when viewed on the side having the greater intensity of illumination? Why does it appear lighter than the rest of the screen when viewed on the side having the lesser intensity of illumination?

12. Sketch a wiring diagram, showing the connections which would be used in making a photometric measurement.

13. Describe the Lummer-Brodhun photometer screen, showing how the light rays from each source are brought to a common eyepiece for comparison. How is the condition of balance determined?

PROBLEMS ON CHAPTER XII

267. A 200-watt, Mazda C lamp has a mean horizontal luminous intensity of 267 candlepower and a mean spherical luminous intensity of 219 candlepower. Determine: (*a*) the watts per mean horizontal candlepower; (*b*) the watts per mean spherical candlepower; (*c*) the spherical reduction factor.

268. A 6.6-amp. series Mazda C street lamp operates at 24.6 volts and has a mean horizontal luminous intensity of 250 candlepower and a mean spherical intensity of 210 candlepower. Determine (*a*), (*b*), and (*c*), problem 267.

269. A Mazda B lamp takes 0.91 amp. at 110 volts and has a mean horizontal luminous intensity of 105 candlepower and a mean spherical luminous intensity of 82 candlepower. Determine (*a*), (*b*) and (*c*), problem 267.

270. A 110-volt, 60-watt, Mazda B lamp, 7 ft. above the floor, has in combination with its reflector a luminous intensity of 40 candlepower in a vertically downward direction. What is the illumination in foot-candles on a table 30 in. high?

271. The top of a drafting table is 4 ft. above the floor and the lighting fixture is such that the lamp filament is 9 ft. above the floor. What must be the candlepower directed vertically downward in order that the intensity of illumination on the table may be 8 foot-candles?

272. The wall of a room has an intensity of illumination of 3 foot-candles at a point 7 ft. above the floor, the height of the lighting unit. The lighting unit is 6 ft. away. What is its horizontal candlepower in the direction of this wall?

273. The distance between the test lamp and the standard lamp in a Bunsen photometer measurement is 4 m. The horizontal candlepower of the standard lamp in the direction of the photometer screen is 21.4 candlepower. A photometric balance is obtained when the screen is 1.6 m. from the standard lamp. (*a*) What is the horizontal candlepower of the test lamp for this position? (*b*) If it is taking 60 watts, what is its efficiency in watts per

mean horizontal candlepower, assuming that the foregoing measurement is practically equal to the mean horizontal candlepower?

274. A measurement of the candlepower of a 100-watt, Mazda B lamp using the photometer of problem 273, gives the balance at 1.35 m. from the standard lamp, whose candlepower is 21.4 in the direction of the photometer screen. The input to the lamp is adjusted to 100 watts. (*a*) What is the candlepower of the test lamp? (*b*) What is its efficiency in watts per mean horizontal candlepower, assuming that the above measurement is practically equal to the average horizontal candlepower?

PROBLEMS IN TRIGONOMETRY

1. The hypotenuse of a right triangle (Fig. 1 (*T*)) is 26 in. and one leg is 12 in. (*a*) What is the sine of angle *A*? (*b*) From the tables, find the value of angle *A* in degrees. (*c*) Determine the cosine of *B* and determine its value in degrees. (*d*) From the sine of *B*, determine side *b* of the triangle.

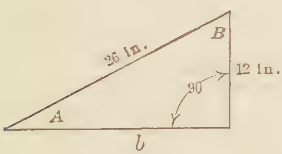


Fig. 1 (*T*).

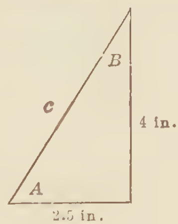


Fig. 2 (*T*).

2. The two legs of a right triangle (Fig. 2 (*T*)) are 4 and 2.5 in. (*a*) Determine angle *A*. (*b*) Determine angle *B*. (*c*) Using sine or cosine functions, determine the hypotenuse *c*.

3. Show that $\sin 30° = 0.5$, $\sin 60° = \sqrt{3}/2 = 0.866$; $\tan 60° = \sqrt{3} = 1.732$; $\sin 45° = 1/\sqrt{2} = 0.707$.

4. Given the isosceles triangle *abc* (Fig. 4 (*T*)), with the side *ab* equal to 24 in. and the angles at *a* and *b* each equal to 68°, find: (*a*) the perpendicular dropped from *c* to *ab*; (*b*) the sides *ac* and *bc*; (*c*) the angle at *c*.

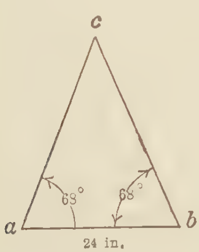


Fig. 4 (*T*).

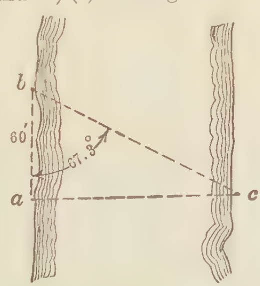


Fig. 5 (*T*).

5. It is desired to measure the width of a river (Fig. 5 (*T*)). From (*a*) on one bank a sight is taken on a small sapling *c* on the opposite bank, the line

ac being practically perpendicular to the river bank. The transit telescope is turned 90° and a point *b*, 60 ft. from *a* is located. The transit is then set up at *b* and the sapling at *c* is again sighted. The angle *abc* is found to be 67.3°. Find the width *ac* of the river.

6. It is desired to determine the height of a precipice *bc* (Fig. 6 (*T*)), the face *bc* being practically vertical. From point *a*, 400 ft. horizontally from *b* at the base of the precipice, a sight is taken on *c*, and the angle *bac* is found to be 54°. Find the height *bc* of the precipice.

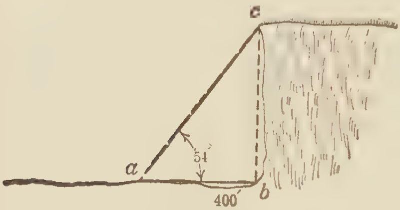


Fig. 6 (*T*).

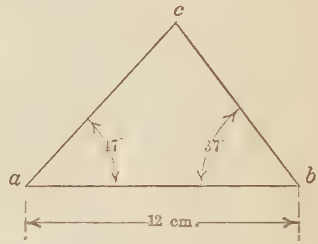


Fig. 7 (*T*).

7. In triangle *abc* (Fig. 7 (*T*)) the side *ab* is 12 cm., the angle *cab* is 47° and angle *abc* is 37°. Find: (*a*) the angle *acb*; (*b*) the side *bc*; (*c*) the side *ac*.

8. The side *ab* of the triangle *abc* (Fig. 8 (*T*)) is 36 cm., the side *bc* is 75 cm., and angle *bca* is 20°. Find: (*a*) the angle *bac*; (*b*) the angle *abc*; (*c*) the side *ac*.

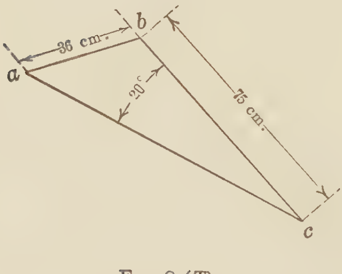


Fig. 8 (*T*).

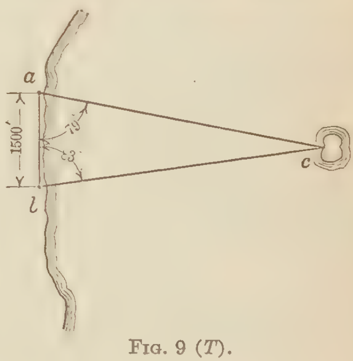


Fig. 9 (*T*).

9. It is desired to determine the distance from the shore line *ab* (Fig. 9 (*T*)) to an off-shore island *c*. A base line *ab* 1,500 ft. long is established along the shore. When *c* is sighted from *a*, the angle *bac* is found to be 79°. When *c* is sighted from *b*, the angle *abc* is found to be 83°. Find the distance of the island *c* from both *a* and *b*.

10. In order to measure the width of a river (Fig. 10 (*T*)), a base line *ab*, 400 ft. long is established on one bank. When a sapling at *c* on the further bank is sighted from *a* the angle *bac* is found to be 68°, and when sighted

from b the angle abc is found to be 59°. Find the width of the river, that is the perpendicular distance from c to ab.

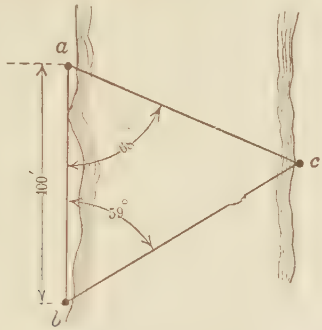


Fig. 10 (*T*).

11. In order to measure the height of a mountain peak c (Fig. 11 (*T*)) above the surrounding plain, a horizontal base line ab, 2,000 ft. long is established, the base line and the peak lying in the same vertical plane.

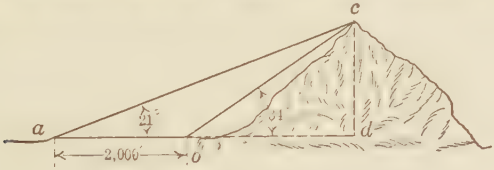


Fig. 11 (*T*).

When the peak is sighted from a, the angle dac is found to be 21°, and when sighted from b, the angle dbc is found to be 34°. Find the height cd of the peak above the surrounding plain.

12. The sides ac and bc of a scalene triangle are equal respectively to 8 and 18.5 in. (Fig. 12 (*T*)) and the included angle is equal to 43°. Find: (*a*) the side ab; (*b*) the angle cab; and (*c*) the angle abc.

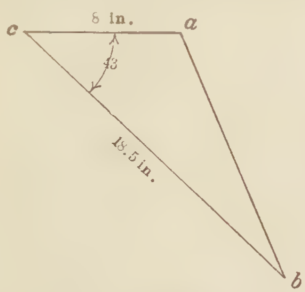


Fig. 12 (*T*).

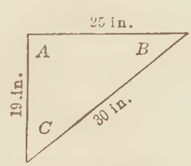


Fig. 13 (*T*).

13. The three sides of a triangle are equal respectively to 19, 25, and 30 in. (Fig. 13 (*T*)). Find the three angles A, B, and C.

14. It is desired to determine the distance *ac* across an inlet (Fig. 14 (*T*)). The lengths of two intersecting lines *ab* and *bc* are known to be 2,200 and

Fig. 14 (*T*).

1900 ft., respectively, and the angle *abc* between *ab* and *bc* is found to be 49.5°. Find the distance *ac*.

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